Project 4: Solve the Quasi-One-Dimensional Euler Equations

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Solve the quasi-1D Euler equations for various finite-volume schemes.

Problem 1 - Solve the quasi one-dimensional Euler equations using a scalar dissipation scheme for the spatial discretization, and a simple Euler explicit scheme for the temporal discretization if the exit static pressure, pexit = 0.8pt . Discretize the nozzle with 50 points. Show plots of the convergence of the density residual, pressure distribution across the channel, and Mach number distribution

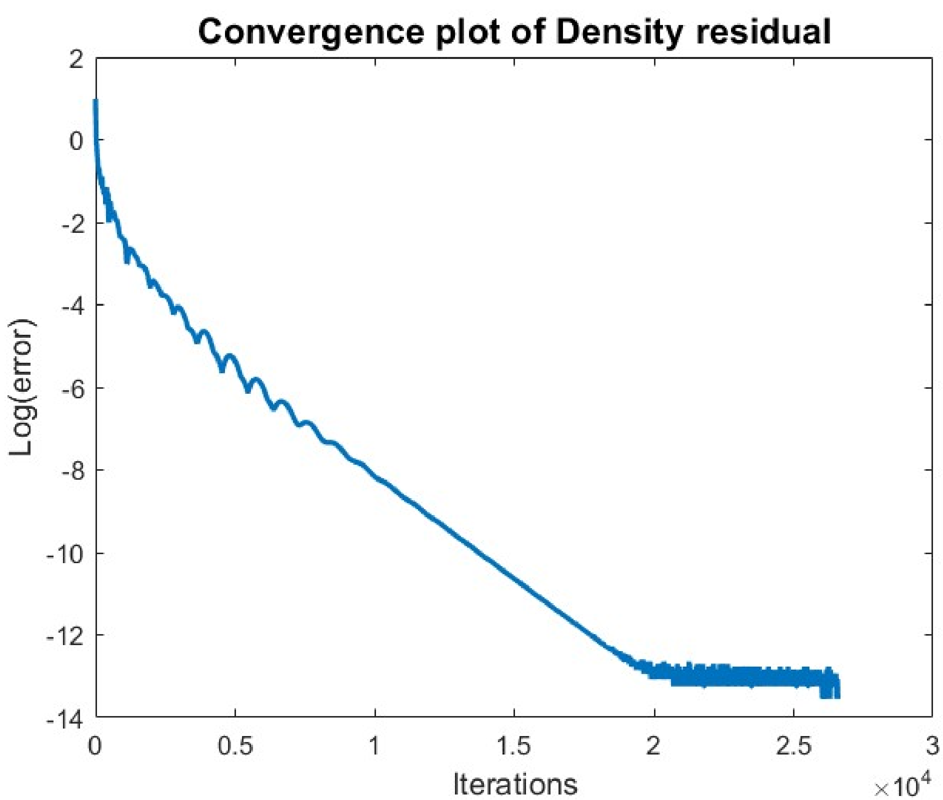


Fig:1 – Density Residual Plot (CFL = 0.3, Eps – 0.2)

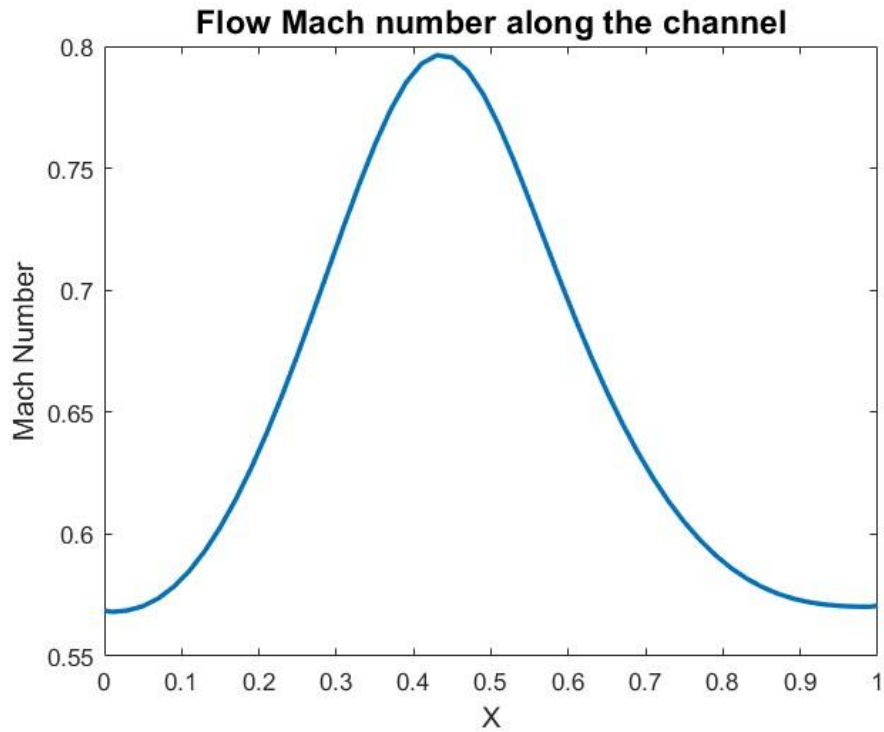


Fig:2 – Flow Mach number Plot (CFL = 0.3, Eps – 0.2)

