DATA SCIENCE
PCA

AGENDA

- Last Class
- Linear Algebra Review
- Dimension Reduction
- PCA
- Kernel PCA
- Python Practice

LINEAR ALGEBRA

Q: What is a matrix?

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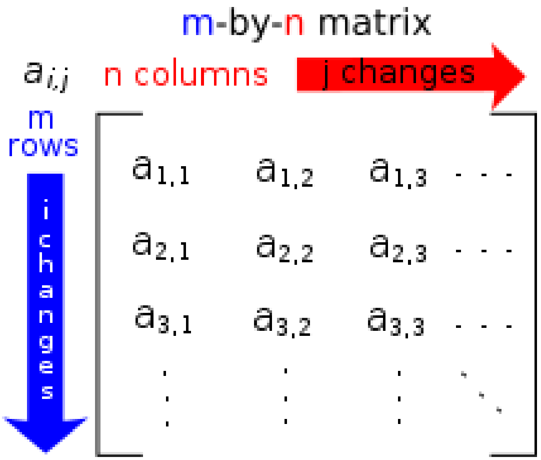
A: Rectangular array of rows and columns.

An m x n matrix has m rows and n columns



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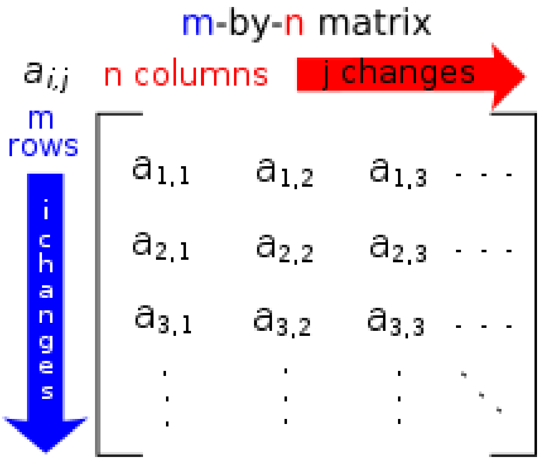

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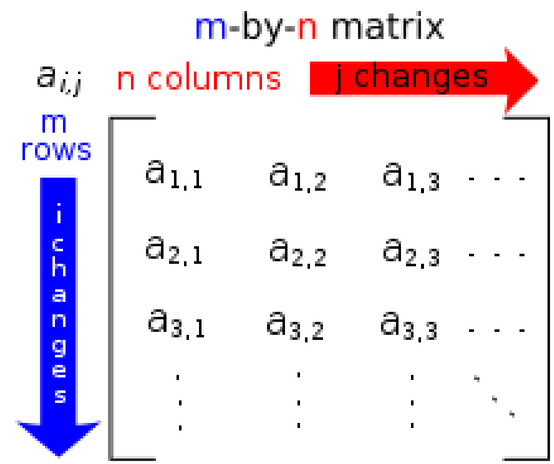

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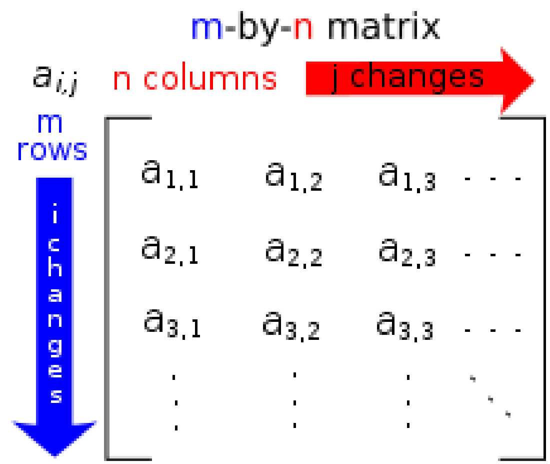

