Project 6: A Coupled Boundary Layer Code and an External Flow Solved by the Murman-Cole Scheme for the Transonic Small Disturbance Equation

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Solve the coupled boundary layer code and the transonic small disturbance (TSD) theory over a circular arc airfoil at various Mach numbers using the Murman-Cole method.

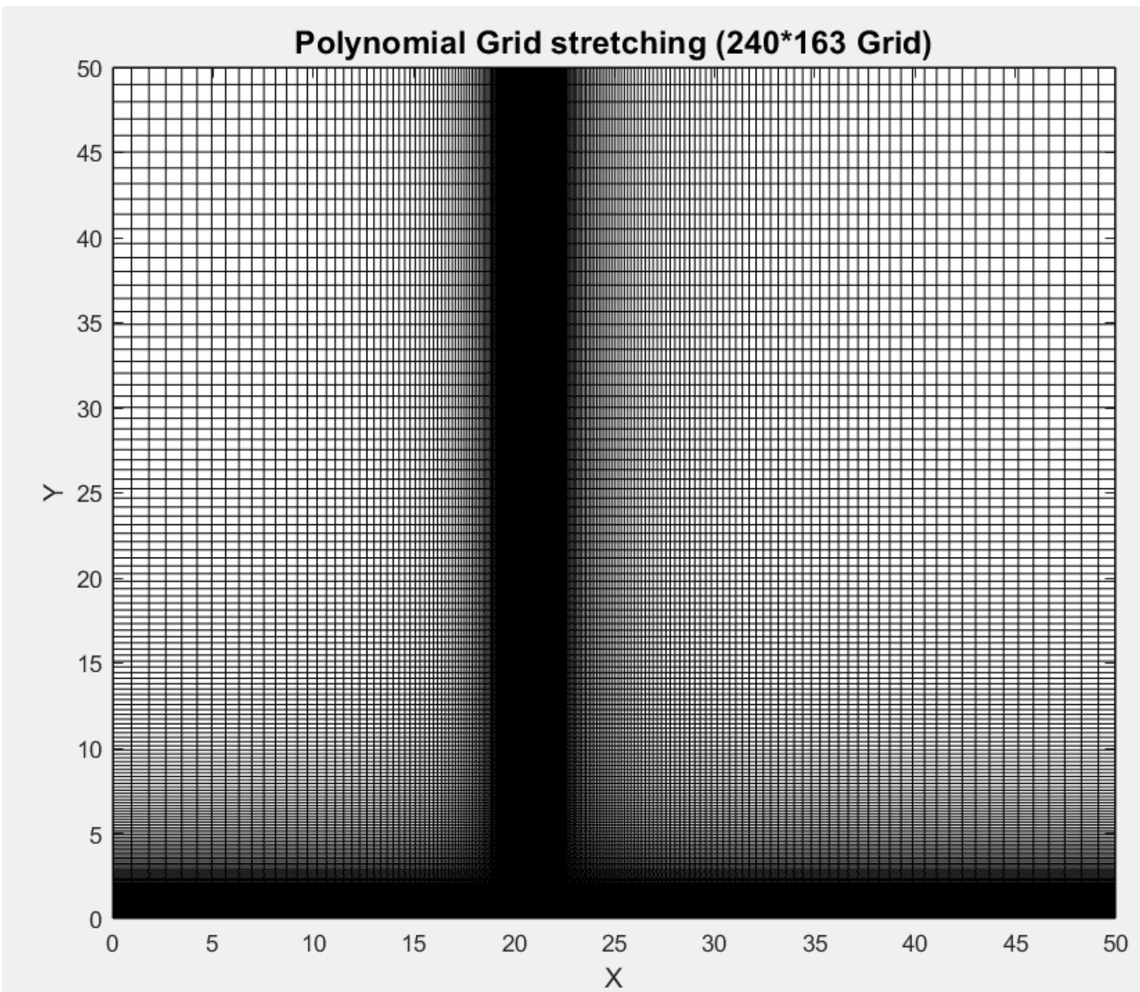


*Fig:1 – The Computational domain (50*50)*

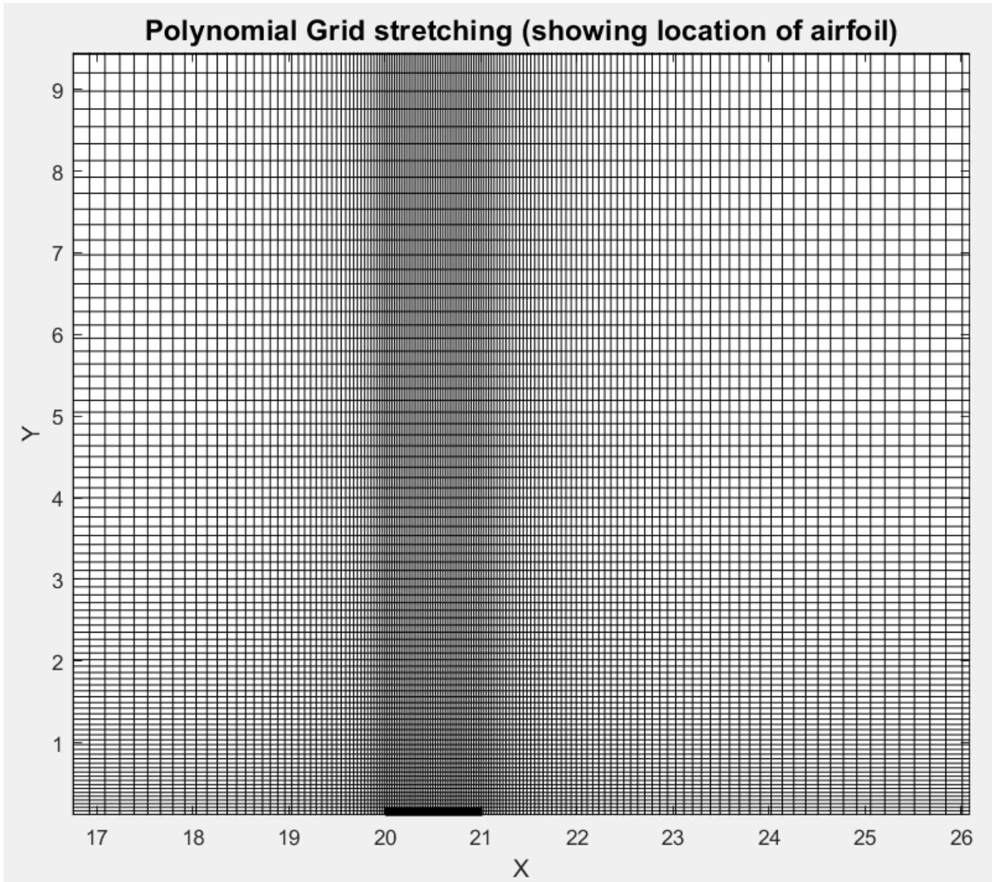


Fig:2 – The computational domain showing the location of the airfoil

Fig 1 shows the computational domain. Polynomial stretching is used to refine the mesh in the near-airfoil zones. Eq 1 shows the equation used to mesh the computational domain after the airfoil's trailing edge. Eq 2 shows the equation used to mesh the computational domain before the airfoil's leading edge. Eq 3 shows the equation used to mesh the computational domain in the y – direction.

$$x(i) = x(i-1) + \{x(i-1) - x(i-2)\} * 1.0286^{\wedge}1.05 \qquad - \quad \text{eq 1}$$

$$x(i-1) = x(i) + \{x(i+1) - x(i)\} * 1.0444^{\wedge}1.05 \qquad - \quad \text{eq 2}$$