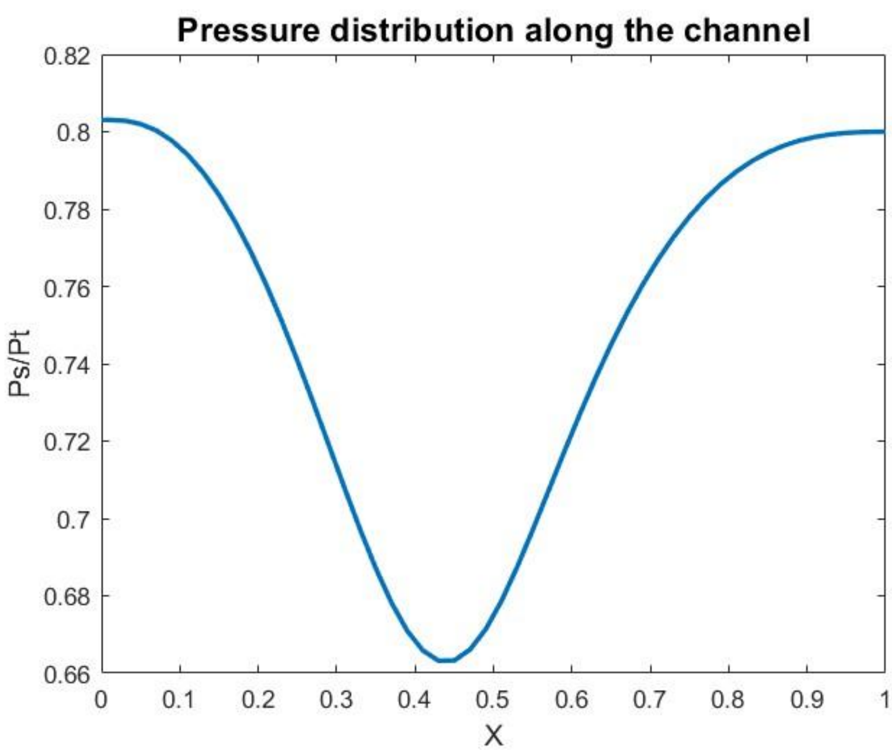
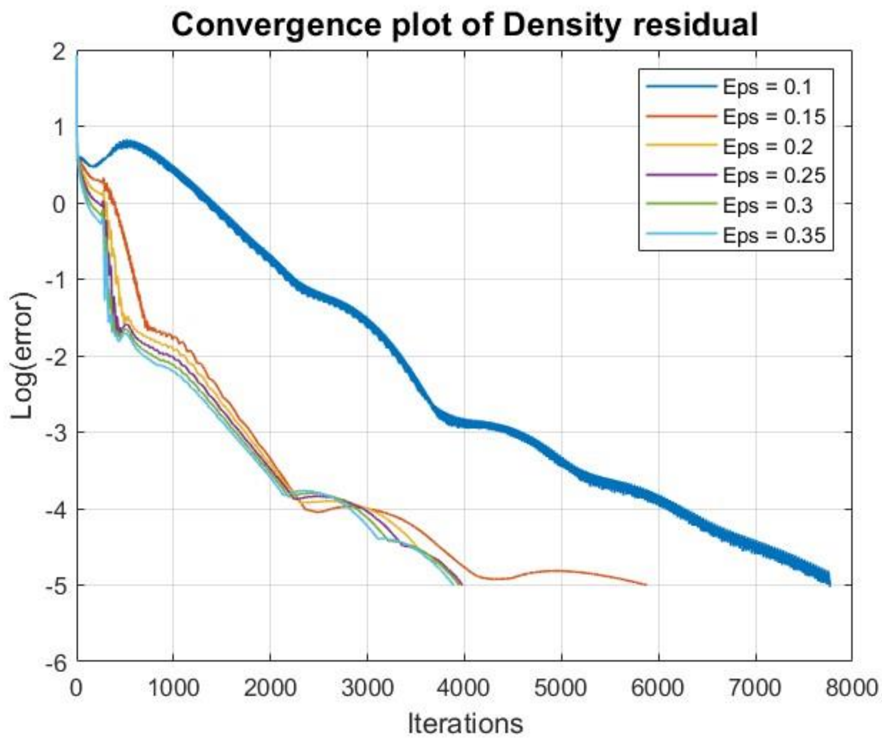


Fig:3 – Pressure ratio Plot (CFL = 0.3, Eps – 0.2)

<u>Problem 2- Exit pressure study</u> - Solve the quasi one-dimensional Euler equations using the Scalar dissipation scheme for four exit static pressure to inlet total pressure ratios of 0.76, 0.72, 0.68, and 0.60 using 50 grid points. Tweek the value of ϵ for the scalar dissipation scheme until at least only two points are observed in the shock for the exit static pressure to inlet total pressure ratio of 0.76. Use the same value of ϵ value for the 0.72, 0.68, and 0.60 cases. Is the code stable at this value ϵ? If not, then what are the values for the other cases? For each exit pressure ratio, show the convergence of the density residual, pressure distribution across the channel, and Mach number distribution.



<u>Fig:4 – Density Residual Plot (CFL = 0.2, P exit = 0.76 P inlet)</u>*

<u>Discussion –</u>

- Fig 4, 5, 6 shows the density residual, Mach number and pressure ratio across the channel, respectively. It can be easily seen in fig 4, that higher epsilon value takes less number of iterations to converge.
- Fig 5 shows the Mach number plot across the nozzle. At higher value of eps, i.e., 0.35, higher dissipation effect is observed in the shock. However lower value of eps such as 0.1 shows the 2- point shock wave. Further lowering the value of eps makes the code unstable.
- The same can be observed in fig 6, which shows the pressure ratio across the channel.
- For the next pressure ratios, eps value of 0.1 is selected. For 0.72 pressure ration, the code is stable and the results are shown in Fig 7, 8, 9. However, for pressure ratio = 0.68, the code is unstable for eps = 0.1. For this, a higher value of eps = 0.12 is selected. The results for this pressure ration are shown in Fig 10, 11, 12.

