

## Project 5: Investigating Subsonic Viscous Aerodynamic Flow over an Airfoil

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**Investigate the subsonic viscous aerodynamic flow over a NACA0012 airfoil at an eight degree angle of attack. The freestream Reynolds number is 2 million, while the Mach number is 0.1.**

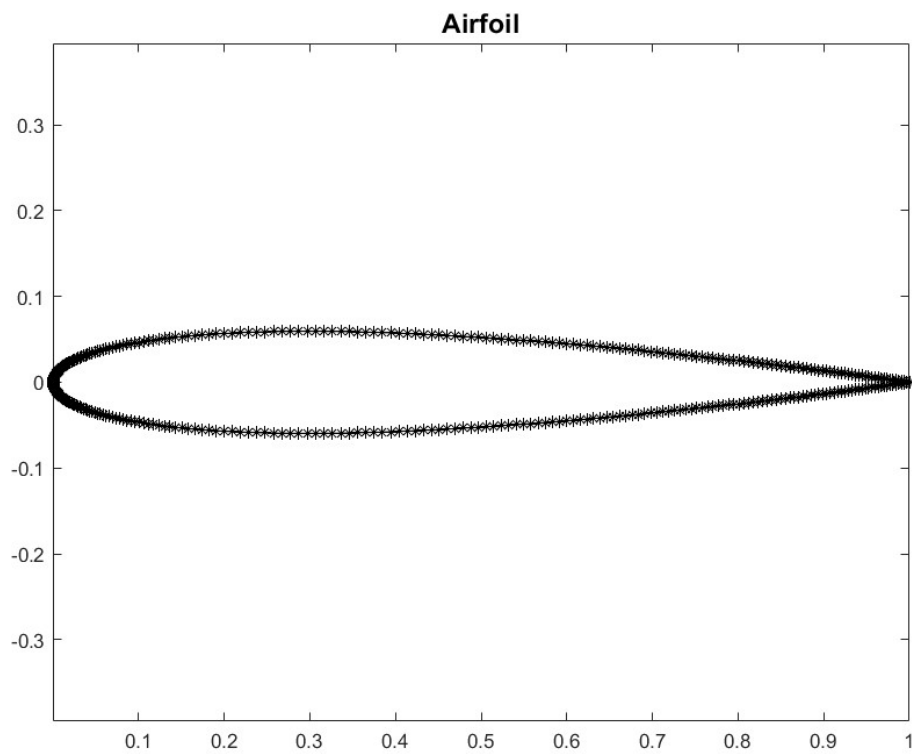
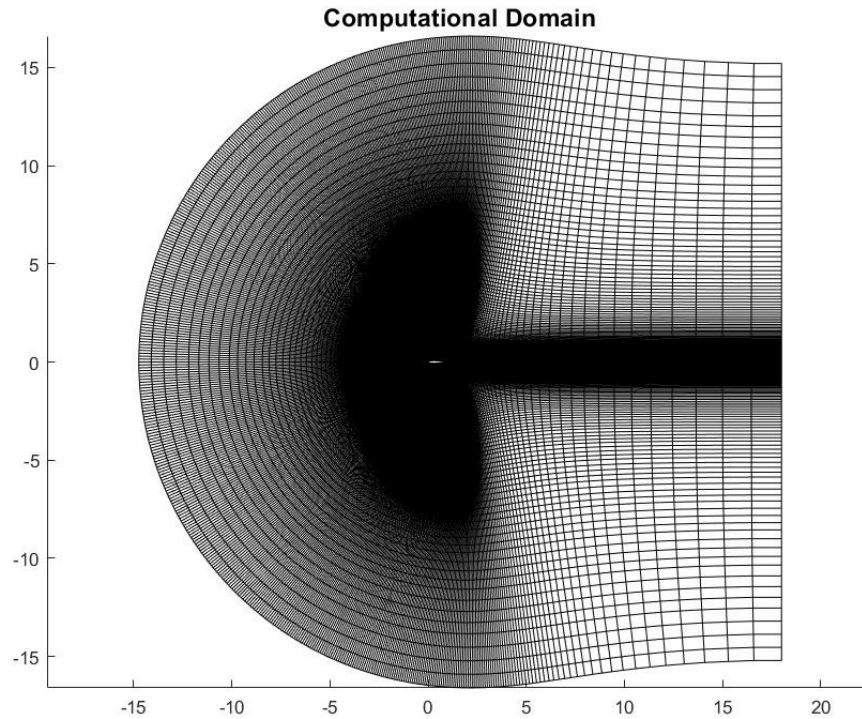


