Data Science

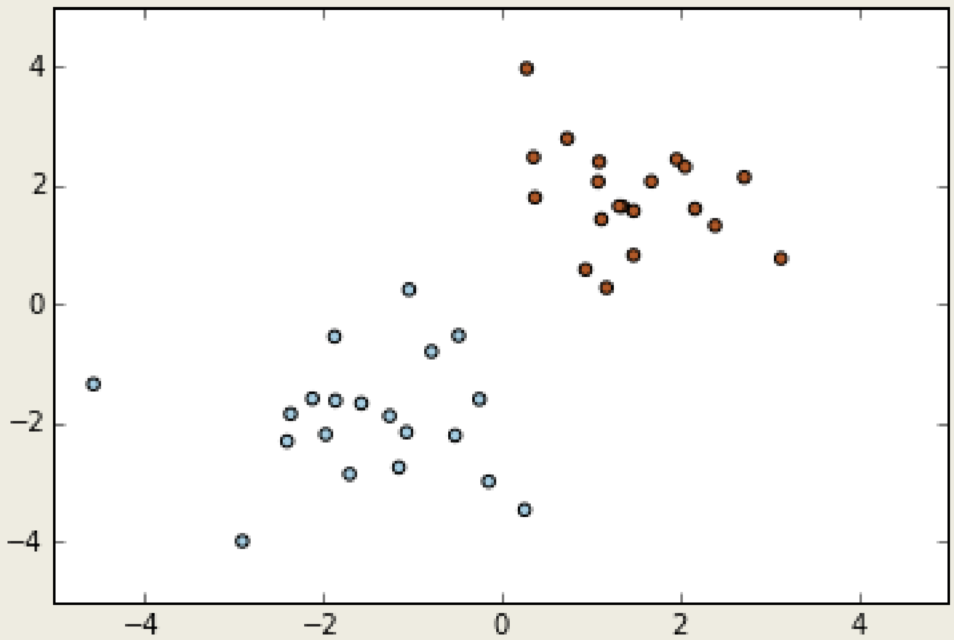
Support Vector Machines

Agenda

- Support Vector Machine
- Non-linear classification, Kernels
- Noisy examples
- SVM Practice in Python

Support Vector Machines

Q: How would you build a classifier for this problem?



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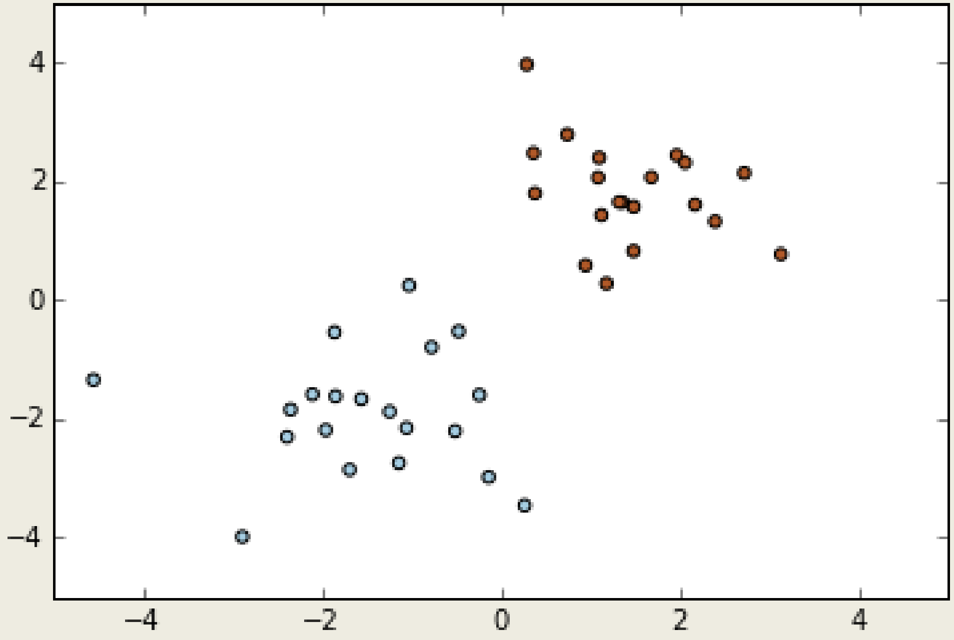

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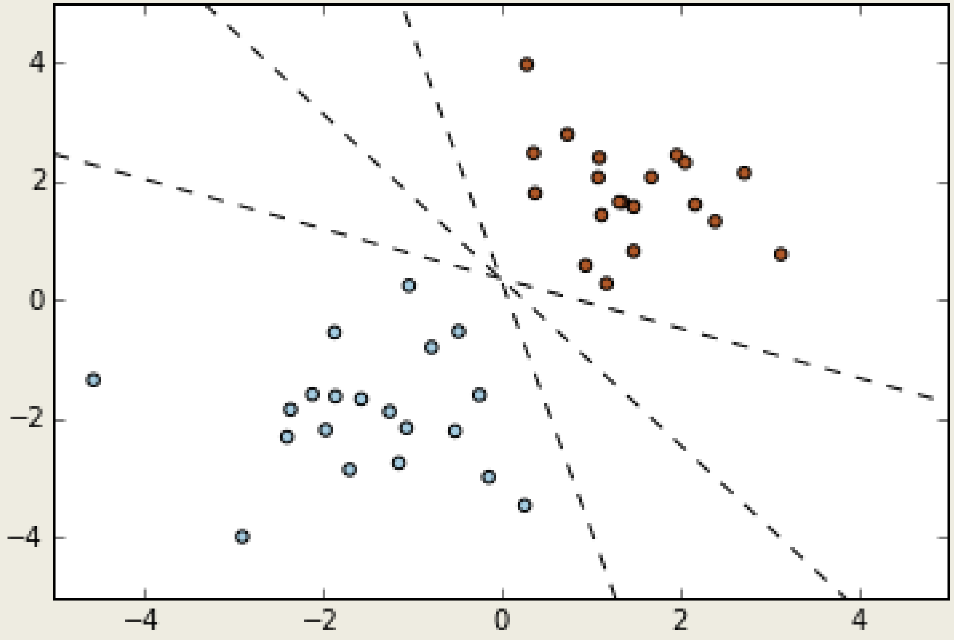

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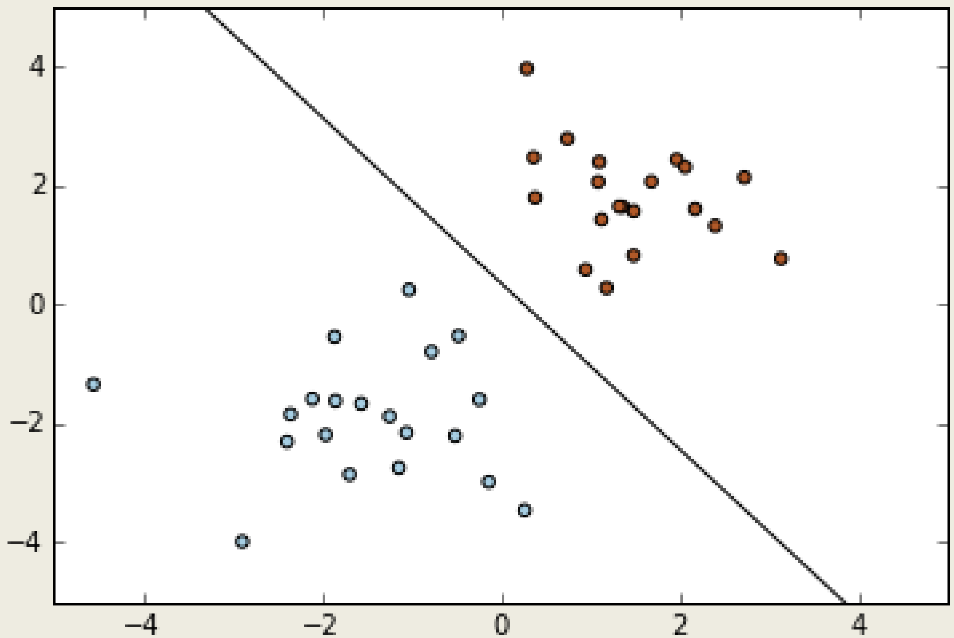

