### Task 1. Numerical experiments with a TVD scheme for the Euler equations

Incorporate a second-order in space and time (on smooth solutions) TVD scheme into the Euler code you have developed for mini-project 2. The problem to be simulated is the same as in the mini-project 2.

**MUSCL-Hancock scheme**

Computational grid = $[-50, 50]$, $dx$ - 1, Time grid = $[0, 25]$, $dt$ - 0.5

**Exercise 1-1. Comparison of solutions obtained with and without a slope limiter.**

In the first question, I have used $\beta$ limiter other than the two cases of arithmetic average of slope and zero slope. As seen in Fig. 1, since it is a first-order scheme, the result shows high numerical dissipation.

**Zero Slope - First order version**

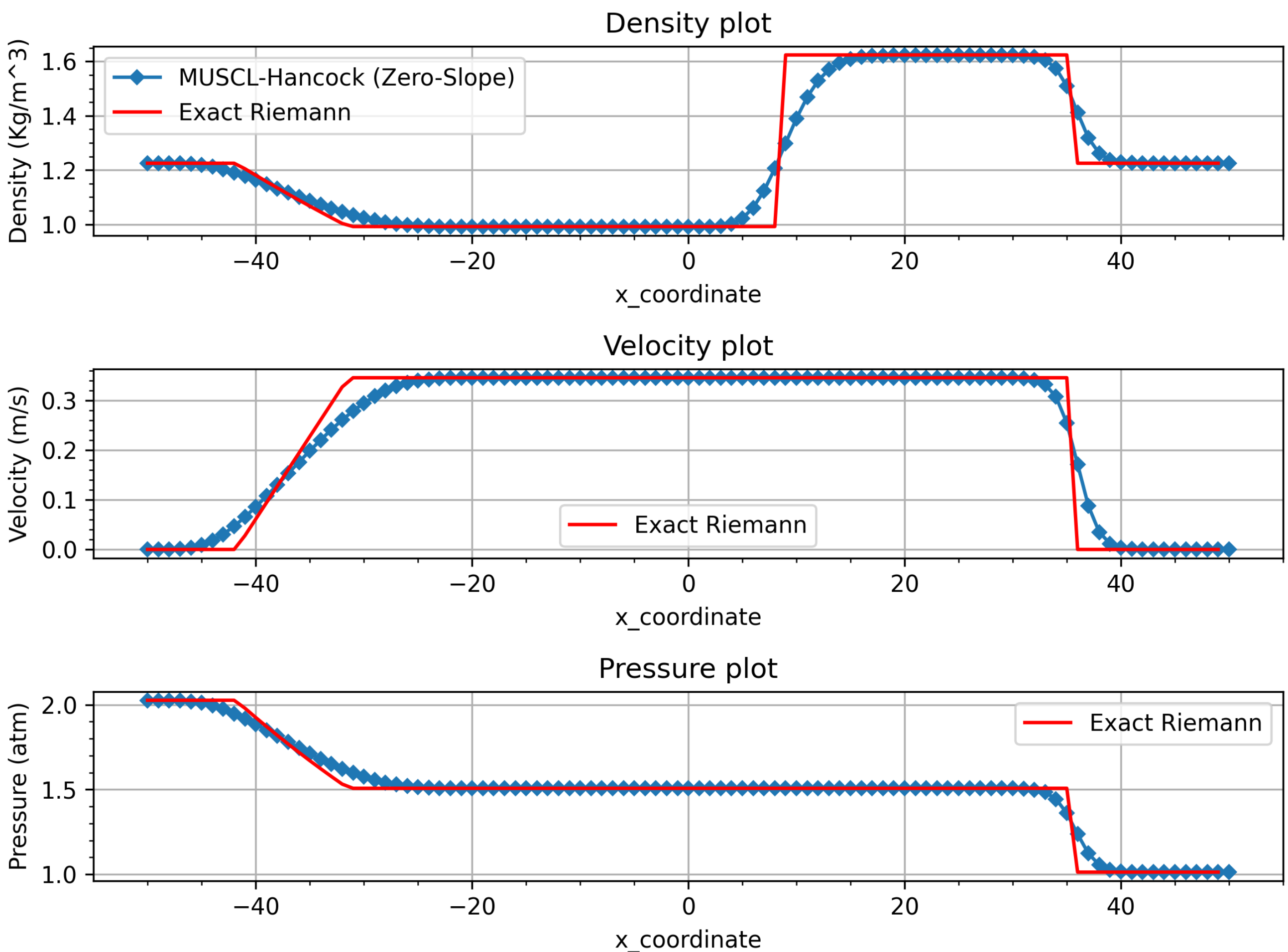


Figure 1: Density, Velocity, and Pressure comparison plot with zero slope

**Arithmetic average of adjacent slope**

Fig. 2 shows the results for the arithmetic average of adjacent solve used as a slope limiter. This method shows some amount of dispersion, but the numerical dissipation is absent near the shock. However, numerical dissipation is present near the contact surface as it can be seen in the density plot.