$$y(i) = y(i-1) + \{y(i-1) - y(i-2)\} * 1.0186^{\wedge}1.1 \qquad - \quad \text{eq 3}$$

Problem 1- Ensure that your TSD code is working based on the expected solutions from Project # 3. Evaluate the pressure drag, Cdp for each Mach number specified in the original Project #3.

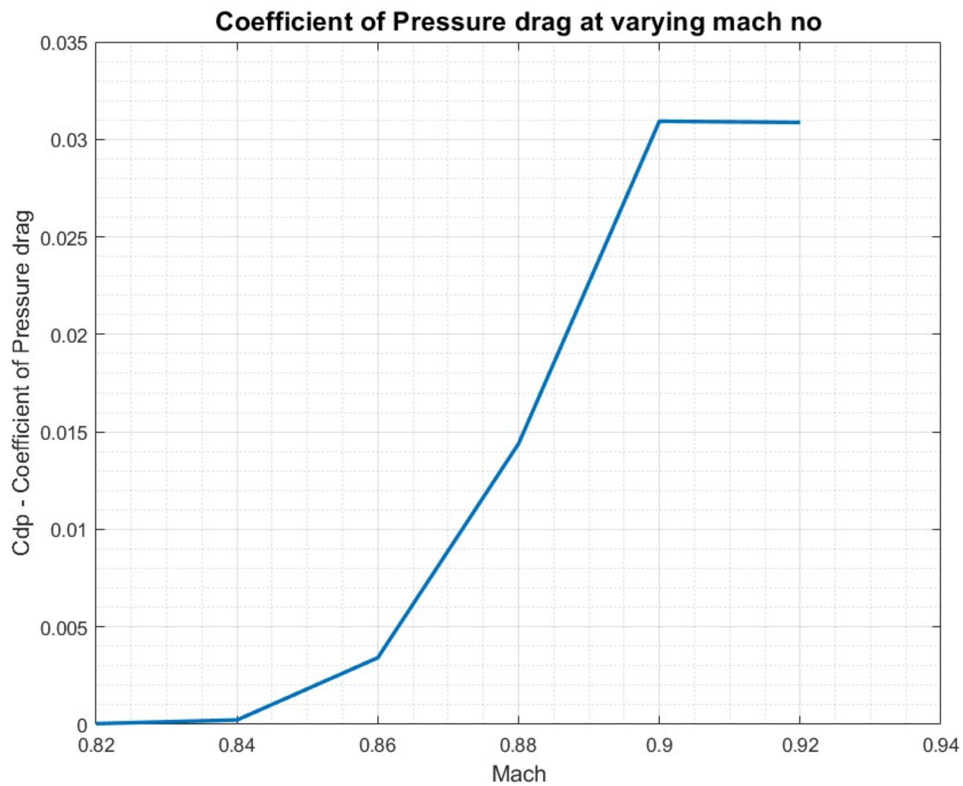


Fig:3 – Coefficient of Pressure at varying mach no (0.82:0.92) for t/c = 0.08

Discussion –

- Fig 3 shows the Coefficient of Pressure plot from TSD code at varying Mach no from 0.82 to 0.92.
- The Coefficient of Pressure increases from Ma – 0.82 till Ma – 0.9. Further increasing the mach no from 0.9 shows a fall in the Coefficient of Pressure.

Problem 2- Complete the Boundary Layer Code by (1) Complete the construction of the linear system matrix; specifically the entries for A, B, C, and D (2) Implement the boundary condition for the upper edge of the boundary, ue based on the output fom the TSD code (3) Implement an additional function to evaluate the displacement thickness at each streamwise location. [Note: ue, is the surface velocity from the TSD code.] Plot x versus ue.