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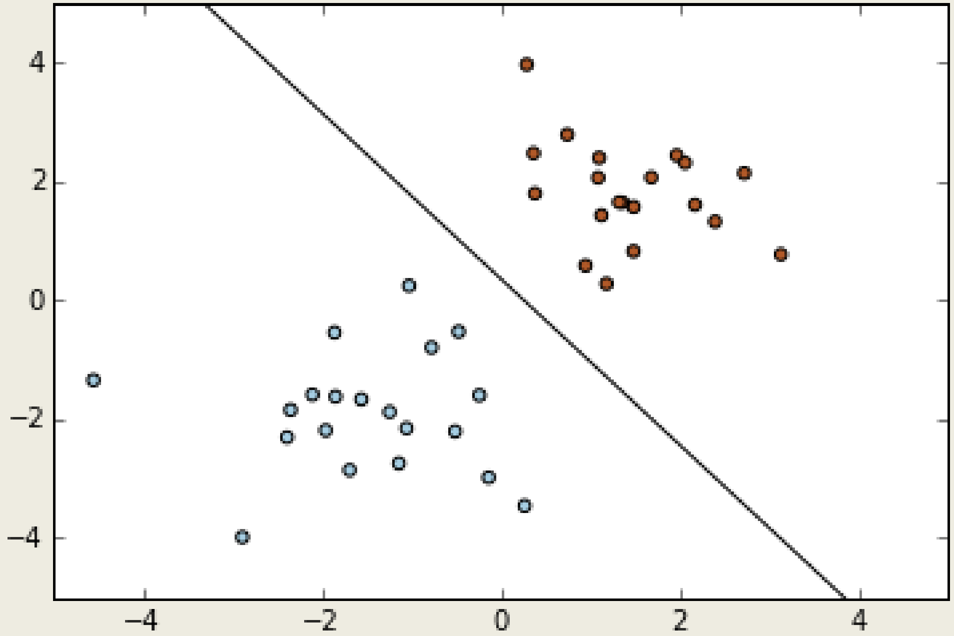

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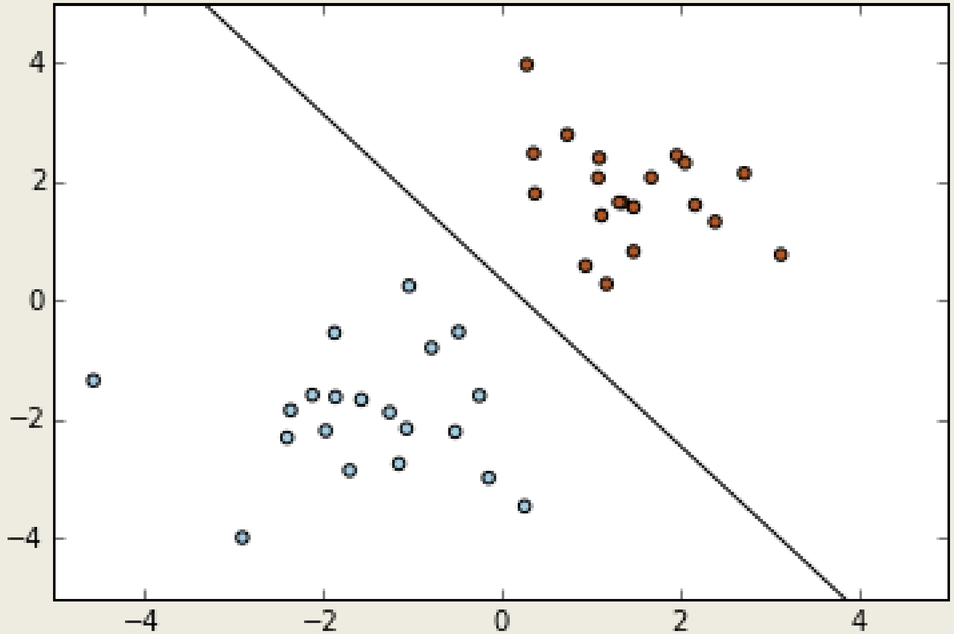

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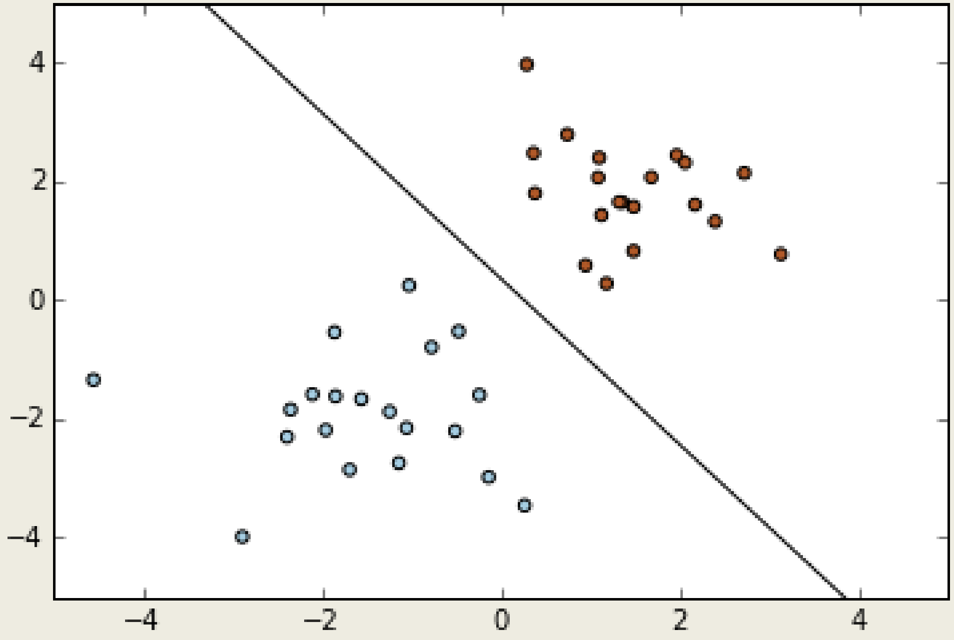