Fig:1 – NACA 0012 airfoil

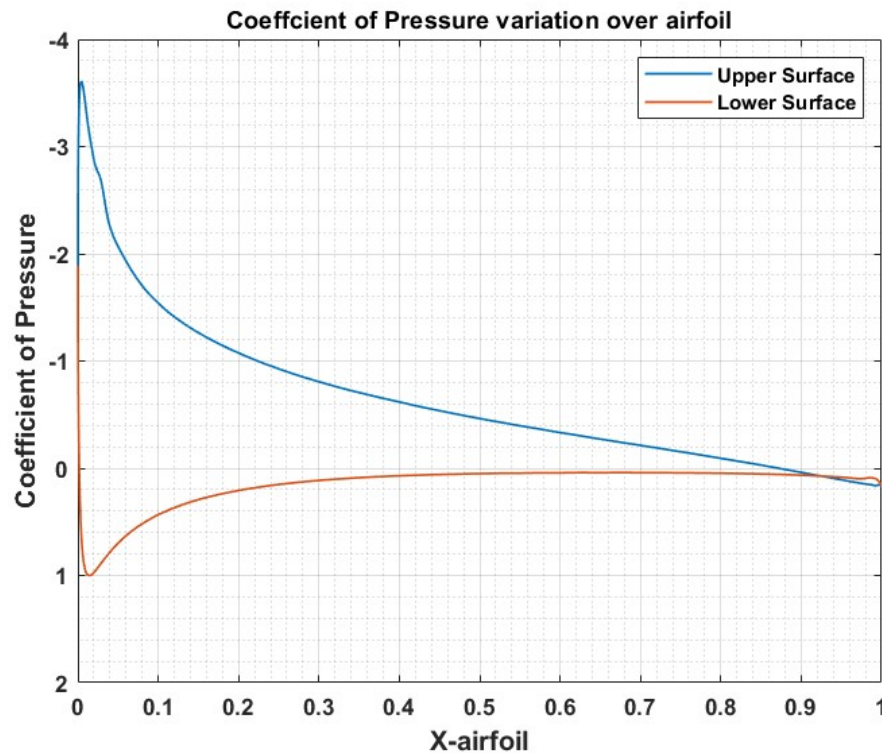


*Fig:2 – The Computational domain*

#### **Discussion –**

- Fig 1 shows the NACA 0012 airfoil used for this project. The Reynolds number and the mach number used is 2 million and 0.1 respectively. The free stream static pressure is 1.0 and the ratio of specific heats = 1.4.
- Fig 2 shows the computational domain. The grid is refined in the near-regions of airfoils to capture the viscous sublayer in the turbulent boundary layer. In the far-away regions, the grid is coarser to reduce computational time.

**Problem 1- Investigate the pressure distribution.** Evaluate the coefficient of pressure,  $c_p$  over the surface of the airfoil and provide a plot of both the upper and lower coefficient of pressure as a function of the x-coordinate. The vertical axis is to be plotted in reverse with the negative axis pointed upwards. Clearly mark the difference between the upper and lower coefficient of pressure distributions. Provide axis labels, titles and a legend. Describe the flow over the airfoil based on the coefficient of pressure distribution.

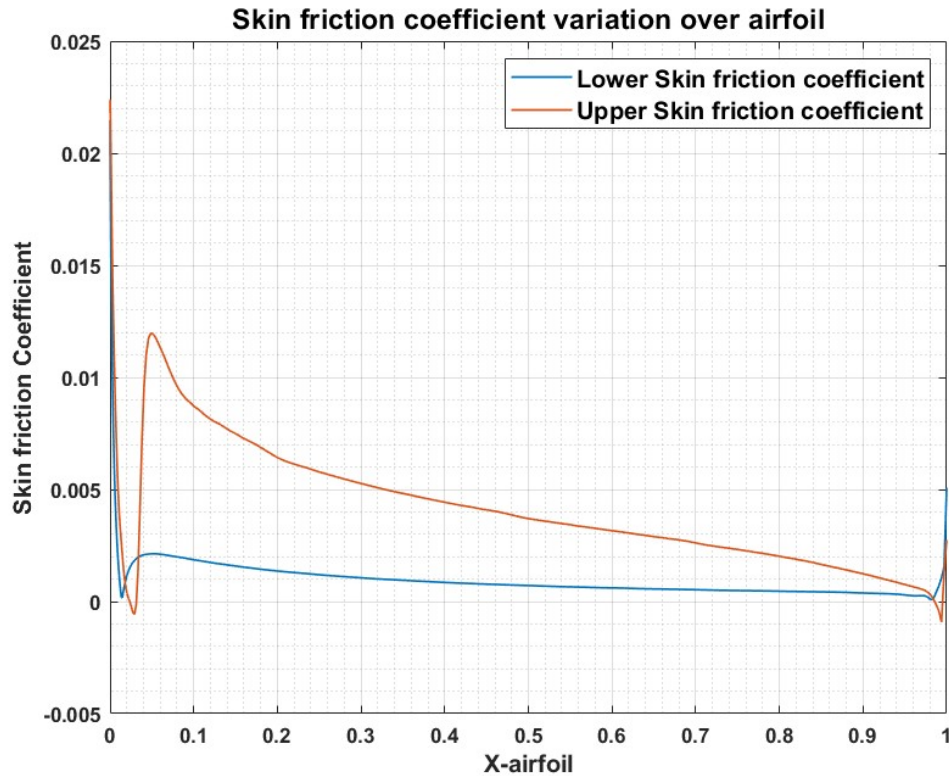


*Fig:3 – Coefficient of Pressure variation over the airfoil*

### Discussion –

- Fig 3 shows the coefficient of Pressure plot for both the upper and lower surface of the airfoil.
- Large negative pressure coefficient is observed at the upper surface of the airfoil near to the leading edge, this simply means the flow has higher velocity in these regions. As the flow moves over the upper surface of the airfoil, the speed reduces and pressure increases and hence the  $C_p$  becomes less negative.
- For the lower surface of airfoil, the trend of  $C_p$  is same. However the absolute magnitude is relatively lower than that of the upper surface at the same location. This means the flow has a relatively lower speed compared to that present on the upper surface. The overall trend of the velocity and pressure variations remains same as that of the upper surface. In beginning, the flow has higher speed, as the flow moves over the airfoil, the speed reduces and pressure increases.