- For pressure ratio = 0.6, the code becomes unstable for eps = 0.12. But increasing the eps value to 0.125 makes the code stable. The results for this pressure ration are shown in Fig 13, 14 & 15.

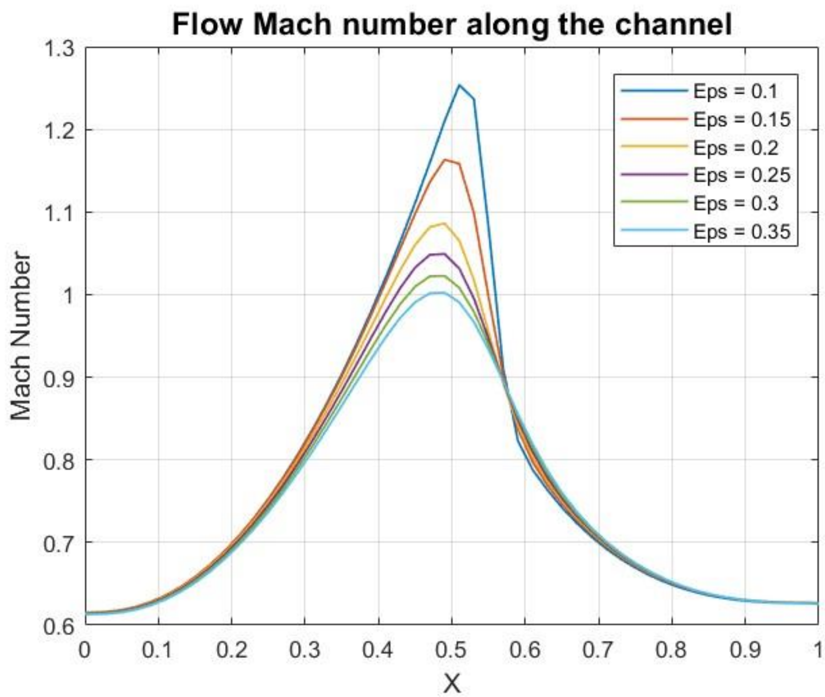


Fig:5 – Flow Mach Number Plot (CFL = 0.2, P exit = 0.76 P inlet)*

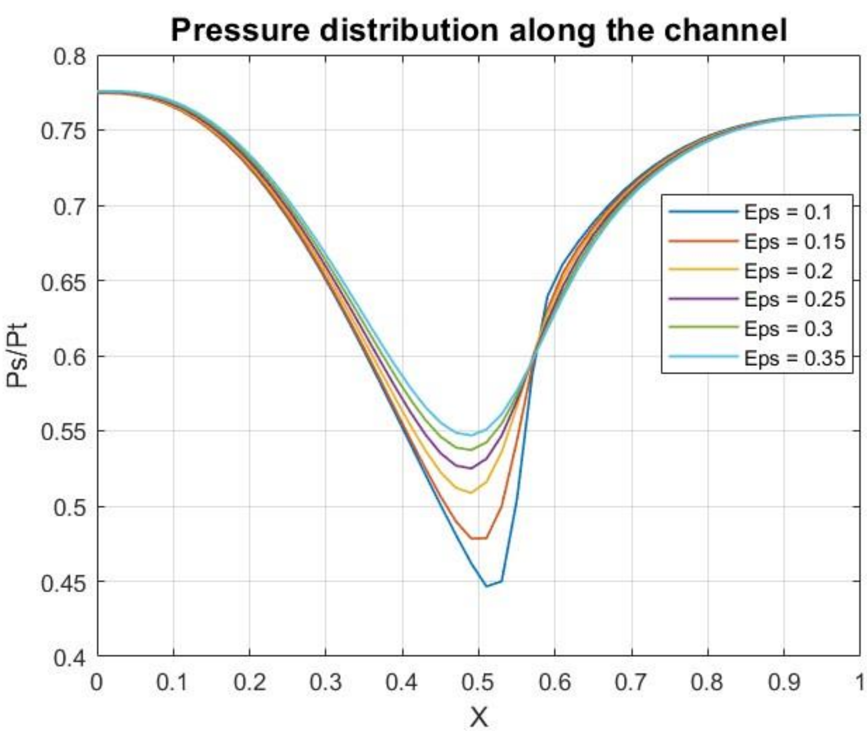


Fig:6 – Pressure ratio Plot (CFL = 0.2, P exit = 0.76 P inlet)*

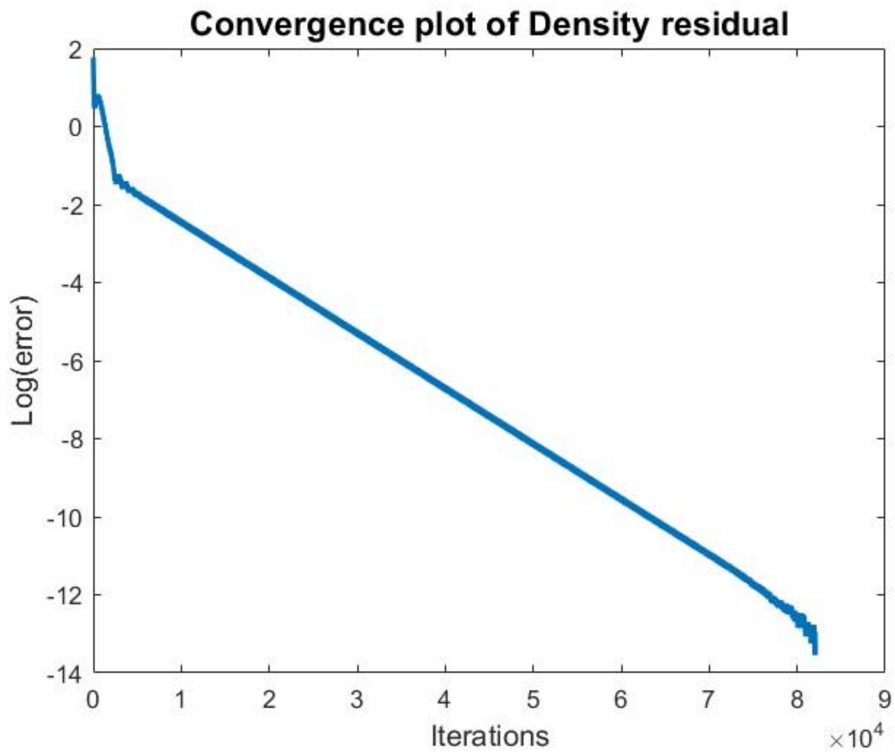


Fig:7 – Density Residual Plot (CFL = 0.2, P exit = 0.72 P inlet, Eps = 0.1)*

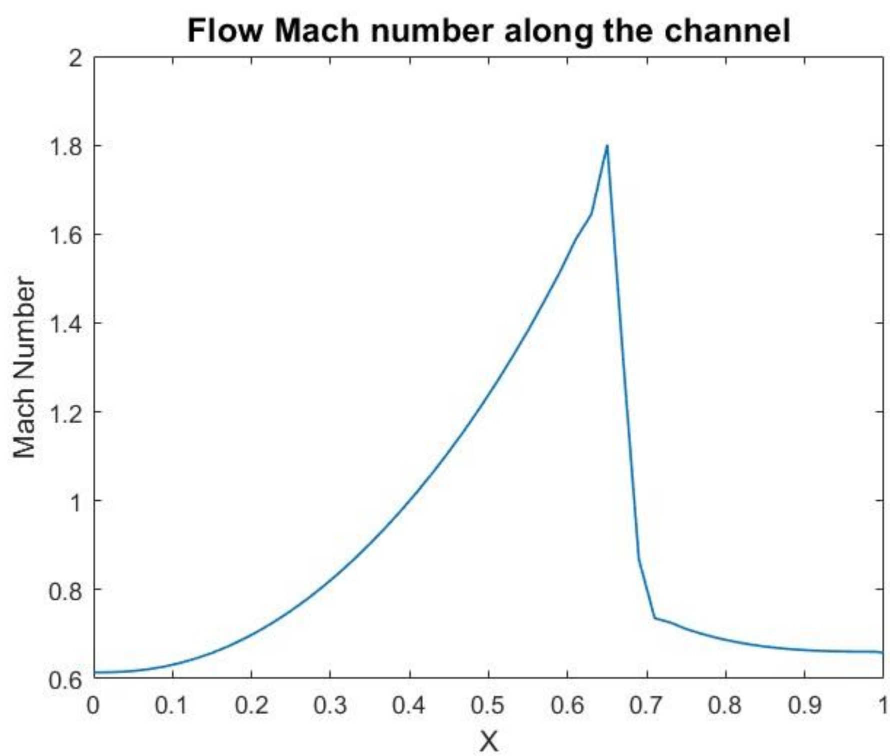


Fig:8 – Flow Mach number Plot (CFL = 0.2, P exit = 0.72 P inlet, Eps = 0.1)*