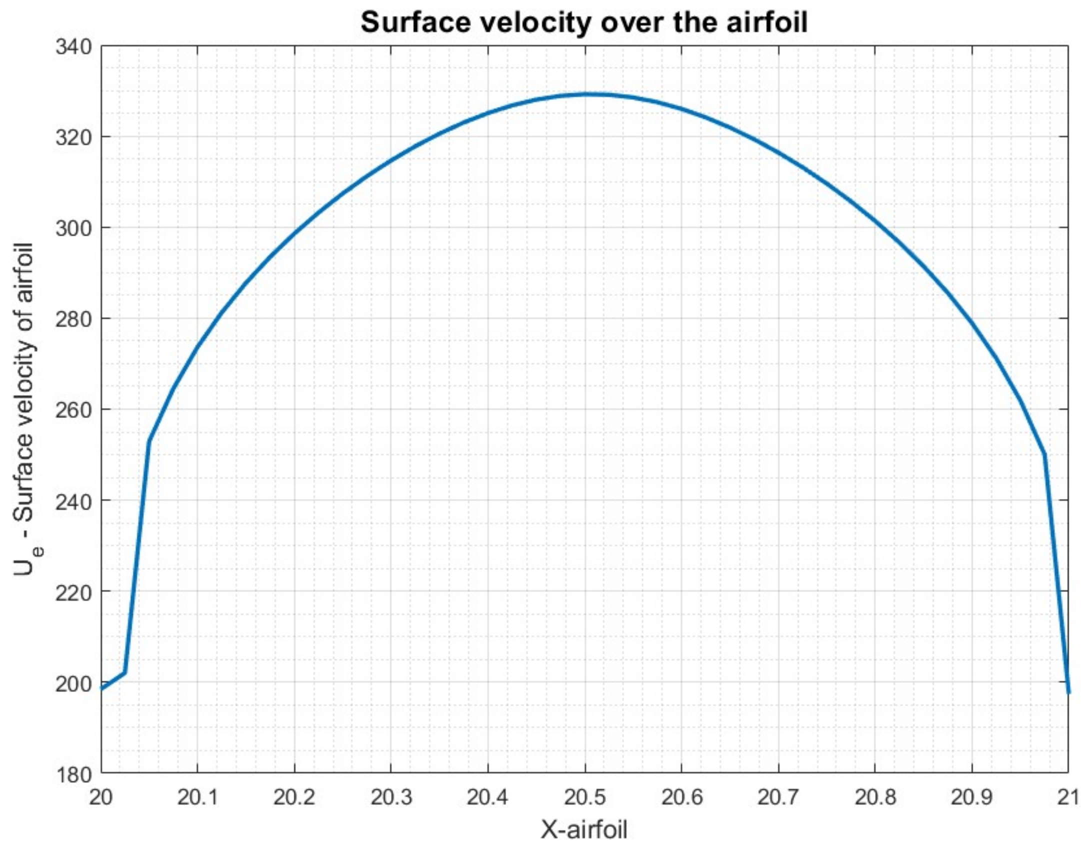


Fig:4 – Surface velocity over the airfoil from TSD code

Discussion –

- Fig 4 shows the Surface velocity over the airfoil from TSD code.

The following steps are taken to couple the Boundary layer code and the TSD code –

- The construction of the Liner system matrix; specifically the entries for A, B, C, and D. The A,B,C & D values are found out from the boundary layer equations.
- The boundary condition for the upper edge of the boundary, Ue based on the output from the TSD code.
- Implementation of an additional function to evaluate the displacement thickness at each streamwise location.

Problem 3- Couple the Boundary Layer Code and the TSD and perform three-coupled iterations between the two codes and report the following final results for a Reynolds number of 4 million.

a) Plot the boundary layer profile, velocity, u versus the y-coordinate at the 25%, 50% and 75% chord-wise stations and discuss your findings.