**Problem 2- Investigate the shear-stress distribution.** Evaluate the skin friction coefficient,  $c_f$  over the surface of the airfoil and provide a plot of both the upper and lower skin friction coefficient as a function of the x-coordinate.



*Fig:4 – Skin Friction coefficient over the surface of airfoil*

#### **Discussion –**

- Fig 4 shows the Skin-friction coefficient plot for both the upper and lower surface of the airfoil.
- For the upper surface Skin-friction coefficient plot, the curve starts with positive, which provides the information that the flow is laminar, and then the curve goes to negative, which means the flow separation occurs in those regions. There are 2 separation and reattachment regions on the upper surface, one closer to the leading edge ( $x = 0.025$  approx.) and one closer to the trailing edge ( $x = 0.985$  approx.).
- For the lower surface, the curve always stays positive. It means the flow does not separate along the lower surface of the airfoil.

**Problem 3- Investigate the aerodynamic performance.** Evaluate the coefficient of lift,  $c_l$ , drag due to the pressure force,  $c_{dp}$ , and the drag due to skin friction,  $c_{df}$ . Provide a table with listing the  $c_l$ ,  $c_{dp}$ ,  $c_{df}$ , and the total drag coefficient,  $c_d = c_{df} + c_{dp}$ . Discuss the magnitude of the values in relation with each other. Compare the values against experimental data. Cite the source for the experimental data. Note any discrepancies between the computational and experimental data.

**Discussion –**

- The experimental data has been obtained from the NASA website - [https://turbmodels.larc.nasa.gov/naca0012\\_val.html](https://turbmodels.larc.nasa.gov/naca0012_val.html)
- The above data contains the  $C_l$  and  $C_d$  values for various degrees of angle of attack.
- Experimental value –  $C_l = 0.8749$ ,  $C_d = 0.0091$

Coefficients	Values
$C_l$	0.8089
$C_{dp}$	0.0048
$C_{df}$	0.0049
$C_d = C_{df} + C_{dp}$	0.0098

- The drag coefficient due to pressure and skin friction is almost the same. The  $C_l$  value is much larger when compared to the  $C_d$  value as it should be in ideal case.
- The error between the experimental data and computed data for  $C_l = 7.53\%$
- The error between the experimental data and computed data for  $C_d = -6.84\%$

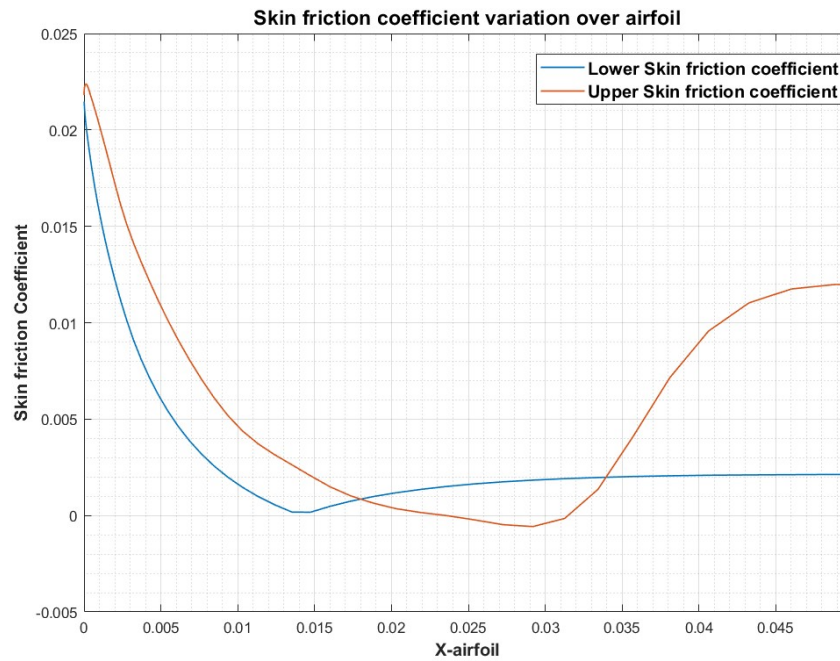
**Problem 4- Investigate the boundary layer.** Using the skin-friction plot from [2.] report on whether any separation exists on the surface of the airfoil. Report on whether the boundary layer is laminar, turbulent, or both might be present at separate locations. Plot the  $u^+$  versus  $y^+$  values in each of the laminar and/or turbulent regions. If there are regions that are separated, then note the point of separation and reattachment.