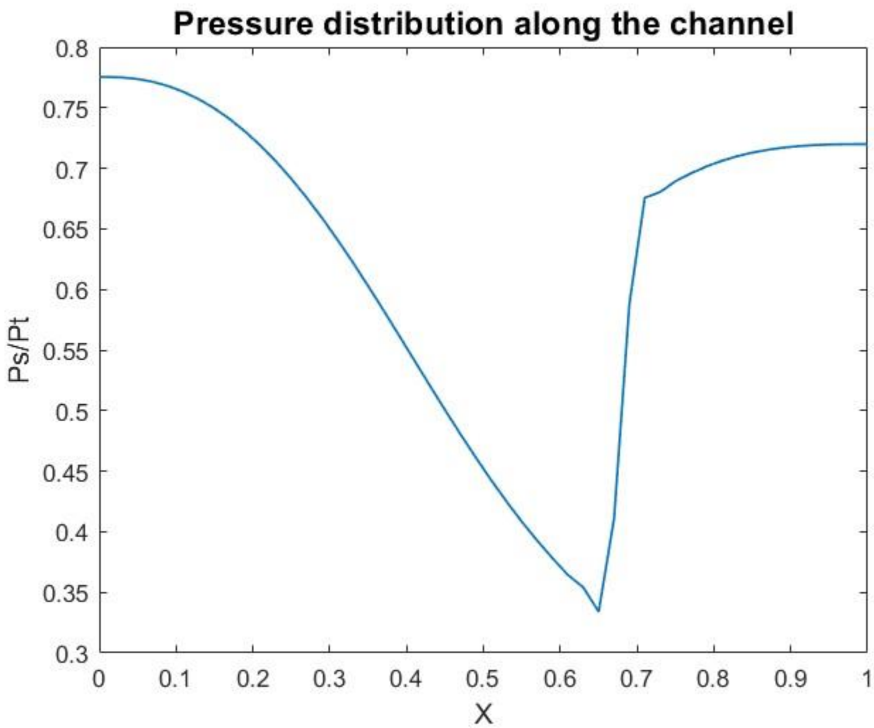


Fig:9 – Pressure ratio Plot (CFL = 0.2, P exit = 0.72 P inlet, Eps = 0.1)*

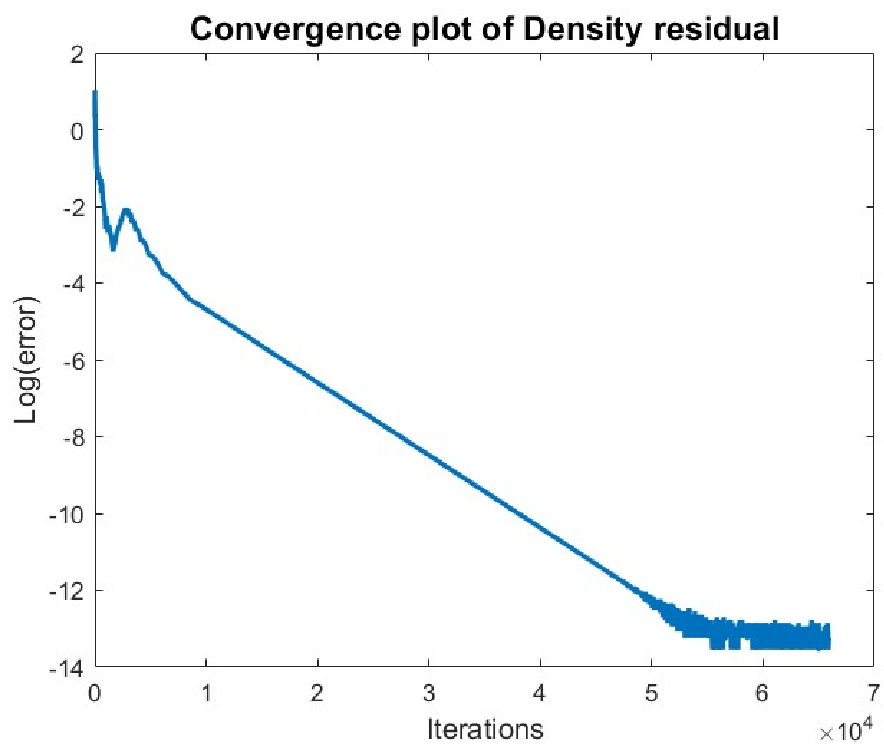


Fig:10 – Density Residual Plot (CFL = 0.2, P exit = 0.68 P inlet, Eps = 0.12)*

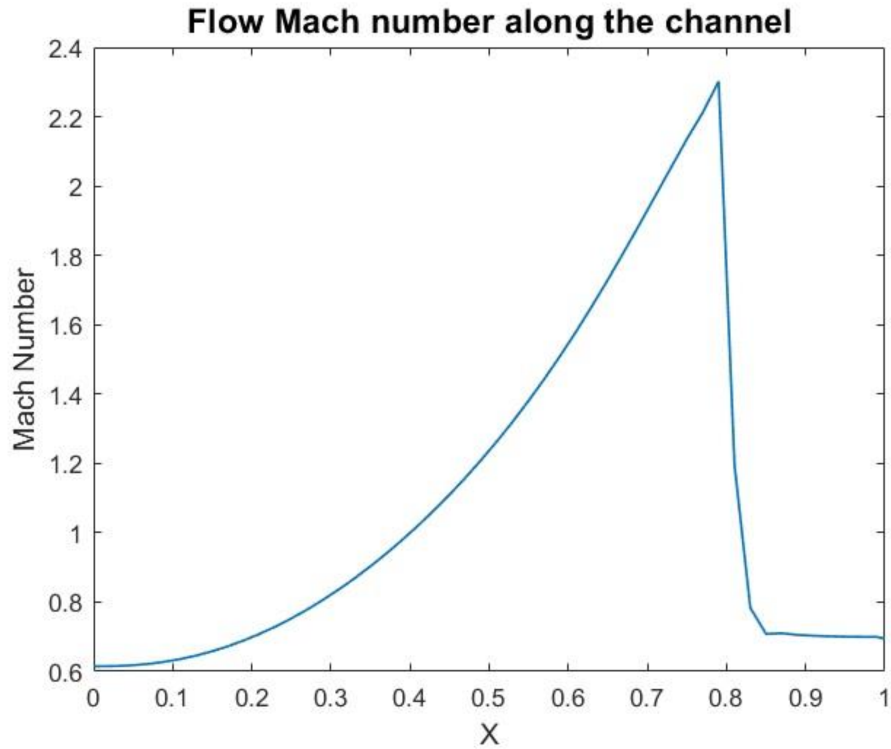


Fig:11 – Flow Mach number Plot (CFL = 0.2, P exit = 0.68 P inlet, Eps = 0.12)*

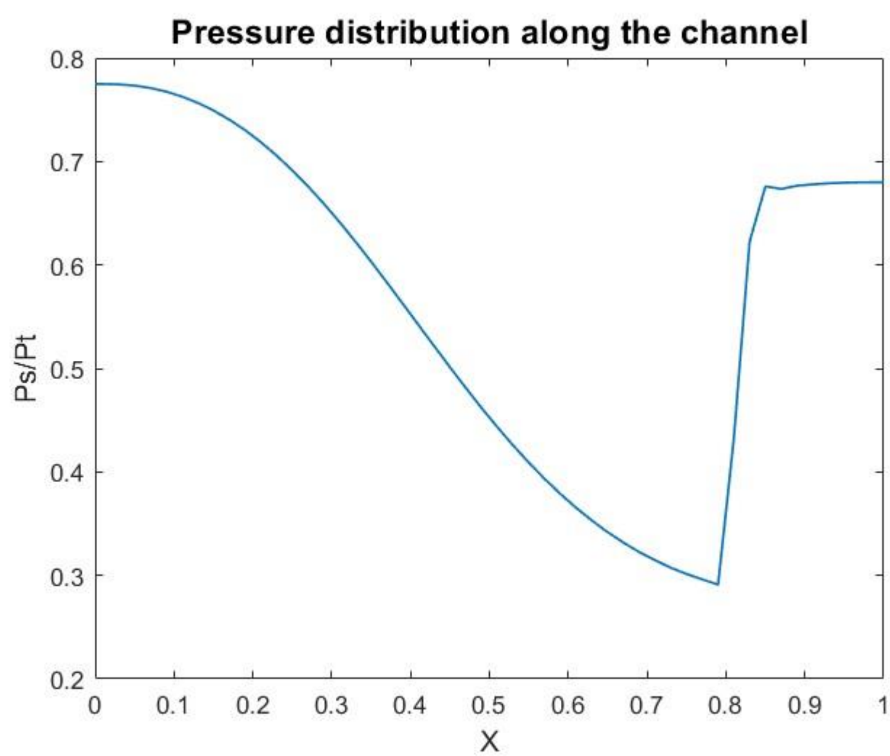


Fig:12 – Pressure ratio Plot (CFL = 0.2, P exit = 0.68 P inlet, Eps = 0.12)*

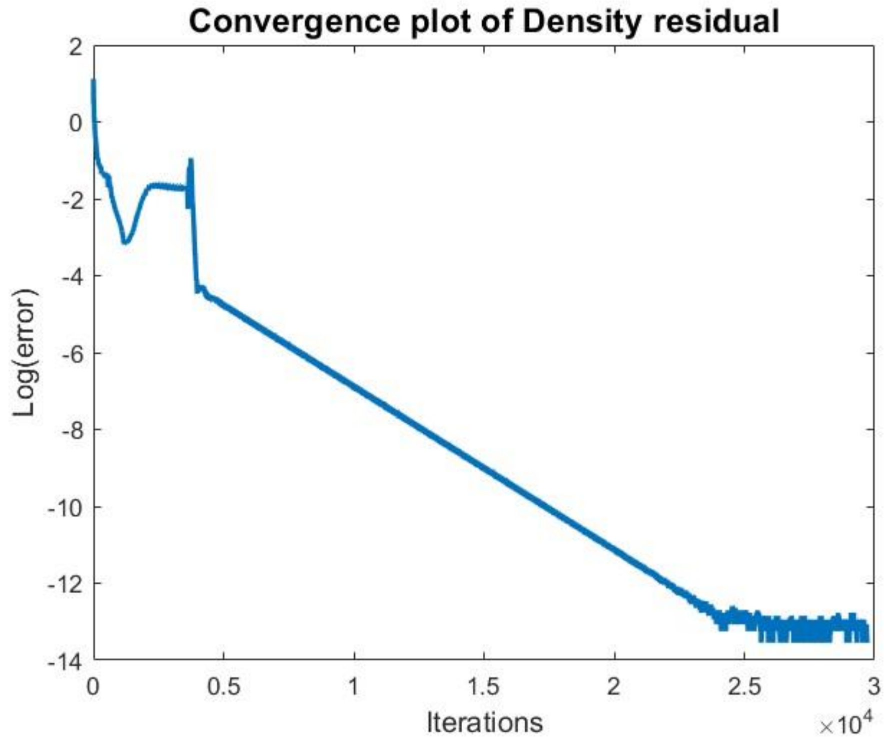


Fig:13 – Density Residual Plot (CFL = 0.2, P exit = 0.6 P inlet, Eps = 0.125)*