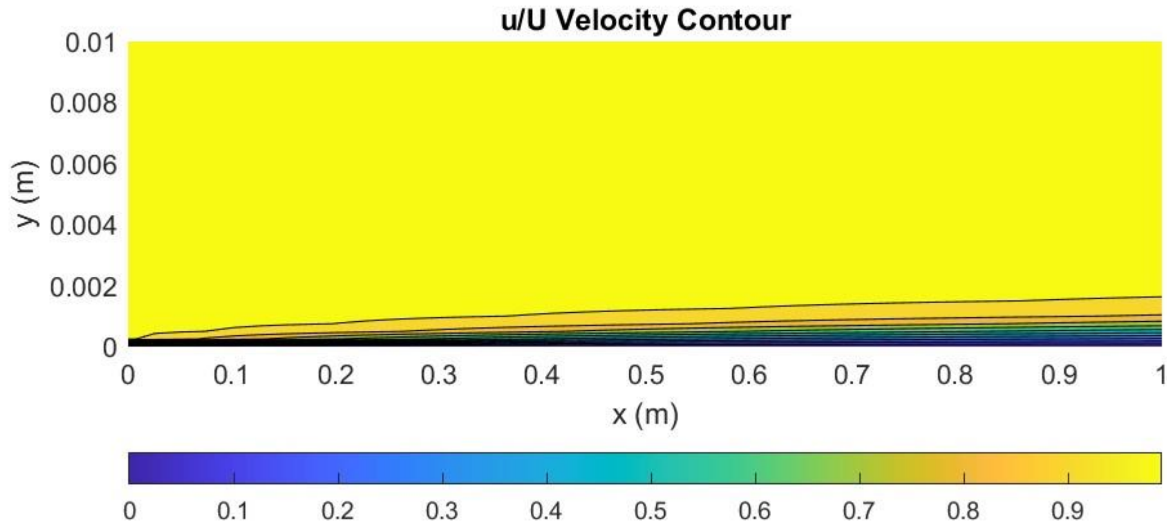


Fig:5 – Boundary Layer Profile over airfoil at Ma – 0.8

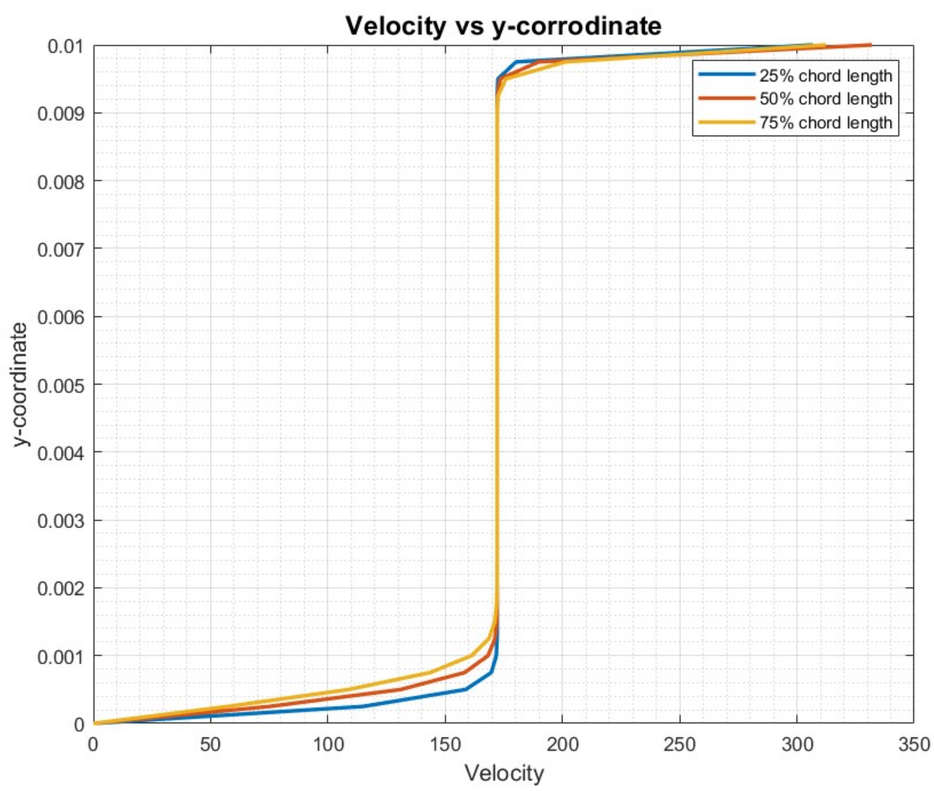


Fig:6 – Velocity (U vs Y-coordinate plot) over airfoil at Ma – 0.8 at different chord length

Disucssion –

- Fig 5 shows the boundary layer profile over the airfoil. The contour represents the velocity magnitude wrt to free stream velocity.
- Fig 6 shows the velocity plot along Y- axis at various chord length. The velocity gradually increases with increase in y, remains constant for further values of y and increases further to the magnitude of free-stream velocity.
- At the lower values of y, the rate of increase in magnitude of velocity is highest at 0.75 chord length and lowest at 0.25 chord length.

b) Plot the displacement thickness, $\delta *$, and skin friction coefficient, cf along the x-coordinate at three Mach numbers 0.80, 0.84, and 0.86 on the same plot and discuss your findings.