**Discussion –**

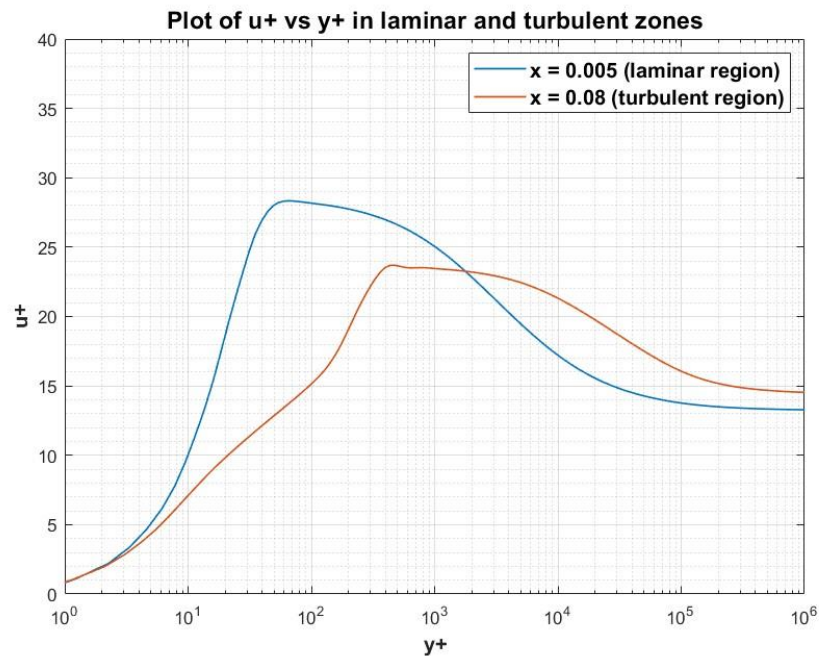
- Fig 7 shows the detailed plot of skin-friction coefficient in the regions closer to the leading edge. The skin friction coefficient value continuously decreases till  $x = 0.02$ . For plotting

$u^+$  and  $y^+$  in laminar regions,  $x = 0.005$  is chosen. For plotting  $u^+$  and  $y^+$  in turbulent regions,  $x = 0.08$  is chosen.

- Fig 8 shows  $u^+$  vs  $y^+$  plot in both the laminar and turbulent zones. The plot in the laminar regions has a large viscous sublayer and buffer zones before turbulent log-law region starts.



*Fig:6 – Detailed plot of skin-friction plot*



*Fig:7 –  $u^+$  vs  $y^+$  plot in the laminar and turbulent regions*

- However in the turbulent regions, a smaller viscous sublayer and buffer zone when compared to large turbulent log-law regions is seen in the plot.

**Problem 5- Further investigate the turbulent boundary layer.** At  $x = 0.5$ , plot the  $u^+$  versus  $y^+$  for the upper surface. Also plot the analytical solutions for the  $u^+$  versus  $y^+$  in both the viscous sub-layer and the log law region of the inner boundary layer.

*Fig:8 – Various zones in turbulent boundary layer*

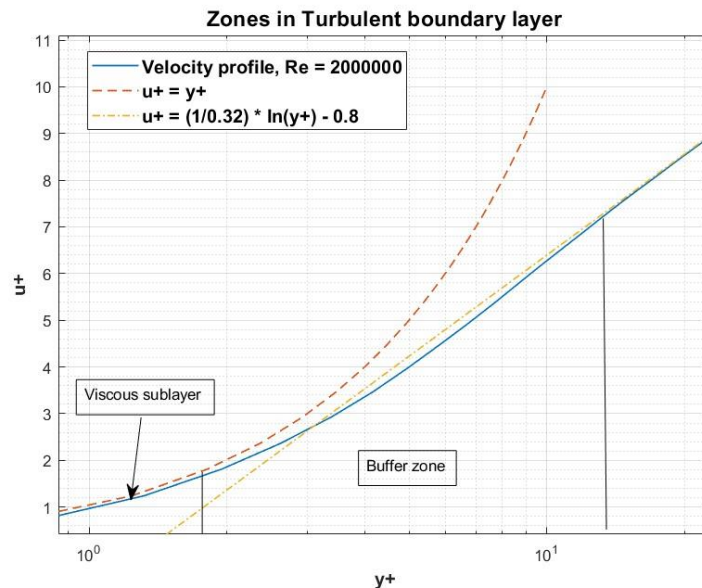
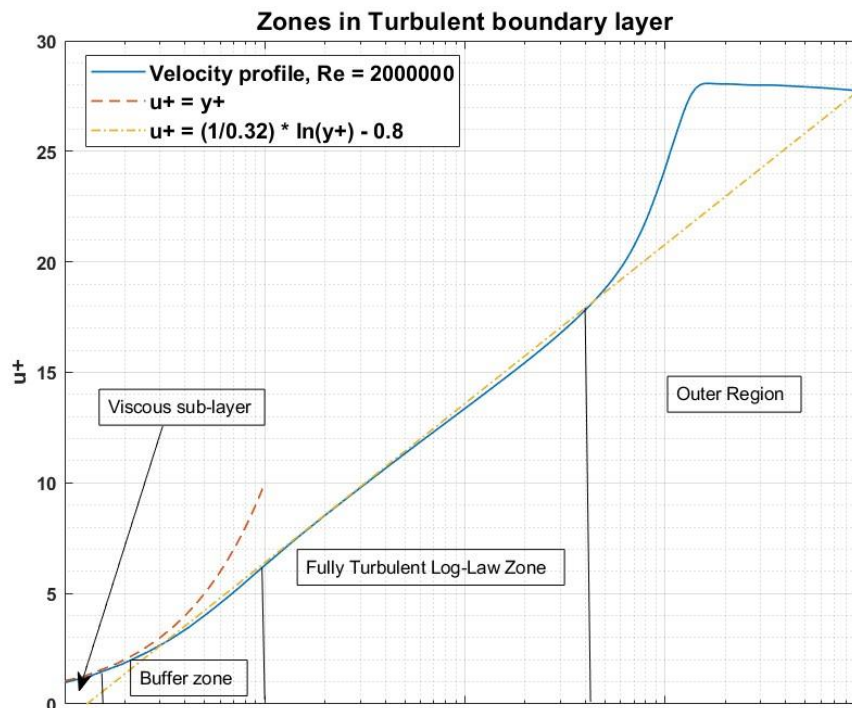