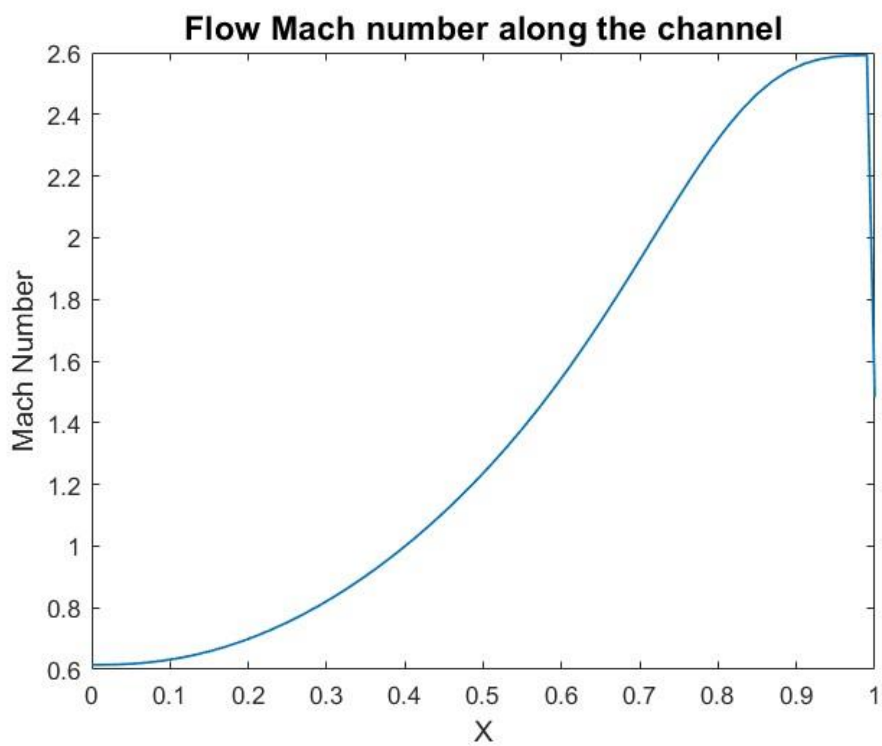


Fig:14 – Flow Mach number Plot (CFL = 0.2, P exit = 0.6 P inlet, Eps = 0.125)*

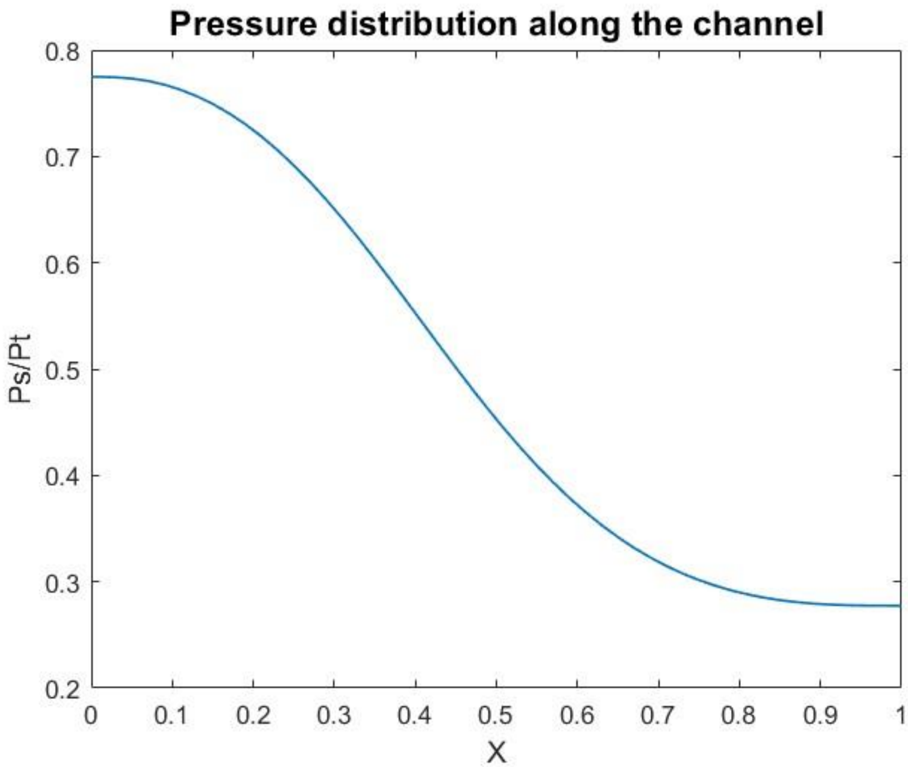


Fig:15 – Pressure ratio Plot (CFL = 0.2, P exit = 0.6 P inlet, Eps = 0.125)*

Discussion –

- For the pressure ratio = 0.6, the shock appears very close to the end of the nozzle. This can be seen in Fig 14 which shows the Mach number across the nozzle length.
- In fig 15, we can not see the shock, as it is really close to the outlet of the nozzle. However it is seen in fig 14.