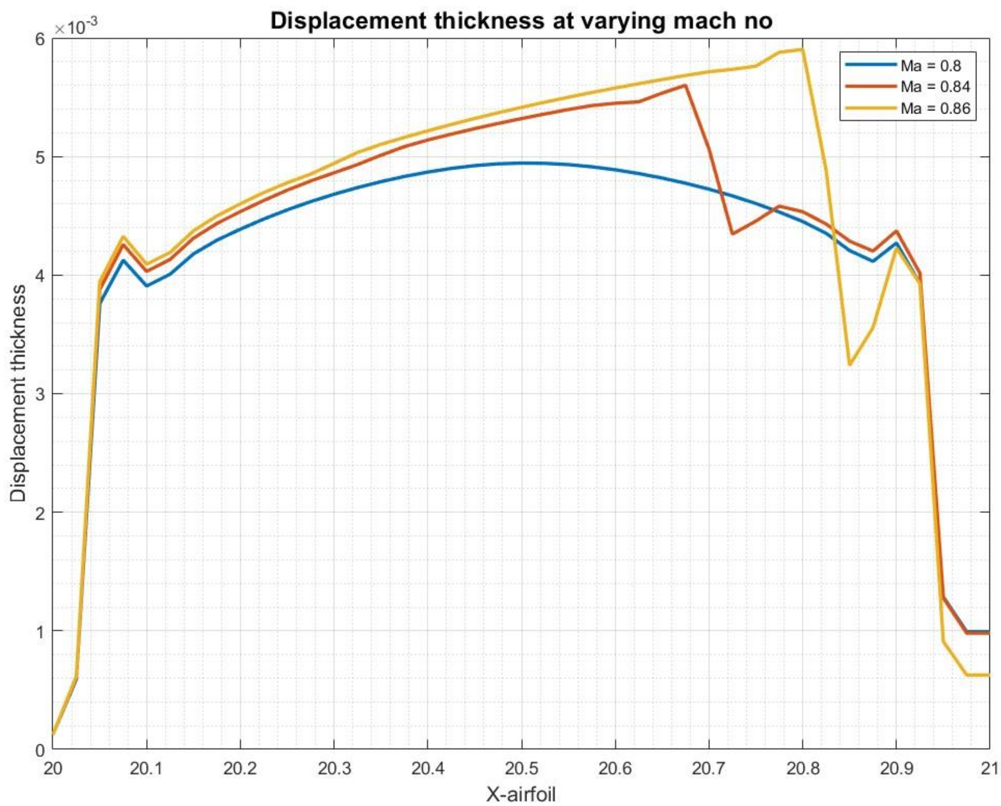


Fig:7 – Displacement thickness plot at varying Mach no

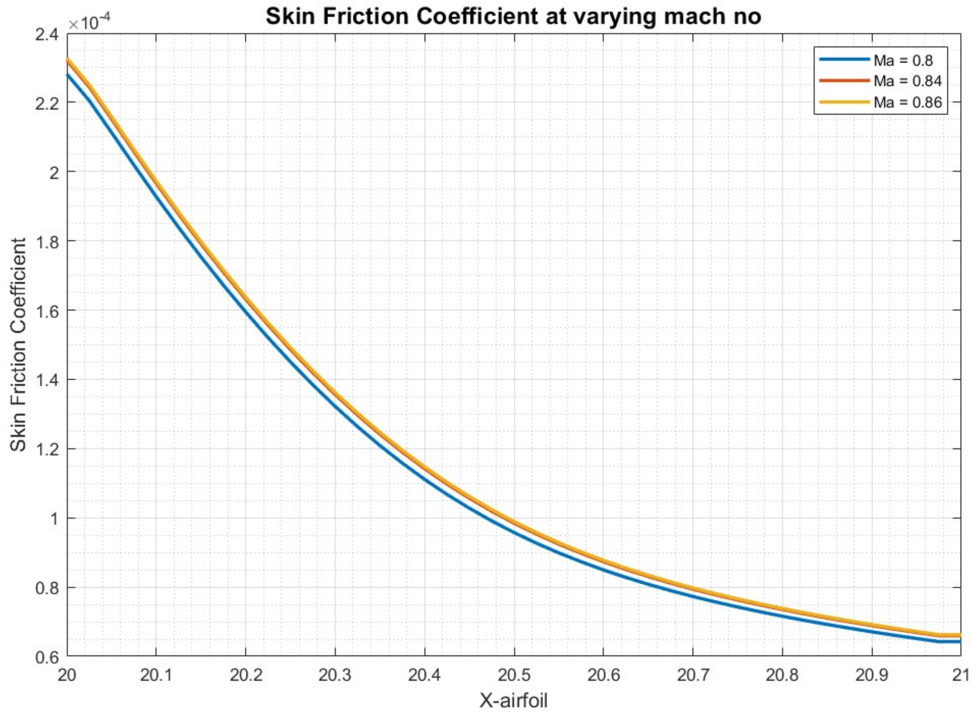


Fig:8 – Skin Friction Drag Coefficient plot at varying Mach no

Disucssion –

- Fig 7 shows the Displacement thickness plot at varying mach no over the airfoil. The displacement thickness value increases first till approx 0.5 chord length and then the value drcreases till the trailing edge. This trend of displacment thickness is seen at all Mach no.
- At higher mach no, we see a sharp drop in displacment thickness. The location of sharp decrease in the thickness is the location of shock wave.
- Fig 8 shows the Skin friction coefficient plot at varying mach no over the airfoil.
- The Cf value is higher at the leading edge and it continues to decrease along the airfoil. This trend is seen for all the Mach no cases.
- The overall difference between the magnitude of Cf at different mach no is very small.

c) Aerodynamic Coefficients: Evaluate the skin friction drag coefficient, cdf and lift coefficient clf at three Mach numbers 0.80, 0.84, and 0.86. Provide a table and list the total lift and drag coefficients, cl and cd as well as their individual contributions, clf , cdf , cdp , and clp .

Coefficients	Values at Ma – 0.8	Values at Ma – 0.8	Values at Ma – 0.8
Clf	0.0000047	0.00000482	0.000004825
Clp	0.1773	0.2142	0.2651
Cl = Clf+ Clp	0.1773	0.2142	0.2651
Cdf	0.0001413	0.0001691	0.0001737
Cdp	0.000009974	0.00055272	0.005
Cd = Cdf+ Cdp	0.000124	0.0006963	0.0051

d) Plot the pressure distribution from the original TSD code and the coupled TSD and Boundary Layer Code on the same plot at three Mach numbers 0.80, 0.84, and 0.86 and discuss your findings.