Fig:9 – Viscous sublayer and buffer zone regions

Discussion –

- The fig 8 shows the  $u^+$  vs  $y^+$  plot, and the analytical solutions for  $u^+$  vs  $y^+$  plot for  $x = 0.5$  at the upper surface of the airfoil. It shows all four zones, namely the viscous sub layer, buffer zone, Fully turbulent log-law zone and the outer region.
- Fig 9 shows the viscous sublayer and buffer zone more closely. As seen in the plot, for the viscous sublayer the analytical solution and the computed values match well. The same can be seen for the Log-Law region and the analytical solution.

**Problem 6- Investigate the momentum deficit in the wake.** Plot the fluid speed profile through the wake regions at the following downstream coordinates: 1/4-chord, 1/2-chord, 1-chord, and 2-chords. Ensure that the complete region of the wake is observable at each downstream coordinate. Discuss how the wake evolves as it convects downstream. Report on the size, magnitude and location of the maximum deficit.

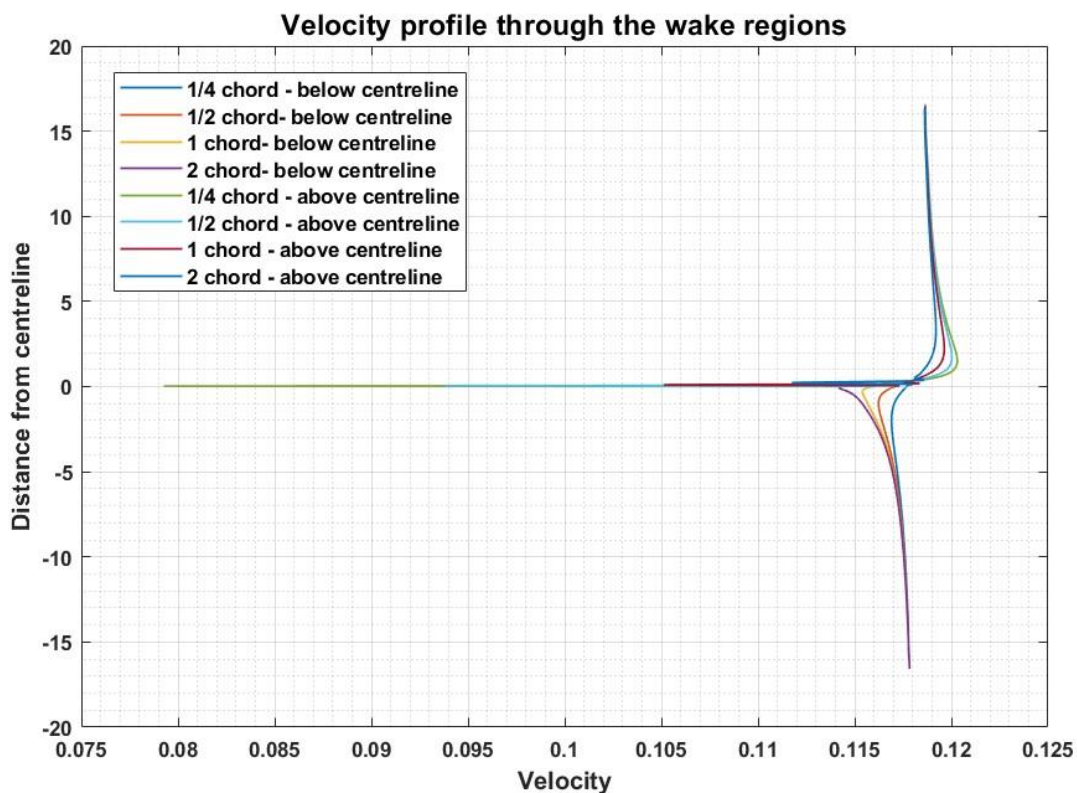


Fig:10 – Velocity Profile through the wake regions



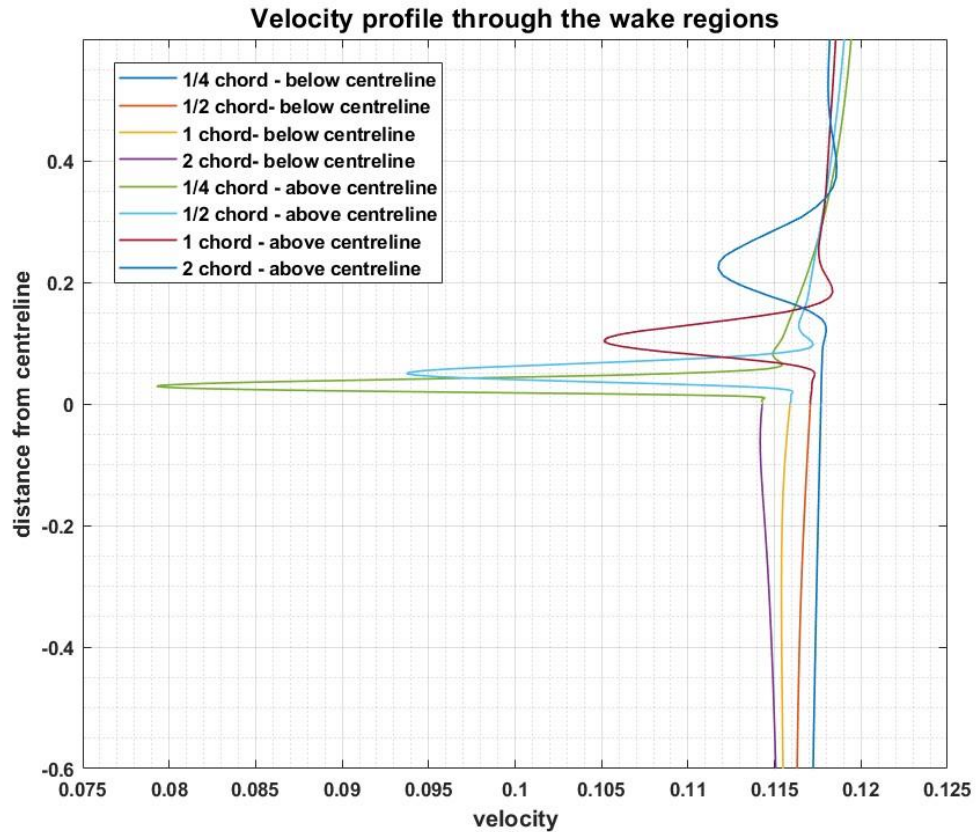


Fig:11 – Velocity Profile through the wake regions- (Detailed)

#### Discussion –

- Fig 10 shows the velocity profile in the wake regions of the airfoil. Fig 11 shows the detailed plot of the velocity profile.
- The wake regions is seen in the flow which is above the centerline in the positive y direction (considering centerline as  $x = 0$ ).
- For the negative y direction, the wake regions are not seen as shown in Fig 10.
- As the distance from the airfoil increases, the vortices gradually lose energy and dissipate due to viscosity of the fluid.

Variables related to wake	$\frac{1}{4}$ Chord length	$\frac{1}{2}$ Chord length	1 Chord length	2 Chord length
Size	0.047	0.0718	0.1109	0.223
Magnitude	0.04	0.025	0.0133	0.00639
Location of maximum deficit	0.02888	0.0494	0.1035	0.222

- The size of the wake is considered as the **difference between the maximum y axis value and minimum y axis value of velocity fluctuations as seen in the velocity profile plot.**
- The magnitude of the wake **double the difference between the lowest speed and the undisturbed speed at that location.**
- The location of maximum deficit is considered as **y axis value of minimum speed for that location.**