Problem 3- Grid study. Solve the equation for an exit pressure ratio of 0.72 for 25, 50, 100, and 200 grid points. Comment on the value of ϵ. Is there a normal shock in the channel? Is the location of the shock the same for all four grid sizes. Explain your answer. For each grid, show the convergence of the density residual as a function of the number of iterations, pressure distribution across the channel, and Mach number distribution on the same plot. Discuss your findings by comparing the solutions. Does it require the same number of iterations to converge the answer for each grid? Explain your answer. Evaluate the total pressure loss by comparing the values at the inlet and the outlet. Compare the the total pressure loss as you refine the grid.

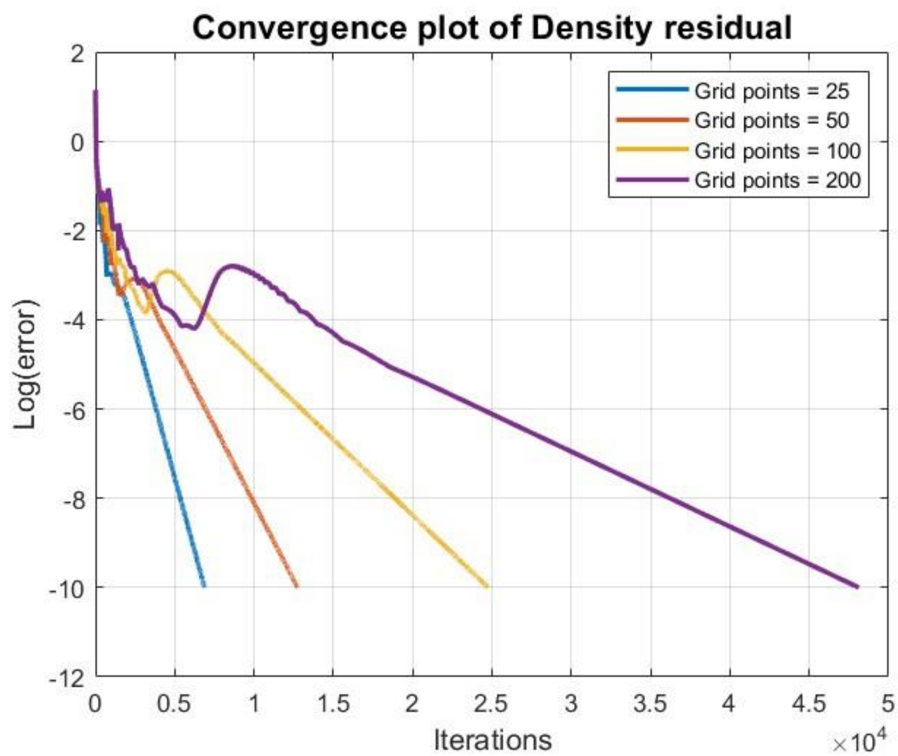


Fig:16 – Density Residual Plot (CFL = 0.3, P exit = 0.72 P inlet, Eps = 0.2)*

Discussion -

- Fig 16, 17, & 18 shows the density residual, Mach number and pressure ratio across the channel, respectively. . It can be easily seen in fig 16, the grid with more number of points takes more iterations to converge. The value of eps is chosen as 0.2 for the grid study.
- All the four grids shows the normal shock in the channel. The shock location remains same for all the four grids. It can be clearly seen in Fig 17 – Flow Mach number plot across the channel.