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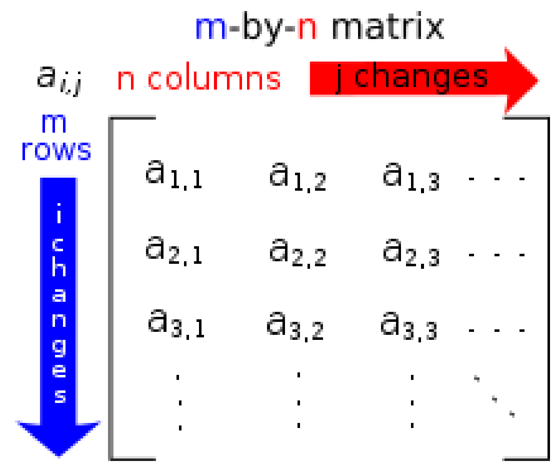

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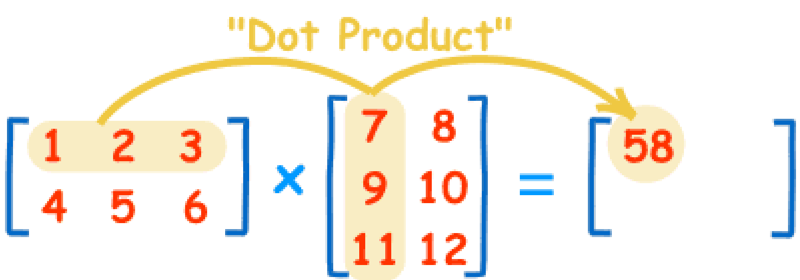

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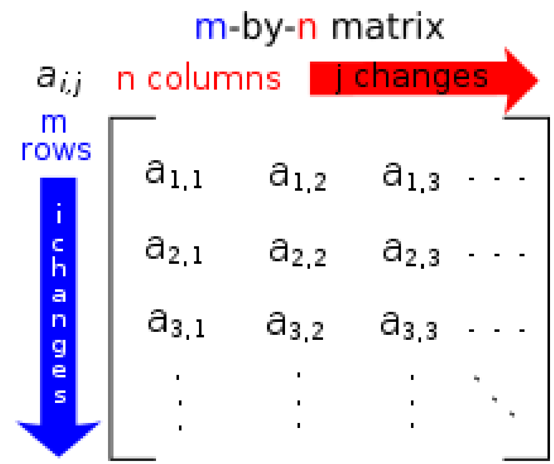

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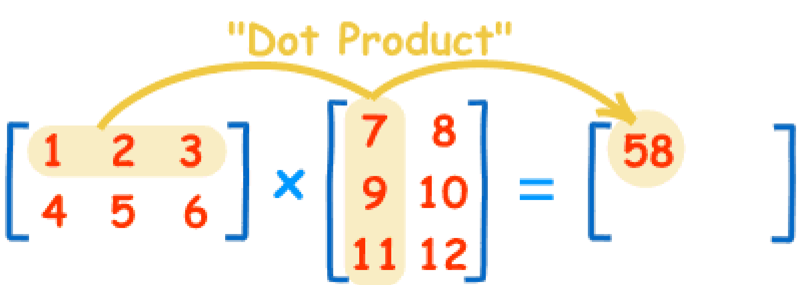

- A is an mxn matrix
- B is an nxp matrix
- AxB is an mxp matrix
- Note:# columns of A = # rows of B

DIMENSIONALITY REDUCTION

Dimensionality reduction is unsupervised learning

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DIMENSIONALITY REDUCTION

Dimensionality reduction is unsupervised learning

Q: What does this mean?
A: We don’t have a y value. We’re trying to find structure, relationships, etc

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- Reduced computation and storage
- Visualization
- Reduce multi-collinearity
- Avoid overfitting (Reduce noise while keeping information)