- As it can be seen from the table the size of the vortices increases but the magnitude of the momentum deficit decreases along the downstream of the airfoil.
- The location of maximum deficit changes along downstream direction as the vortices moves in the positive y direction.

## MATLAB CODE

```
close all
clear
clc

load NACA0012_flowfieldv2.dat

imax = 513;
jmax = 257;
TE_start = 65; % Trailing Edge Lower Point
TE_end = 449; % Trailing Edge Upper Point
LE = 257; % Leading Edge Point
gamma = 1.4;
mach = 0.1;
p0 = 1.; % Freestream static pressure
alpha = 8; % Angle of Attack
Re = 3e6; % Reynolds number

k = 1;
for j=1:jmax
    for i=1:imax
        x(i,j) = NACA0012_flowfieldv2(k,1);
        y(i,j) = NACA0012_flowfieldv2(k,2);
        k = k +1;
    end
end

% Plot the Airfoil Surface
figure(1)
plot(x(TE_start:TE_end,1),...
     y(TE_start:TE_end,1),'k*-')
axis equal
title('Airfoil','FontSize',14)

% Plot the Computational Domain.
figure(2)
axis equal
hold on
for j=1:jmax
    plot(x(1:imax,j),y(1:imax,j),'k-')
end
for i=1:imax
    plot(x(i,1:jmax),y(i,1:jmax),'k-')
end
title('Computational Domain','FontSize',14)
```

```

%
% % Plot the Computational Domain.
% figure(3)
% axis equal
% hold on
% for j=1:jmax
%     plot(x(1:imax,j),y(1:imax,j),'k-')
% end
% for i=1:imax
%     plot(x(i,1:jmax),y(i,1:jmax),'k-')
% end
% axis([-0.1 1.1 -0.2 0.2])

Nvariables = 7;
% Transfer all other flow properties and evaluate Pressure
k = 1;
for j=1:jmax
    for i=1:imax
        for n=1:Nvariables
            w(i,j,n) = NACA0012_flowfieldv2(k,2+n);
        end
        k = k + 1;
    end
end

% *****%
% Plot Surface Pressure
% *****%
for j=1:jmax
    for i=1:imax
        pressure(i,j) = (gamma - 1)*(w(i,j,4) ...
            -.5*(w(i,j,2)^2 + w(i,j,3)^2)/w(i,j,1));
        velocity(i,j) = sqrt((w(i,j,2)/w(i,j,1))^2 ...
            +(w(i,j,3)/w(i,j,1))^2);
    end
end

% Evaluate Shear Stress.
% for i=TE_start:TE_end
%     dxi = x(i,1) - x(i-1,1); % dx/dxi
%     dyi = y(i,1) - y(i-1,1); % dy/dxi
%     dxj = x(i,2) - x(i,1); % dx/deta
%     dyj = y(i,2) - y(i,1); % dy/deta
%     dsj = 1/(dxi*dyj - dyi*dxj);
%
%     dui = w(i,1,2)/w(i,1,1) - w(i-1,1,2)/w(i-1,1,1); % du/dxi
%     dvi = w(i,1,3)/w(i,1,1) - w(i-1,1,3)/w(i-1,1,1); % dv/dxi
%     duj = w(i,2,2)/w(i,2,1) - w(i,1,2)/w(i,1,1); % du/deta
%     dvj = w(i,2,3)/w(i,2,1) - w(i,1,3)/w(i,1,1); % dv/deta
%