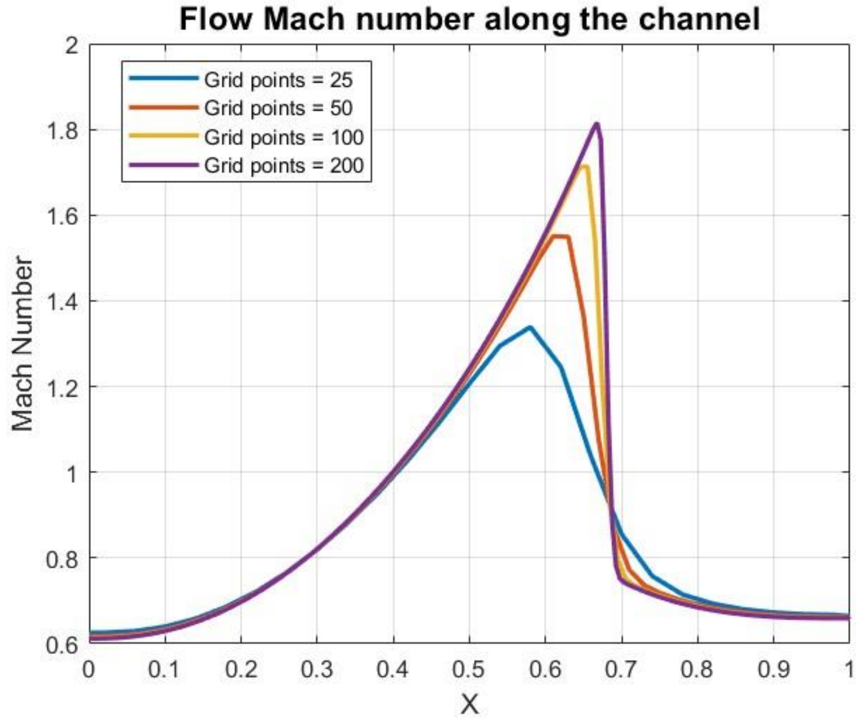


Fig:17 – Flow Mach number Plot (CFL = 0.3, P exit = 0.72 P inlet, Eps = 0.2)*

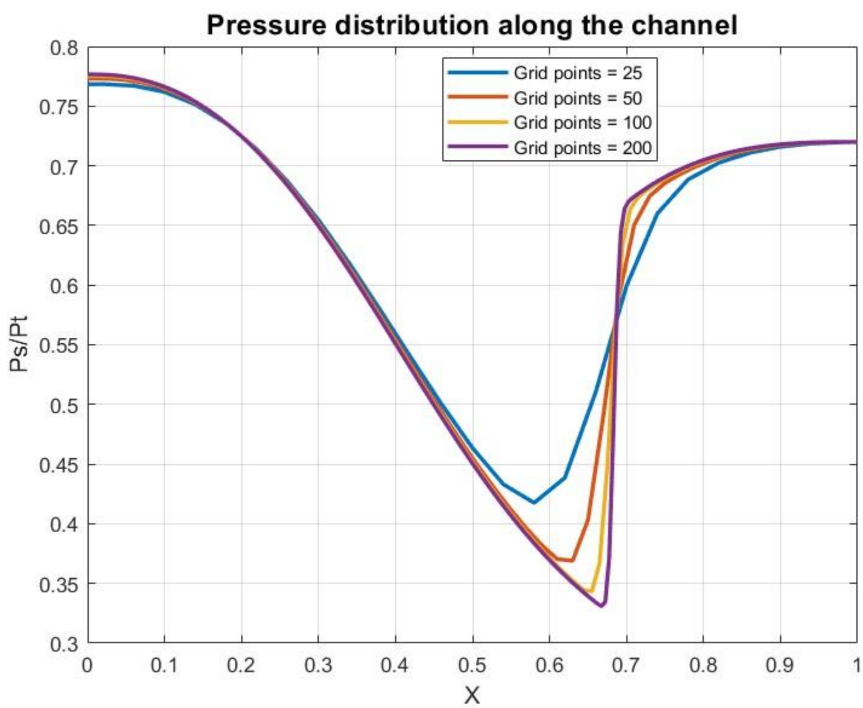


Fig:18 – Pressure ratio Plot (CFL = 0.3, P exit = 0.72 P inlet, Eps = 0.2)*

- The coarse grid (25 points) shows the shock with higher dissipation. As the Grid points in the computational domain increases, the dissipative nature decreases and at grid (200 points), a sharp shock front can be seen in Fig 17 & Fig 18.
- The total pressure loss comparison is presented in the table given below for all the four computational grid. It can be seen in the total pressure loss column, that as the grid size is increased, the total pressure loss across the inlet and outlet increases. The highest pressure loss is observed in 200 grid.

Total Pressure Loss Comparison -

Grid points	**Pressure Inlet**	**Pressure outlet**	**Total Pressure loss**
25	77879.27 Pa	72981 Pa	4898.27 Pa
50	78360.96 Pa	72981 Pa	5379.96 Pa
100	78599.68 Pa	72981 Pa	5618.68 Pa
200	78717.21 Pa	72981 Pa	5736.21 Pa