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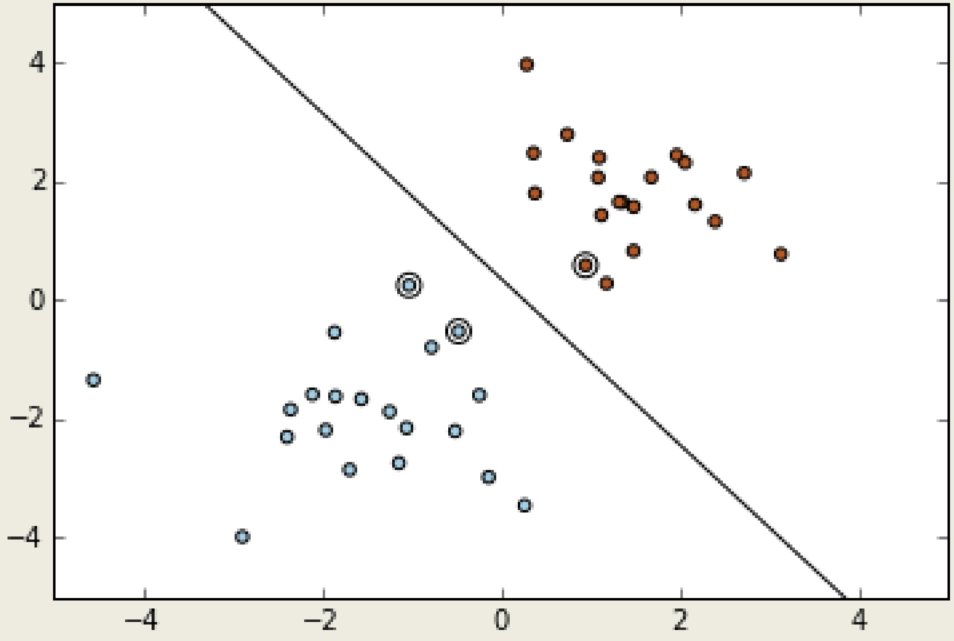

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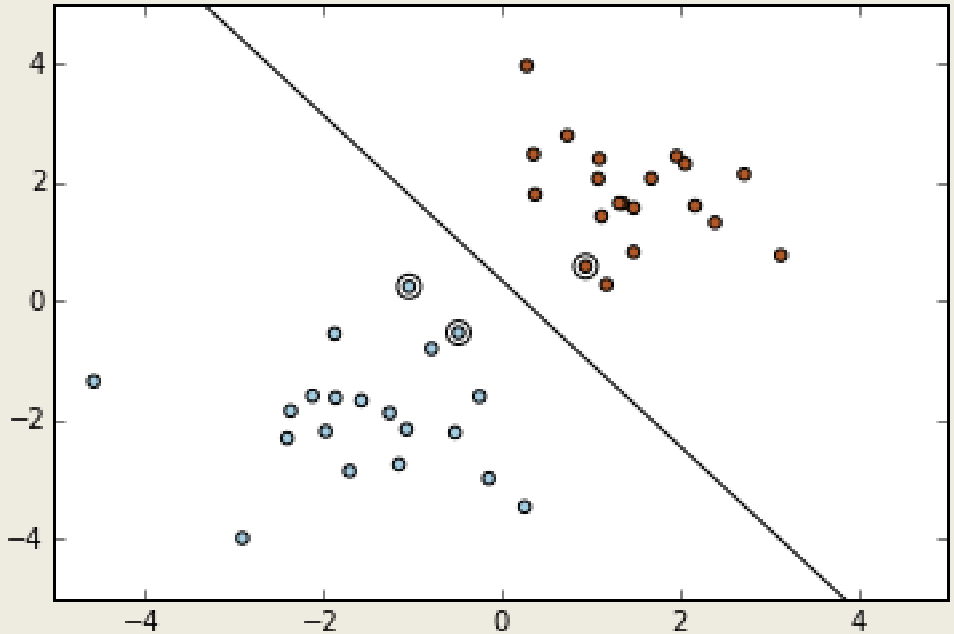

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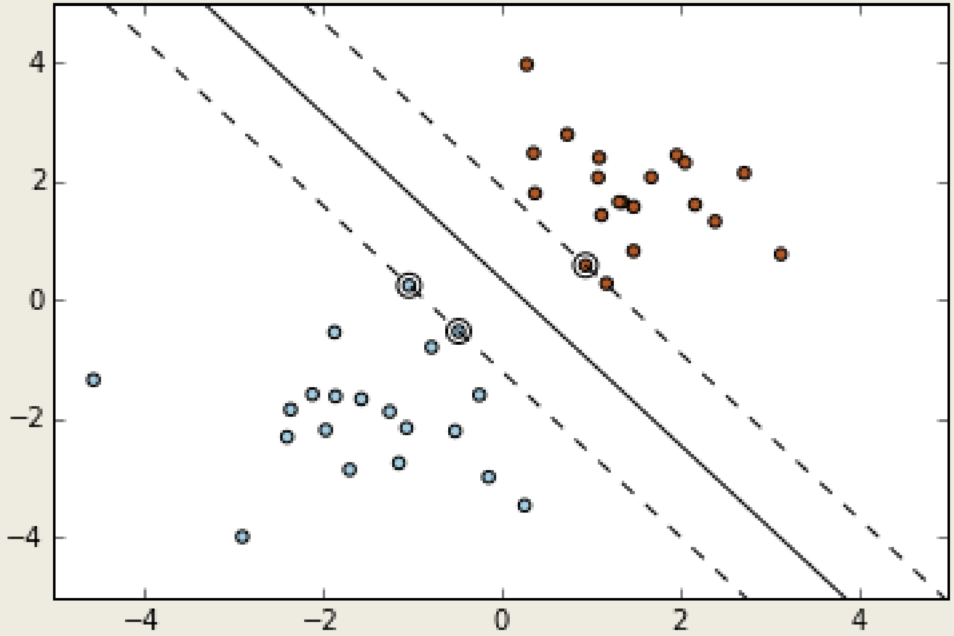

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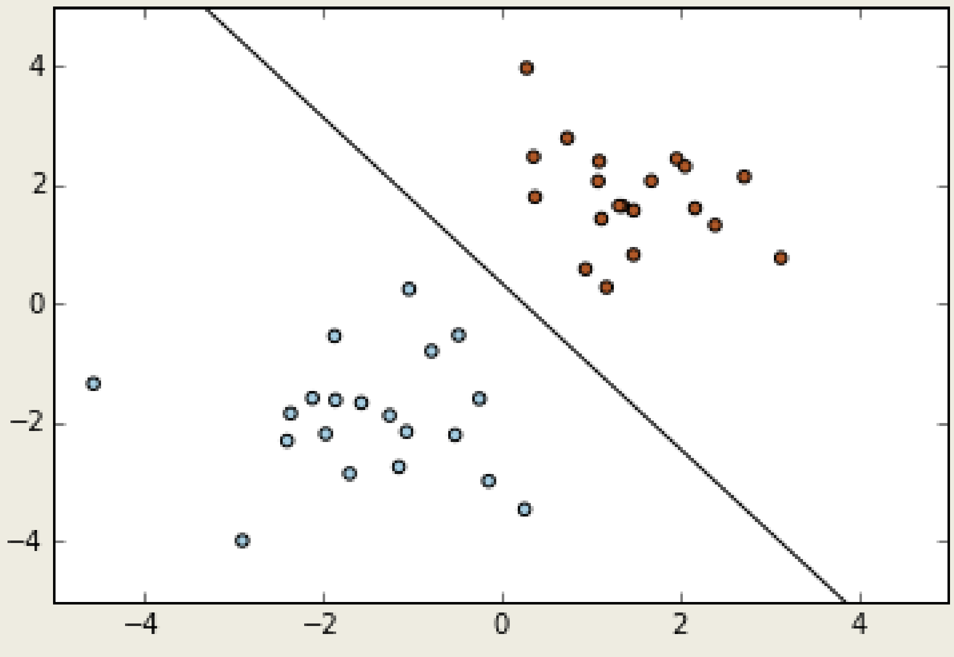