```

```

%     dux = (dui*dyj -duj*dyi)*dsj; % du/dx
%     dvx = (dvi*dyj -dvj*dyi)*dsj; % dv/dx
%     duy = (duj*dxj -dui*dxj)*dsj; % du/dy
%     dvy = (dvj*dxj -dvi*dxj)*dsj; % dv/dy
% end

%% Question 1
Cp = ((pressure((65:449),1)/p0)-1)/(0.5*gamma*(mach)^2);
figure(4)
plot(x(257:449),Cp(193:385),'LineWidth',1)
hold on;
plot(x(65:257),Cp(1:193),'LineWidth',1)
set(gca,'YDir','reverse');
ylabel('Coefficient of Pressure')
xlabel('X-airfoil')
grid on;
grid minor;
title('Coefficient of Pressure variation over airfoil')
legend('Upper Surface', 'Lower Surface')

%% Question 2
% mew = NACA0012_flowfielddv2(TE_start:TE_end,7)+...
%     NACA0012_flowfielddv2(TE_start:TE_end,8);
% Txx = (mew*2*dux)-((2/3)*mew*(dux+dvy));
% Txy = mew*(duy+dvx);
% Tyy = (mew*2*dvy)-((2/3)*mew*(dux+dvy));
Cf = zeros(TE_end-TE_start,1);
Tw = zeros(TE_end-TE_start,1);
for i = TE_start:TE_end
    dyp = 0.5*(y(i+1,1) +y(i,1));
    dym = 0.5*(y(i,1) +y(i-1,1));
    dxp = 0.5*(x(i+1,1) +x(i,1));
    dxm = 0.5*(x(i,1) +x(i-1,1));

    dx = dxp - dxm;
    dy = dyp - dym;
    dA = sqrt((dx)^2+(dy)^2);
    cos_a = dx/dA; sin_a = dy/dA;

    dxi = x(i,1) -x(i-1,1); % dx/dxi
    dyi = y(i,1) -y(i-1,1); % dy/dxi
    dxj = x(i,2) -x(i,1); % dx/deta
    dyj = y(i,2) -y(i,1); % dy/deta
    dsj = 1/(dxi*dyj -dyi*dxj);

    dui = w(i,1,2)/w(i,1,1) -w(i-1,1,2)/w(i-1,1,1); % du/dxi
    dvi = w(i,1,3)/w(i,1,1) -w(i-1,1,3)/w(i-1,1,1); % dv/dxi
    duj = w(i,2,2)/w(i,2,1) -w(i,1,2)/w(i,1,1); % du/deta
    dvj = w(i,2,3)/w(i,2,1) -w(i,1,3)/w(i,1,1); % dv/deta

    dux = (dui*dyj -duj*dyi)*dsj; % du/dx
    dvx = (dvi*dyj -dvj*dyi)*dsj; % dv/dx
    duy = (duj*dxi -dui*dxj)*dsj; % du/dy
    dvy = (dvj*dxi -dvi*dxj)*dsj; % dv/dy

```

```

mew = NACA0012_flowfieldv2(i,7) + NACA0012_flowfieldv2(i,8);

Txx = (mew*2*dux)-((2/3)*mew*(dux+dvy));
Txy = mew*(duy+dvx);
Tyy = (mew*2*dvy)-((2/3)*mew*(dux+dvy));

Tx = -Txx*sin_a + Txy*cos_a;
Ty = -Txy*sin_a + Tyy*cos_a;
Tw(i-TE_start+1) = Tx*cos_a + Ty*sin_a;
Cf(i-TE_start+1) = Tw(i-TE_start+1)/(0.5*p0*(mach)^2*gamma);
end
figure(5)
plot(x(65:256),abs(Cf(1:192)),'LineWidth',1)
hold on;
plot(x(257:449),Cf(193:385),'LineWidth',1)
ylabel('Skin friction Coefficient')
xlabel('X-airfoil')
title('Skin friction coefficient variation over airfoil')
grid on;
grid minor;
legend('Lower Skin friction coefficient ','...
       'Upper Skin friction coefficient')

%% Question 3
Cpx = 0;
Cpy = 0;
Cfx = 0;
Cfy = 0;
alpha_r = alpha*pi/180;
for i = TE_start:TE_end
    dyp = 0.5*(y(i+1,1) +y(i,1));
    dym = 0.5*(y(i,1) +y(i-1,1));
    dxp = 0.5*(x(i+1,1) +x(i,1));
    dxm = 0.5*(x(i,1) +x(i-1,1));

    dx = dxp - dxm;
    dy = dyp - dym;
    dA = sqrt((dx)^2+(dy)^2);
    cos_a = dx/dA; sin_a = dy/dA;
    n1 = -sin_a; n2 = cos_a;
    t1 = cos_a; t2 = sin_a;

    Cpi = ((pressure(i,1)/p0)-1)/(0.5*gamma*(mach)^2);
    Cpx = Cpx - (Cpi*n1*dA);
    Cpy = Cpy - (Cpi*n2*dA);
    Clp = -Cpx*sin(alpha_r) + Cpy*cos(alpha_r);
    Cdp = Cpx*cos(alpha_r) + Cpy*sin(alpha_r);

    Cfx = Cfx + Cf(i-TE_start+1,1)*t1*dA;
    Cfy = Cfy + Cf(i-TE_start+1,1)*t2*dA;
    Clf = -Cfx*sin(alpha_r) + Cfy*cos(alpha_r);
    Cdf = Cfx*cos(alpha_r) + Cfy*sin(alpha_r);

