NAÏVE BAYES

AGENDA

- Intro to Probability
  - Sample Space and Events
  - Joint Probability and Independence
  - Conditional Probability
- Bayes theorem
- Naïve Bayes

INTRO TO PROBABILITY

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- A: {HH},{TT},{HT},{{TH}

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- Q: Given a set of fair dice
  - What is the sample space?
  - What is the joint probability of each event?
- A: All combinations of {1,2,3,4,5,6}x{1,2,3,4,5,6} with a joint probability of 1/36.

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  - Probability of an event given another event has occurred. $P(A|B)$
  - $P(A \mid B)$: Probability of A given B has occurred
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- Q: What if B doesn’t influence A?
- A: Independence
  - $P(A,B) = P(A)P(B)$
  - $P(B) = P(B|A)$

Summary:

- Probability: P(x) = Probability of x
  - $\sum_i P(x_i) = 1$
  - 0 <= P(x)<= 1
- Joint Probability: P(AB)= Probability of A and B
- Conditional Probability: P(A|B)= Probability of A given B
- Independence:
  - P(B) = P(B|A)
  - P(AB) = P(A)P(B)

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  - Test returns correct positive result 98% of the time
  - Correct negative result in only 97% of cases where don’t have disease
  - 0.008 of entire population have cancer
  - Patient gets test and comes back positive. .What is the probability the patient has cancer?

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Use Bayes rule: $P(cancer|+) = P(+|cancer)*P(cancer)/P(+)$

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P(+) = P(+,C) + P(+,-C) = P(+|C)P(C) + P(+|-C)P(-C)

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  - Assume independence:
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- Notes:
  - Don't add to one since we didn't normalize
  - Doesn't work well at estimating probabilities since independence probably doesn't hold but still works good as a classifier

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- A:
  - Take the logarithm. We sum the logs of probabilities
  - Add pseudo counts. Assume we've seen every case once initially.