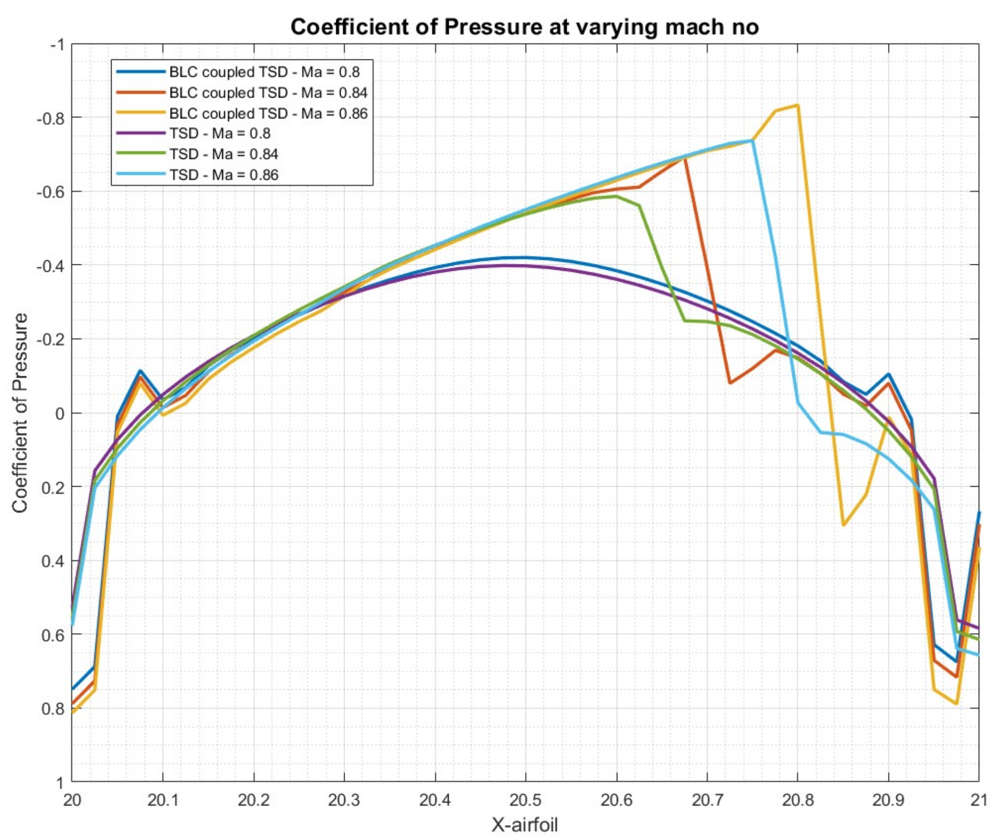


Fig:9 – Skin Friction Drag Coefficient plot at varying Mach no

Discussion –

- Fig 9 shows the Coefficient of Pressure distribution at different mach nos for both the TSD code and TSD coupled BLC.
- For mach no 0.8, the Cp curve is symmetric for both TSD and BLC coupled TSD. However a sharp change in values is seen at both the upstream and downstream side of the airfoil for BLC coupled TSD code.
- For mach 0.84 and 0.86, the effect of shock is clearly seen in both TSD and BLC coupled TSD. However the shock location is different in both the codes for the same mach no. This shows the effect of coupling BLC with TSD code.
- Fluctuations in the magnitude of Cp is seen at the downstream side of airfoil in the case of BLC coupled TSD.

e) Plot the displacement thickness, $\delta *$ after each iteration during the three-iteration coupled solution on the same plot and discuss your findings.

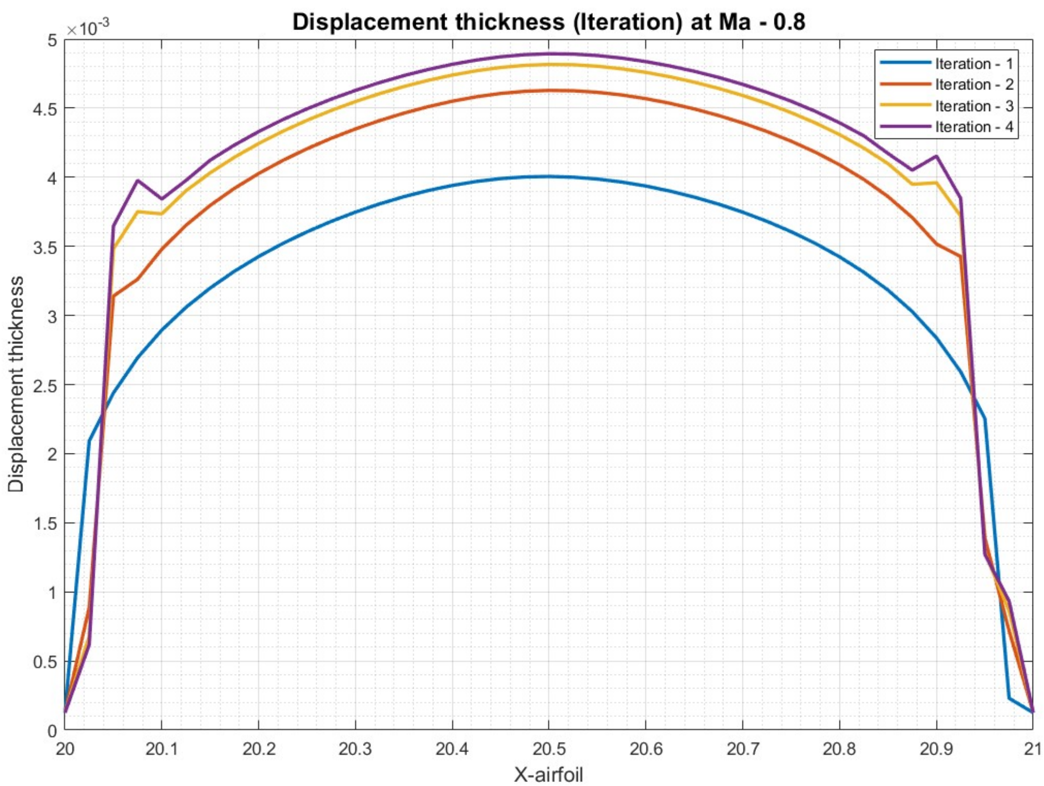


Fig:10 – Displacement thickness after each iteration at Mach no 0.8

Discussion –

- Fig 10 shows the Displacement thickness at each BLC coupled TSD iteration till it reach convergence. Similar to explanation in question 3b, the displacement thickness value increases first till approx 0.5 chord length and then the value drcreases till the trailing edge.
- With each iteration, the difference between the magnitude of Displacement thickness decreases. It can be seen from the plot. The overall magnitude increases with each iteration.
- The sharp fluctuation in the value at both the upstream and the downstream side is seen to be increasing with each iteration.

f) Plot the drag coefficients, cdf , cdp , and cd as a function of Mach number for the three Mach numbers stated above and discuss your findings.