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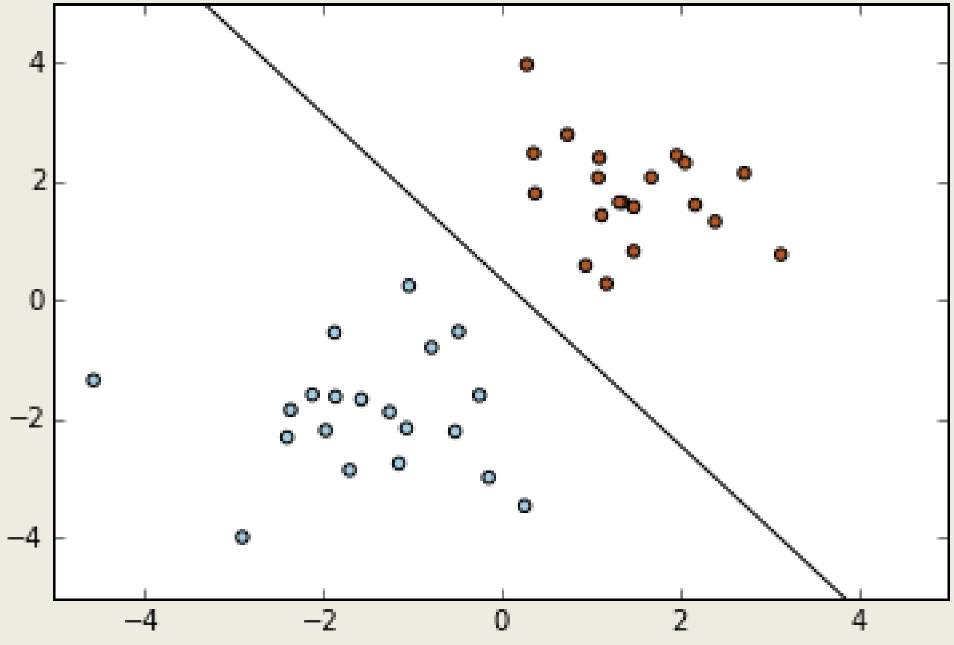

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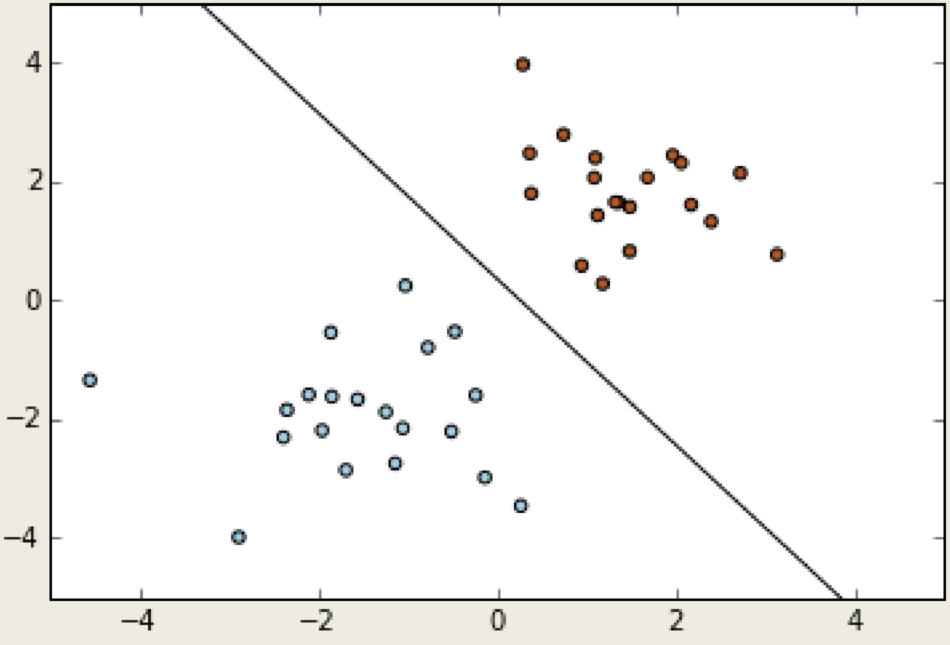

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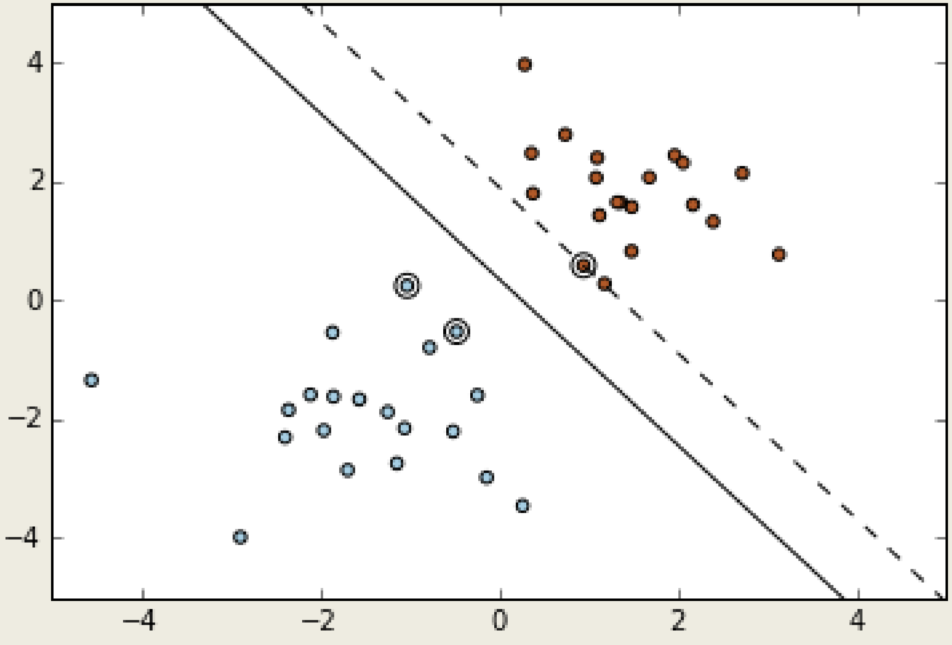

