Assignment

Swadeepta Mandal

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1 Minimum Background Test

1.1 Vector and Matrices

(1) $y^T x = 11$

(2) $Xy = \begin{bmatrix} 14 \\ 10 \end{bmatrix}$

(3) $X^{-1} = \begin{bmatrix} \frac{3}{2} & -2 \\ -\frac{1}{2} & 1 \end{bmatrix}$

(4) $rank(X) = 2$ as the columns are linearly independent.

1.2 Calculus

(1) $\frac{dy}{dx} = 3x^2 + 1$

(2) $\nabla f(x) = (^{(1-x_1)sin(x_2)e^{-x_1}}_{x_1 cos(x_2)e^{-x_1}})$

1.3 Probability and Statistics

(1) Sample mean $= \frac{3}{5}$

(2) Sample variance $= \frac{6}{25}$

(3) Probability of observing this data $= \frac{1}{32}$

(4) Let p be the probability of 1,
We would like to maximise:

$$\Pi_{i=1}^{n} p^{x_i}(1-p)^{(1-x_i)}$$

in our case,

$$p^3(1-p)^2 \qquad 0 \leq p \leq 1$$

So computing we get $p = \frac{3}{5}$.

(5) $p(z = T \text{ AND } y = b) = 0.1$
and, $p(z = T|y = b) = \frac{0.1}{0.1+0.15} = 0.4$

1.4 Big-O Notation

(1) Both, as $lg(n) = ln(n).lg(e)$ where $lg(e)$ is constant.

(2) $g(n) = O(f(n))$, 3^n grows faster than n^{100}.

(3) $g(n) = O(f(n))$, 3^n grows faster than 2^n.

(4) $f(n) = O(g(n))$, n^3 grows faster than n^2.

2 Medium Background Test

2.1 Algorithm

In given scenario, **binary search** will work as it has run time of $O(logn)$ and as all 0's occurs 1's. We will look for if the middle element 0, if yes then move to the right side and find the middle term there or else if it's 1 then we check if the element next to it(on the left side) is) then that is required element. It only need only $logn$ time as it removes (excludes it from search) half of the array each time.

2.2 Probability

(a) False

(b) True

(c) False

(d) False

(e) True

Discrete and Continuous Distributions

Multivariable Gaussian : $\frac{1}{\sqrt{(2\pi)^d |\sum|}} exp(-\frac{1}{2} - (x-\mu)^T \sum^{-1}(x-\mu))$

Bernoulli : $p^x(1-p)^{1-x}$

Uniform : $b-a$ when $a \leq x \leq b$; 0 otherwise

Binomial : $\binom{n}{x} p^x(1-p)^{n-x}$

2.3 Mean, Variance and Entropy

(a) To prove : $\mathbb{E}[(X - \mathbb{E}(X))^2] = \mathbb{E}(X^2) - \mathbb{E}(X)^2$.
LHS $= \mathbb{E}[X]^2 - 2\mathbb{E}[X\mathbb{E}(X)] + \mathbb{E}[(\mathbb{E}[X]^2)] = \mathbb{E}[X]^2 - 2\mathbb{E}[X^2] + (\mathbb{E}[X]^2) =$ RHS.

(b) Mean : p,
Variance : $p(1-p)$,
Entropy : $-plog(p) - (1-p)log(1-p)$.

2.4 Law of Large Numbers and Central Limit Theorem

(a) IF the die is unbaised then it is true because if Law of Large Numbers.

(b) LHS $\rightarrow$ RHS, when $n \rightarrow \infty$ from Central Limit Theorem.

2.5 Linear Algebra

2.5.1 Vector norms

2.5.2 Geometry

(a)

(b)

2.6 Programming Skills

refer pogskill.ipynb