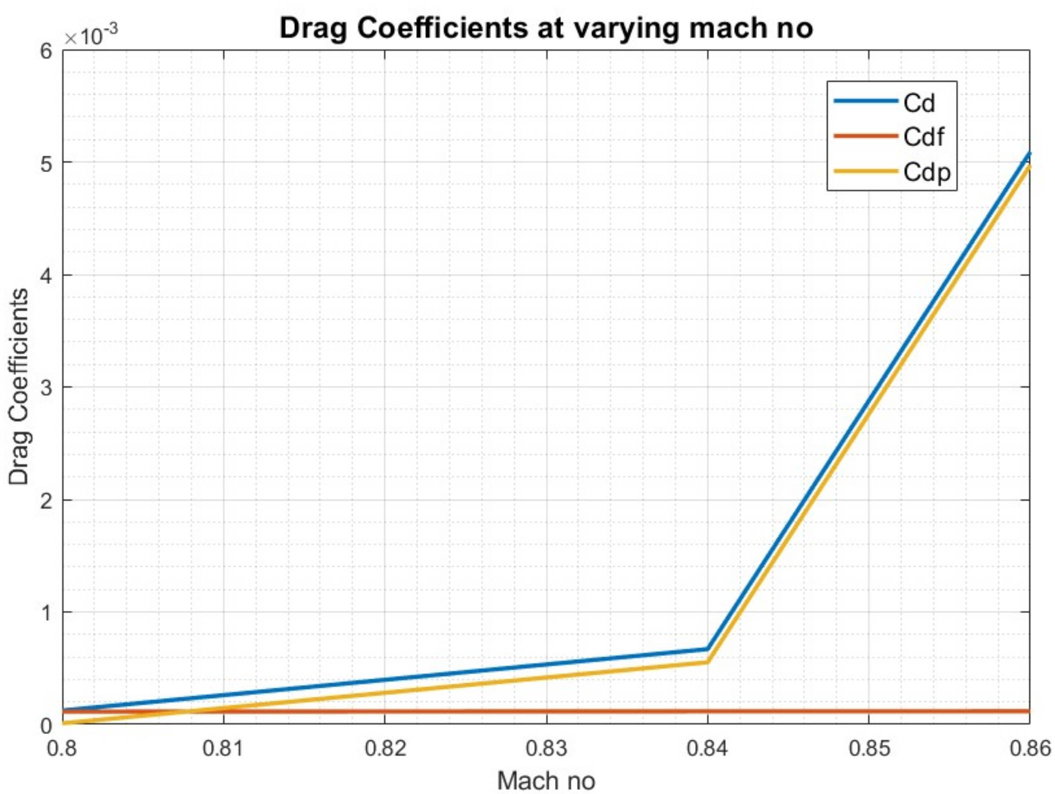


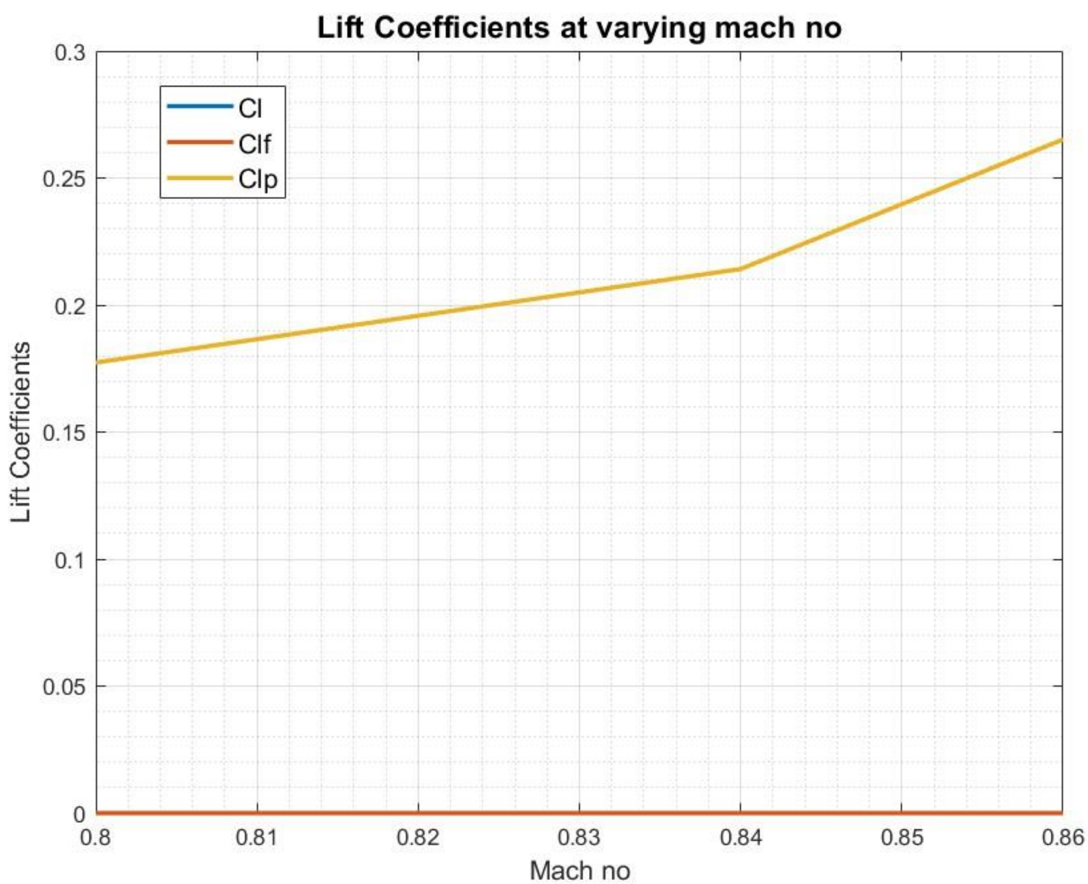
Fig:11 – Drag Coefficients plot at varying Mach no

Discussion –

- Fig 11 shows the Drag coefficients plot at varying Ma for the BLC coupled TSD code.