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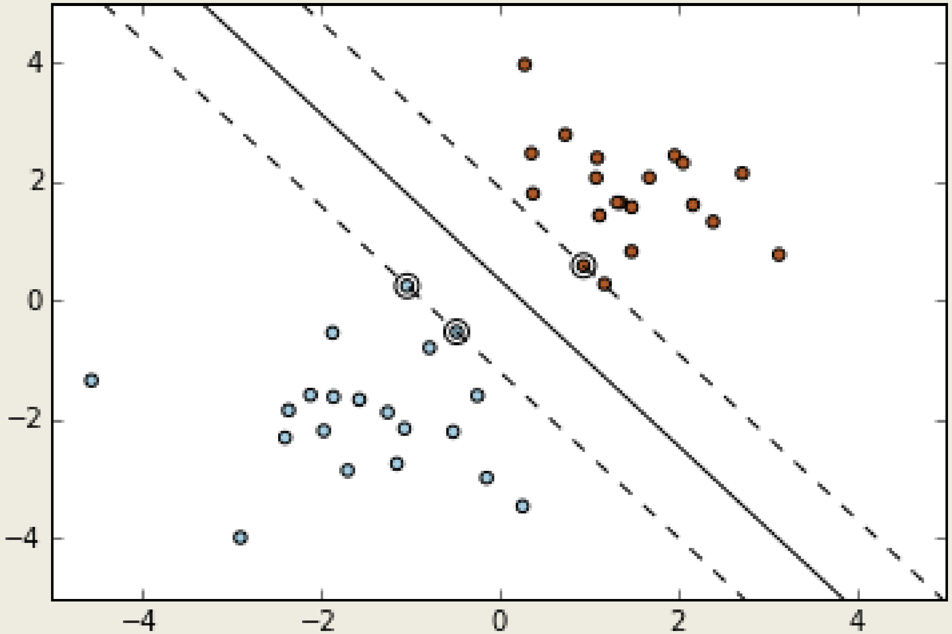

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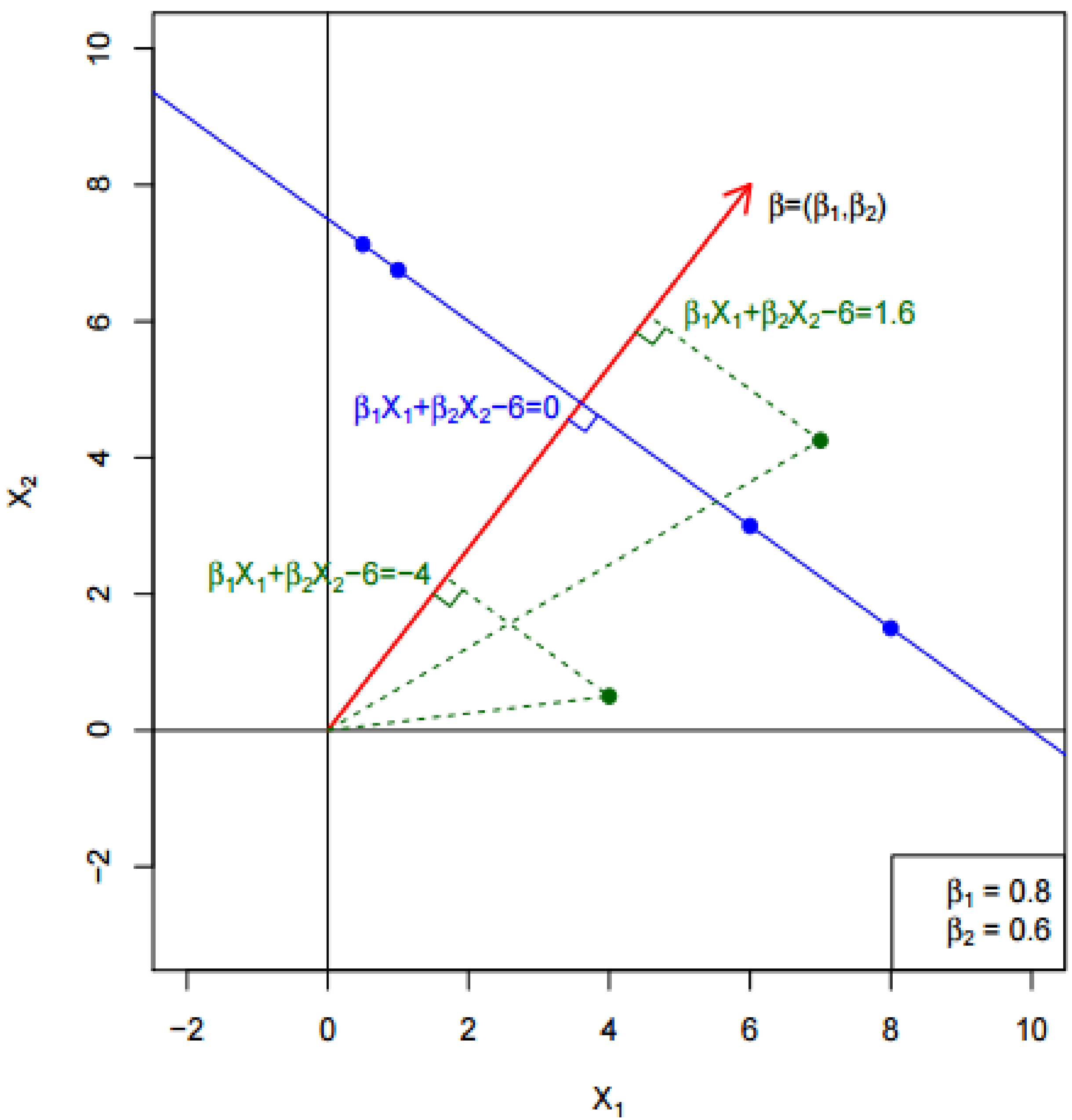

$\beta=(\beta_1,\beta_2)$
$\beta_1X_1+\beta_2X_2-6=1.6$
$\beta_1X_1+\beta_2X_2-6=0$
$\beta_1X_1+\beta_2X_2-6=-4$
$\beta_1 = 0.8$
$\beta_2 = 0.6$
X_1
X_2
−2
0
2
4
6
8
10

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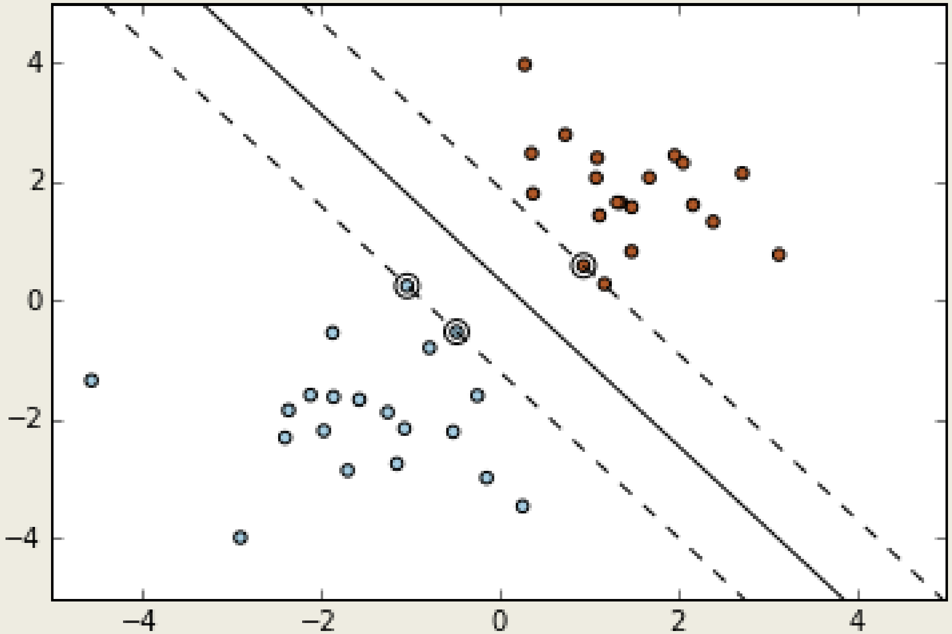