```

```

Cl = Clf + Clp;
Cd = Cdf + Cdp;
end
cl_actual = ((1.12074 - 0.870758)/(10.1891-7.96346))*(8-7.96346) + 0.870758;
cd_actual = 0.00915707;
error_cl = ((cl_actual-Cl)/cl_actual)*100;
error_cd = ((cd_actual-Cd)/cd_actual)*100;

%% Q4
ui_plusx1 = zeros(2,jmax);
yi_plus = zeros(2,jmax);
k = 1;
for i = [276,313]
for j = 1:jmax
    u_friction = sqrt(abs(Tw((i-TE_start+1)))/w(i,1,1));
    ui_plusx1(k,j) = velocity(i,j)/u_friction;

    mew =w(i,1,5) + w(i,1,6);
    vw = mew/w(i,1,1);
    y_normal = sqrt(w(i,j,7));
    yi_plus(k,j) = y_normal*u_friction/vw;
%    ui_plus1 = (1/0.32)*log(yi_plus)-0.8;
end
k = k+1;
end
yi_plusx = ui_plusx1(1:27);
figure(6)
semilogx(yi_plus(1,:),ui_plusx1(1:27),'LineWidth',1)
axis([1 1000000 0 40])
grid on;
grid minor;
hold on;
semilogx(yi_plus(2,:),ui_plusx1(2:27),'LineWidth',1)
ylabel('u+')
xlabel('y+')
title('Plot of u+ vs y+ in laminar and turbulent zones')
legend('x = 0.005 (laminar region)', 'x = 0.08 (turbulent region)');

%% question 5
ui_plus = zeros(jmax,1);
ui_plus1 = zeros(jmax,1);
yi_plus = zeros(jmax,1);
for j = 1:jmax
    i = 363;
    u_friction = sqrt(abs(Tw((i-TE_start+1)))/w(i,1,1));
    ui_plus(j) = velocity(i,j)/u_friction;

    mew =w(i,1,5) + w(i,1,6);
    vw = mew/w(i,1,1);
    y_normal = sqrt(w(i,j,7));
    yi_plus(j) = y_normal*u_friction/vw;
    ui_plus1 = (1/0.32)*log(yi_plus)-0.8;
end
yi_plus1 = ui_plus1(1:27);

```



```

figure(7)
semilogx(yi_plus,ui_plus,'LineWidth',1)
axis([1 10000 0 30])
grid on;
grid minor;
hold on;
semilogx(yi_plus1,ui_plus(1:27),'LineStyle','--','LineWidth',1);
hold on;
semilogx(yi_plus,ui_plus1,'LineStyle','-.','LineWidth',1);

ylabel('u+')
xlabel('y+')
title('Zones in Turbulent boundary layer')
legend('Velocity profile, Re = 200000', 'u+ = y+',...
       'u+ = (1/0.32) * ln(y+) - 0.8')

%% Question 6
k = 1;
v = zeros(8,jmax);
y_ver = zeros(4,jmax);
%for i = [336,363,449,485]
for i = [22,29,36,42,472,478,485,492]
    for j = 1:jmax
        v(k,j) = velocity(i,j);
        y_ver(k,j) = y(i,j)-y(i,1);
    end
    k = k+1;
end
figure(8)
plot(v(1,:),y_ver(1:,:), 'linewidth',1)
hold on;
plot(v(2,:),y_ver(2:,:), 'linewidth',1)
plot(v(3,:),y_ver(3:,:), 'linewidth',1)
plot(v(4,:),y_ver(4:,:), 'linewidth',1)
plot(v(5,:),y_ver(5:,:), 'linewidth',1)
plot(v(6,:),y_ver(6:,:), 'linewidth',1)
plot(v(7,:),y_ver(7:,:), 'linewidth',1)
plot(v(8,:),y_ver(8:,:), 'linewidth',1)
axis([0.075 0.125 -0.6 0.6])
grid on;
grid minor;
ylabel('Distance from centreline')
xlabel('Velocity')
title('Velocity profile through the wake regions')
legend('1/4 chord - below centerline', '1/2 chord- below centerline',...
       '1 chord- below centerline', '2 chord- below centerline',...
       '1/4 chord - above centerline', '1/2 chord - above centerline',...
       '1 chord - above centeline', '2 chord - above centerline')

```