- As seen in the plot, the contribution of Pressure drag in the overall Drag value is negligible.
- The overall contribution in the Coefficient of drag comes from Skin fricition drag coefficient.
- The values of these drag coefficients can be seen in the table presented in question 3c.
- The overall value of drag coefficient increases with increase in the mach no. This can be clearly seen in the plot.

g) <u>Plot the lift coefficients, clf , clp , and cl as a function of Mach number for the three Mach numbers stated above and discuss your findings.</u>



<u>Fig:12 – Lift Coefficients plot at varying Mach no</u>

<u>Discussion –</u>

- Fig 12 shows the Lift coefficients plot at varying Ma for the BLC coupled TSD code.

- As seen in the plot, the contribution of skin friction component in the overall Coefficient of Lift is negligible.
- The overall contribution in the Coefficient of Lift comes from pressure component of Lift.
- The values of these drag coefficients can be seen in the table presented in question 3c.
- The overall value of Lift coefficient increases with increase in the mach no. This can be clearly seen in the plot.

h) Plot the drag polar, cl versus cd for the three Mach numbers and discuss your findings.

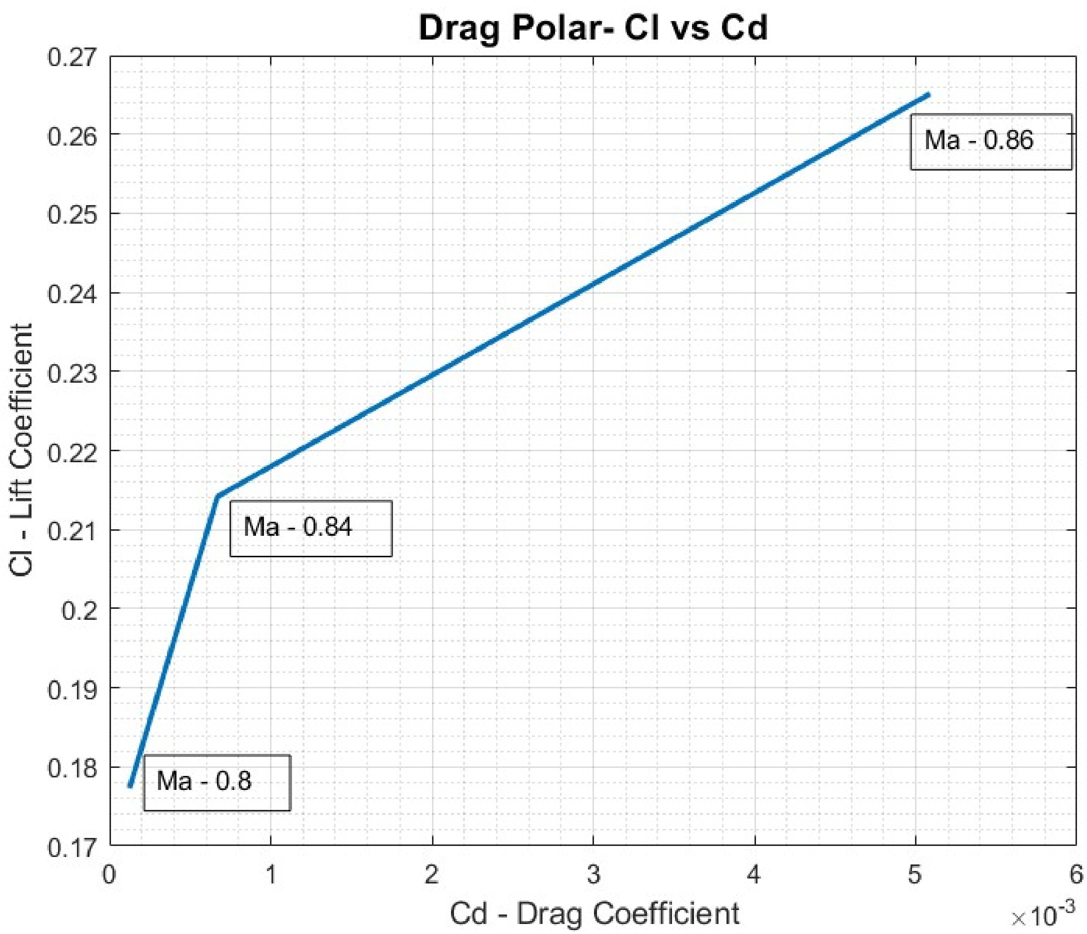


Fig:13 – Drag Polar (Cl vs Cd) plot at varying Mach no

Discussion –

- Fig 13 shows the Drag Polar plot of the circular arc airfoil at varying Ma for the BLC coupled TSD code.
- Drag Polar is an important tool in the design optimization of an aircraft. It provides a lot of data regarding the overall performace of the wing.

- In this case, the angle of attack is 0. As presented in the plot, both the Cd value and Cl value increases with increase in Mach no .
- The change in Cl value from Ma – 0.8 to Ma – 0.84 is pretty large when compared to that in Cd.