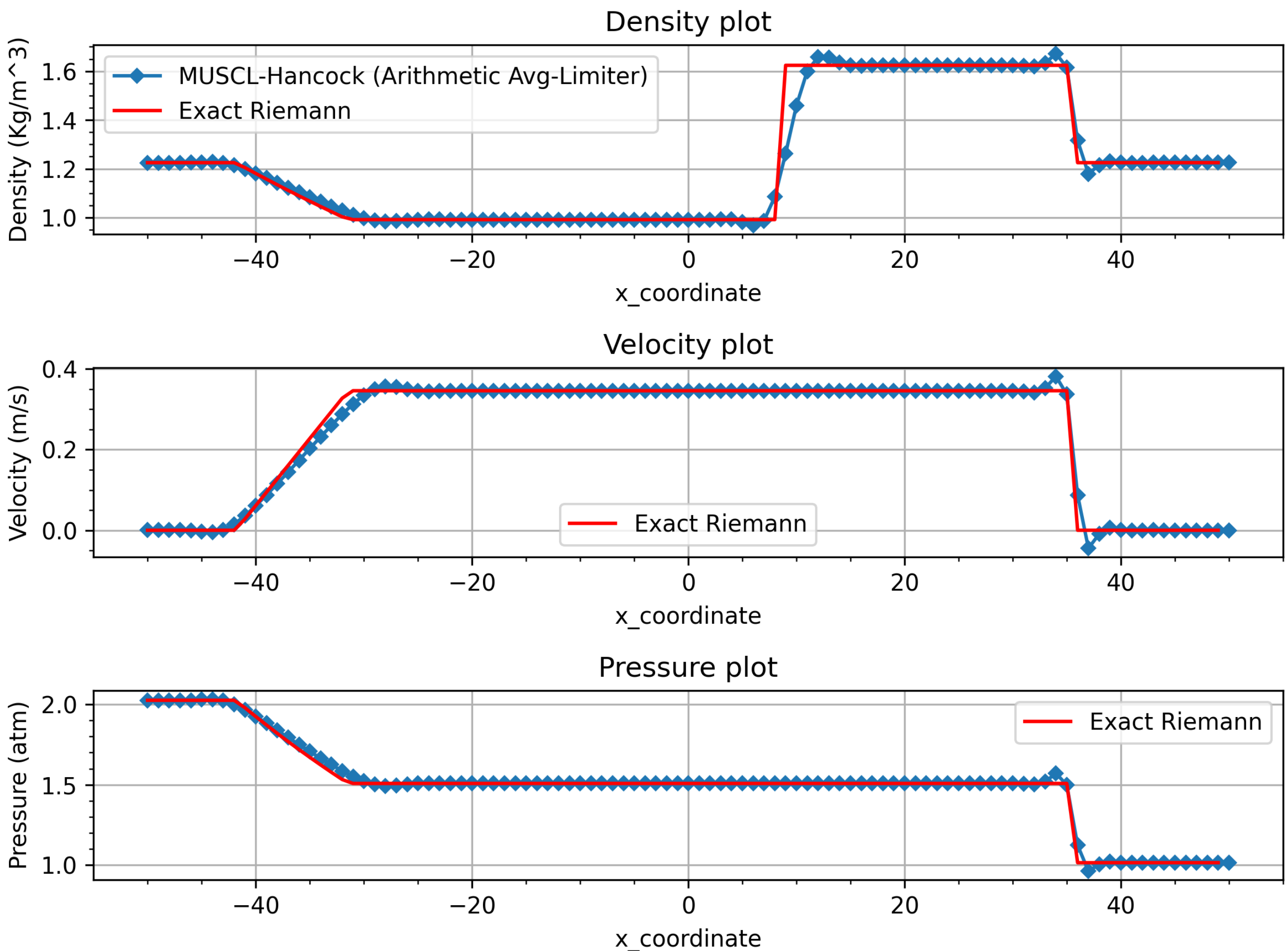


Figure 2: Density, Velocity and Pressure comparison plot with Arithmetic average of adjacent slope

**$\beta$ Limiter**

Fig. 3 shows the MUSCL-Hancock scheme with $\beta$ limiter. As seen from the plot, the result accuracy is better than the zero-slope and arithmetic average slope. The method does not show any numerical dissipation or dispersion near the shock. However, the accuracy in predicting the contact surface is less.