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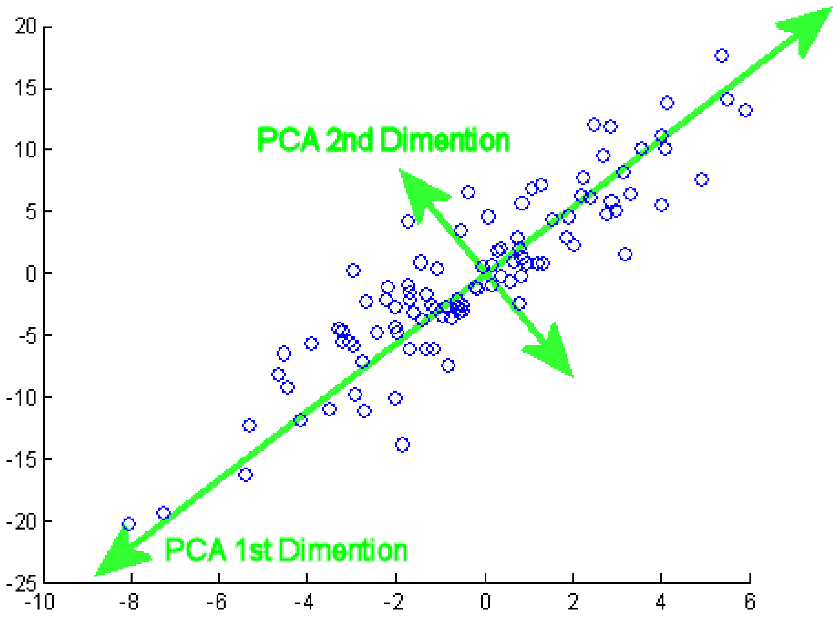

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Let’s dig a little deeper

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A: Variance only measures this on one dimension. Covariance is a measure of how much different dimensions change together

$$\sigma(X,Y) = \mathrm{E}\left[(X-\mathrm{E}[X])(Y-\mathrm{E}[Y])\right],$$

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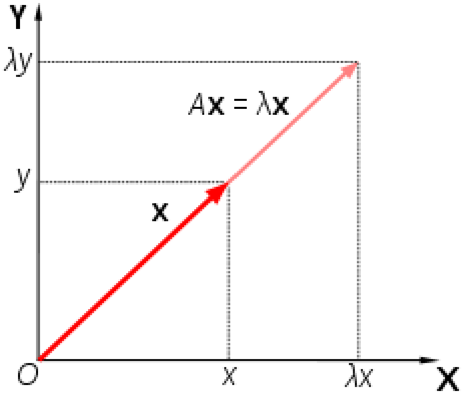
Anyone know about Eigenvalues and Eigenvectors?

$$A\mathbf{v} = \lambda\mathbf{v},$$



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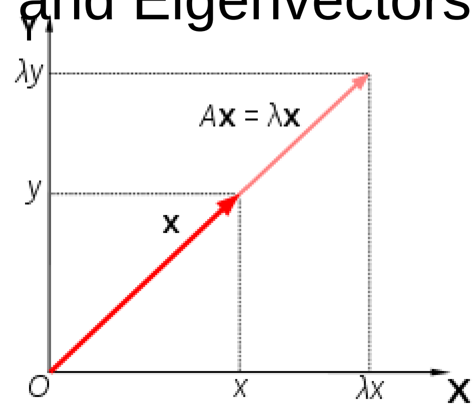

It turns out that the eigenvectors of the covariance matrix are the principal components where the eigenvalues tell us the magnitude

If A is a square nxn matrix with n linearly idependent eigenvectors then we can factorize A as such

$$\mathbf{A} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{-1}$$

Where Q is made up of the eigenvectors and LAMBDA a diagonal matrix of eigenvalues

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Covariance matrix: $$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \\ \sigma_{n1} & \sigma_{n2} & & \sigma_{nn} \end{pmatrix}$$

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Steps:

- Standardize the dataset
- Construct the covariance matrix
- Find eigenvalues and eigenvectors
- Select k eigenvectors corresponding to the k largest eigenvalues
- Construct a projection matrix W from the top k eigenvectors
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Q: How can we make this nonlinear?

A: Similar to SVMs we use a kernel

Questions?