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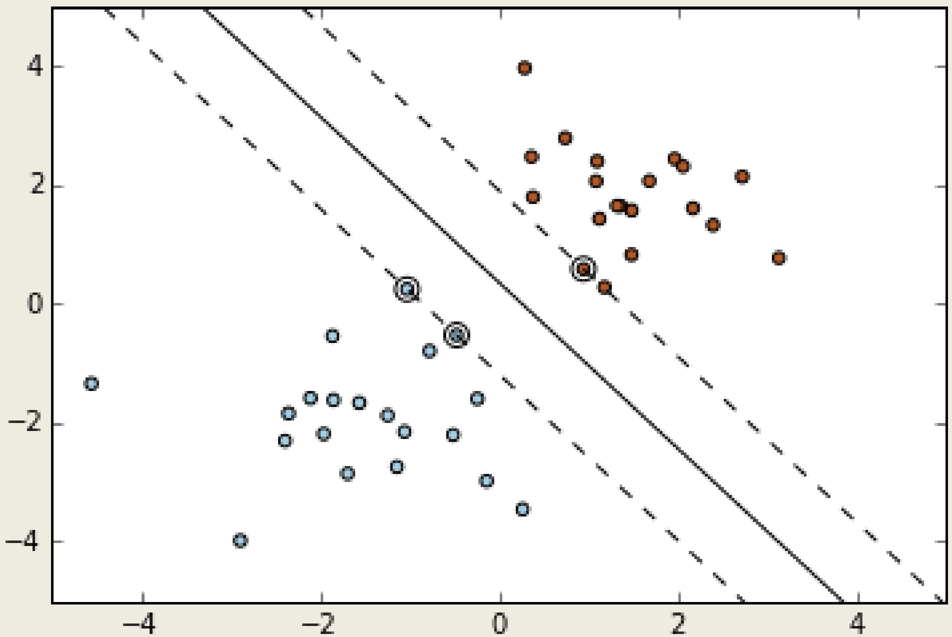

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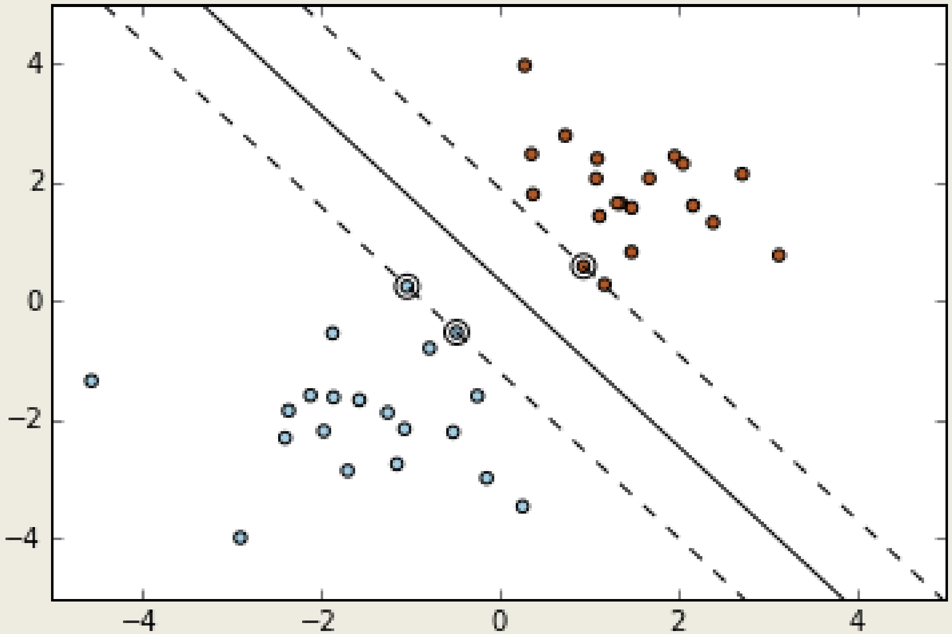

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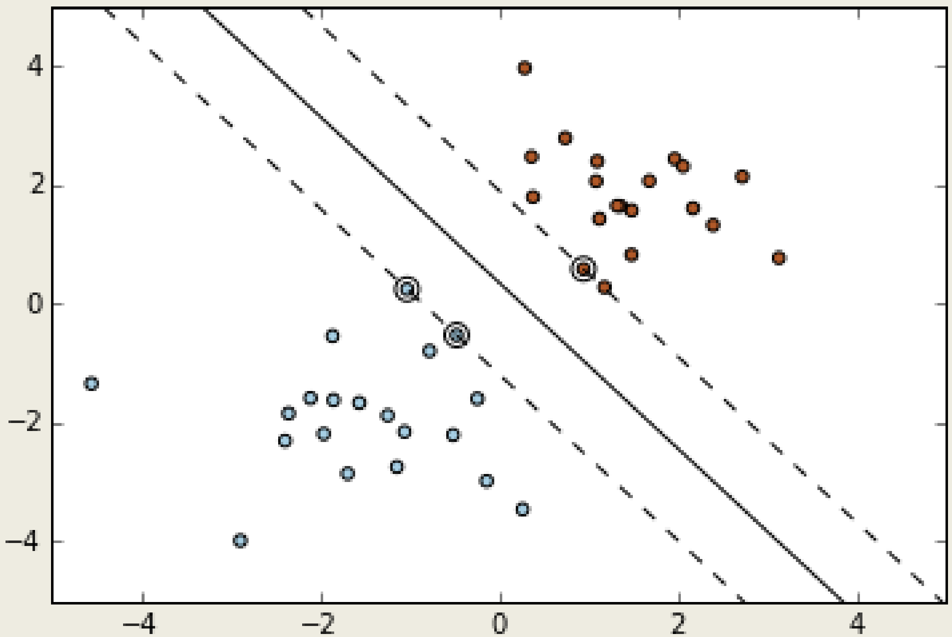

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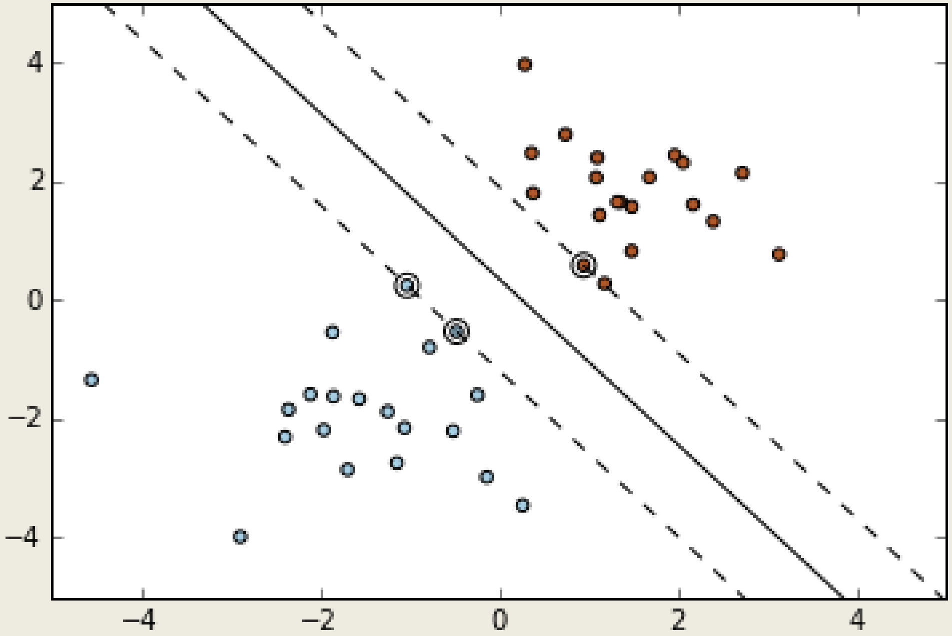

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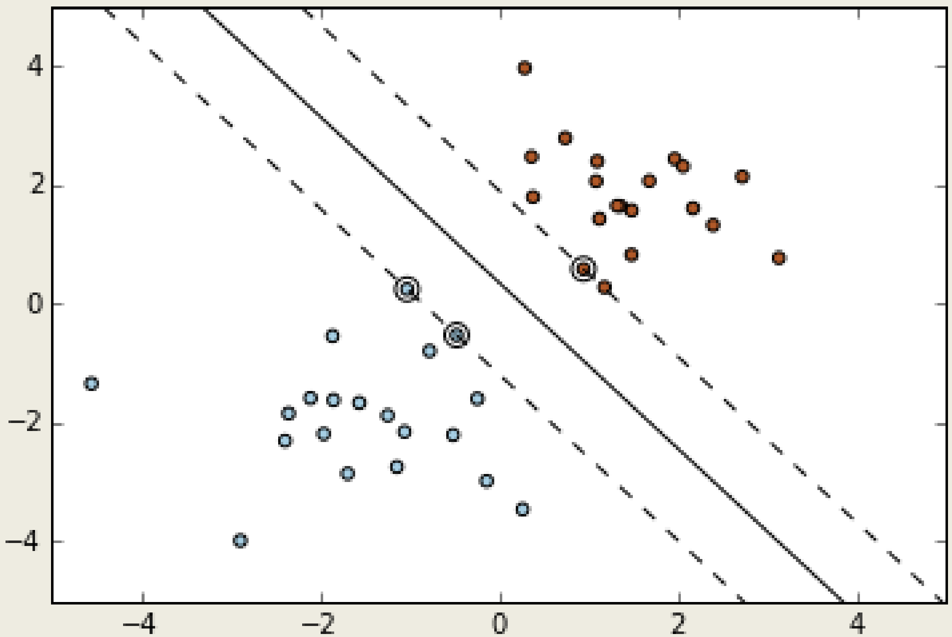

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- Minimize $||w||^2/2$
- s.t. $y_i(w^Tx_i + b) >= 1$
- **Classify:**$f(x)=\text{sign}(w^Tx+b)$

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- Objective:
  - Minimize $||w||^2/2$
  - S.T. $y_i(w^T x_i + b) >= 1$
- It turns out that we can reformulate this problem to:
  - Max $\Sigma_i \alpha_i - 1/2\ \Sigma_{ij} \alpha_i \alpha_j y_i y_j x_i^T x_j$
  - s.t. $\alpha_i >= 0$ and $\Sigma_i \alpha_i y_i = 0$
- Then $w = \Sigma_i \alpha_i y_i x_i$
- It turns out that $\alpha_i = 0$ for non support vectors
- Now we can classify by: $f(x) = \text{sign}(\Sigma_i \alpha_i y_i <x_i, x>)$
- Note that this is line Nearest Neighbors where the neighbors have been selected for you

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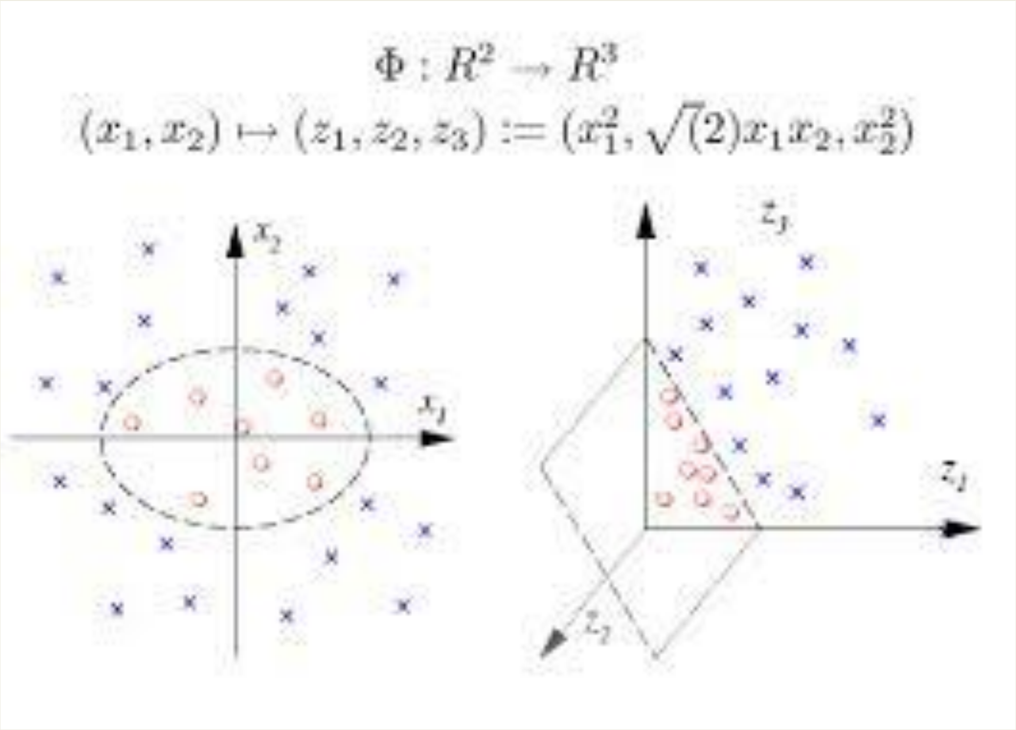

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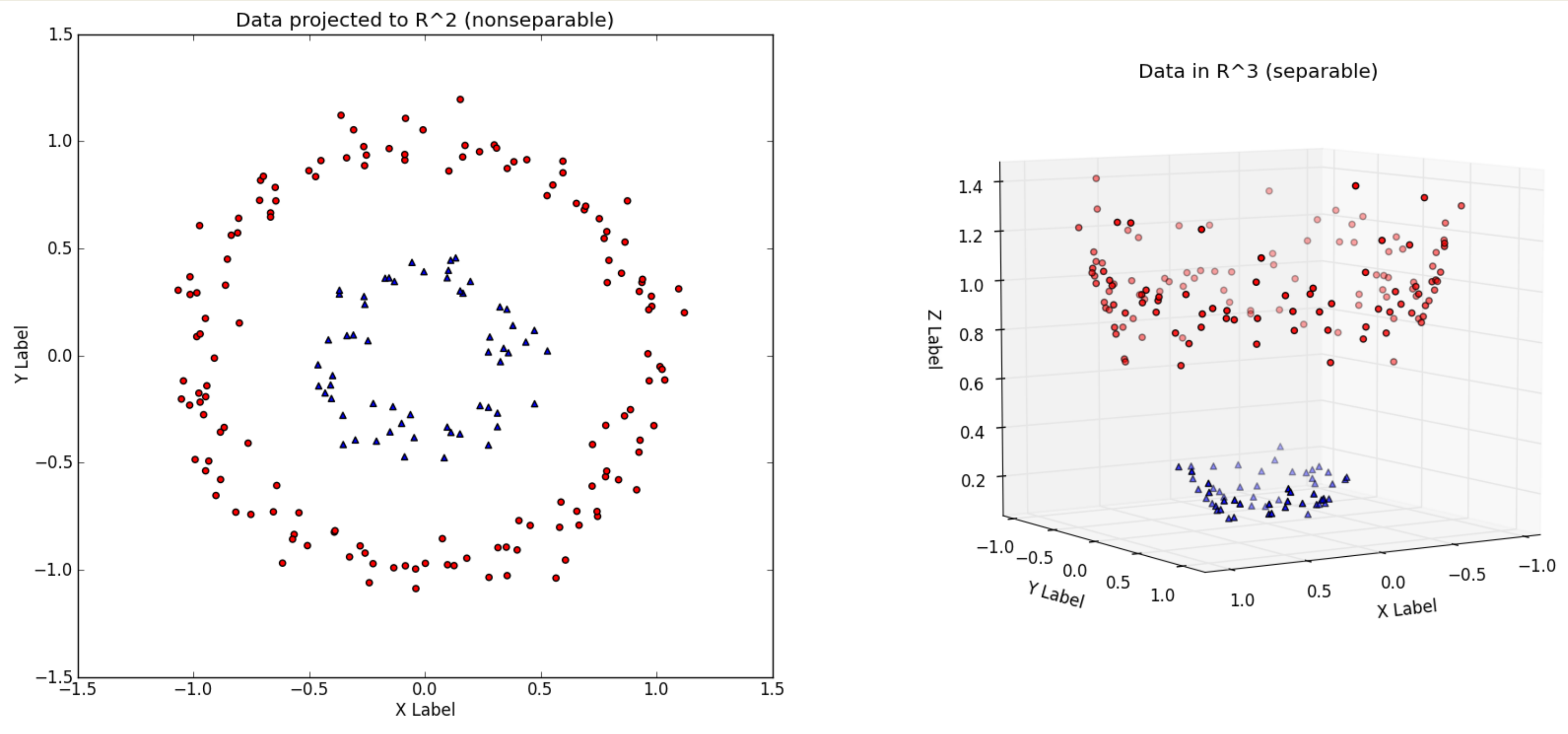

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  - Polynomial: $K(x_1,x) = \exp(\langle x_1,x \rangle + a)^r$
- **Our new classifier is now $f(x) = \mathrm{sign}(\Sigma_i \alpha_i y_i K(x_i,x))$**

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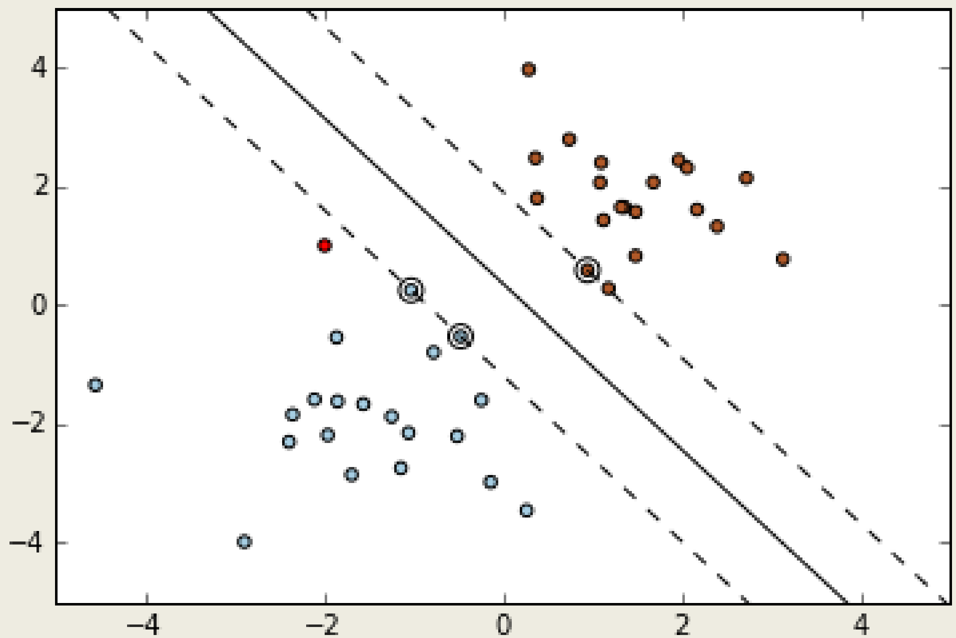

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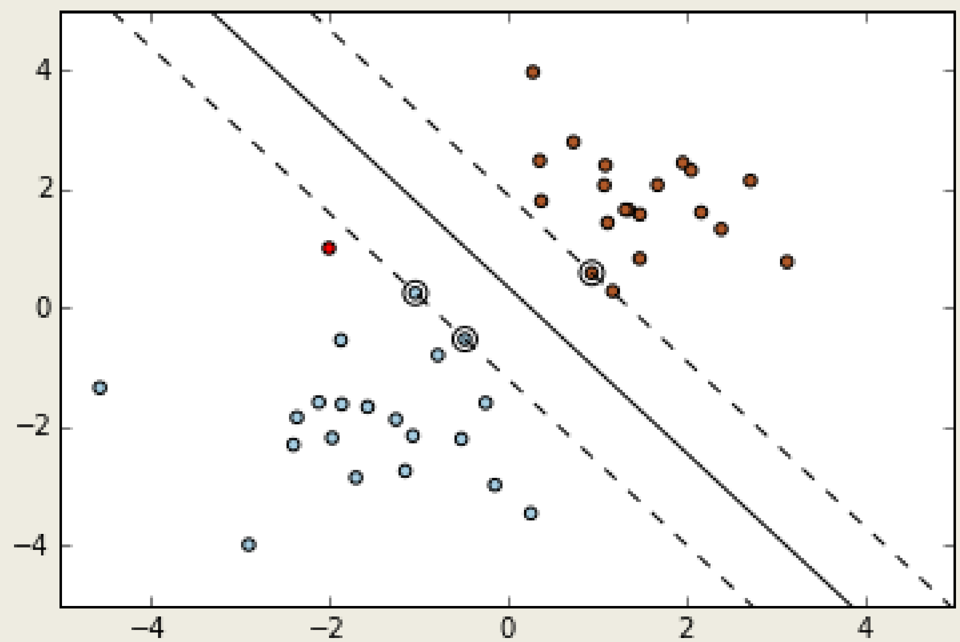
A: We introduce the idea of a slack variable:

- ζ_i is a slack variable
- Now solve min $||w||^2 + C\Sigma_i \zeta_i$
- Now the constraint is $y_i(w^T x_i + b) >= 1-\zeta_i$ where $\zeta_i >= 0$



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- We can classify new points with $\mathbf{f(x) = sign(\Sigma_i \alpha_i y_i K(x_i,x))}$

Questions?