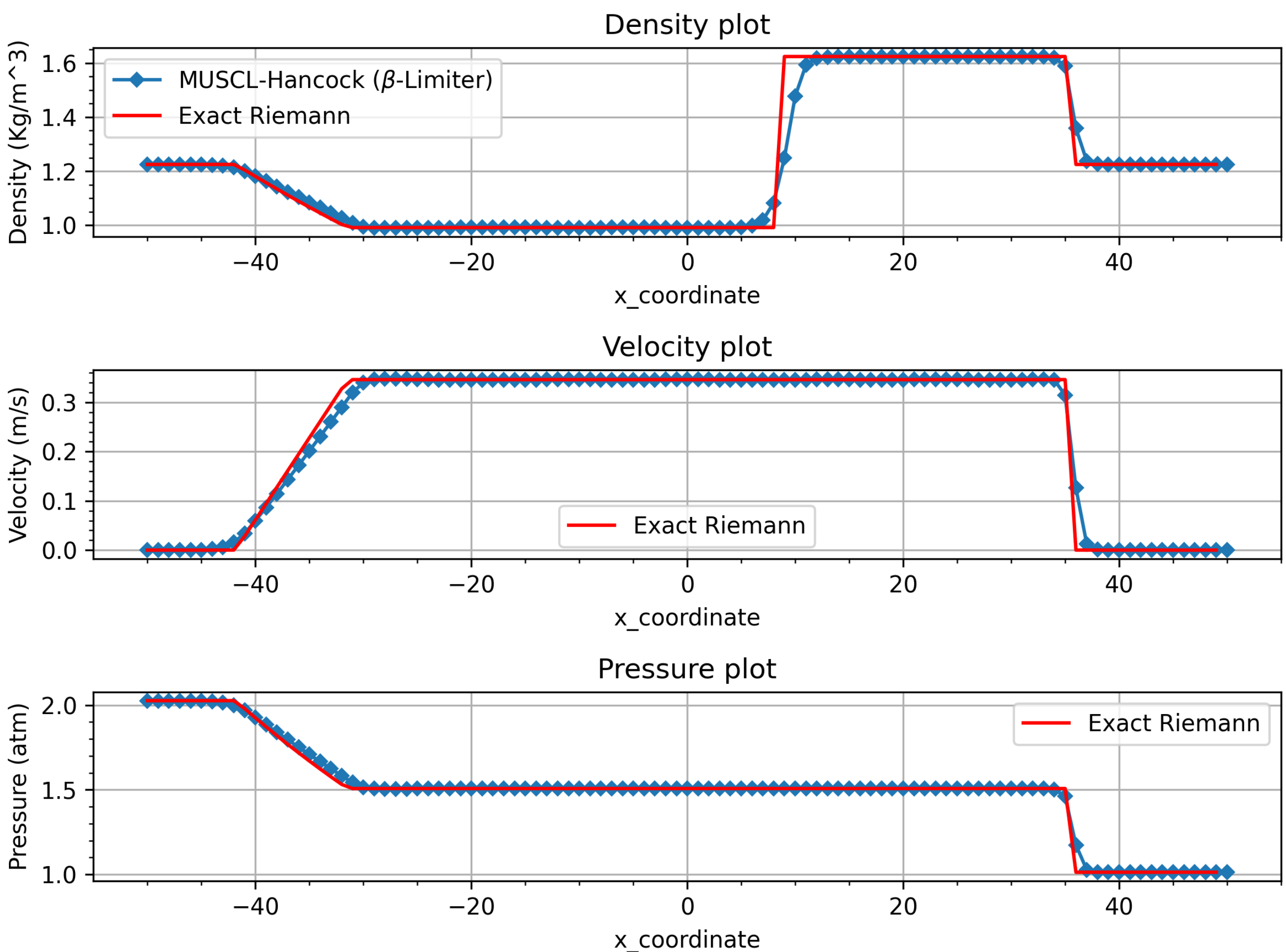


Figure 3: Density, Velocity and Pressure comparison plot with $\beta$ limiter

**Exercise 1-2. Study of different limiters.**

For this exercise, I have used MINMOD Limiter, $\beta$ Limiter, and SUPER-BEE Limiter. Fig. 4 shows the MUSCL-Hancock scheme with the MINMOD limiter. As seen from the plot, MINMOD limiter shows a little dissipation around the shock and higher dissipation around the contact surface.

**MINMOD Limiter**

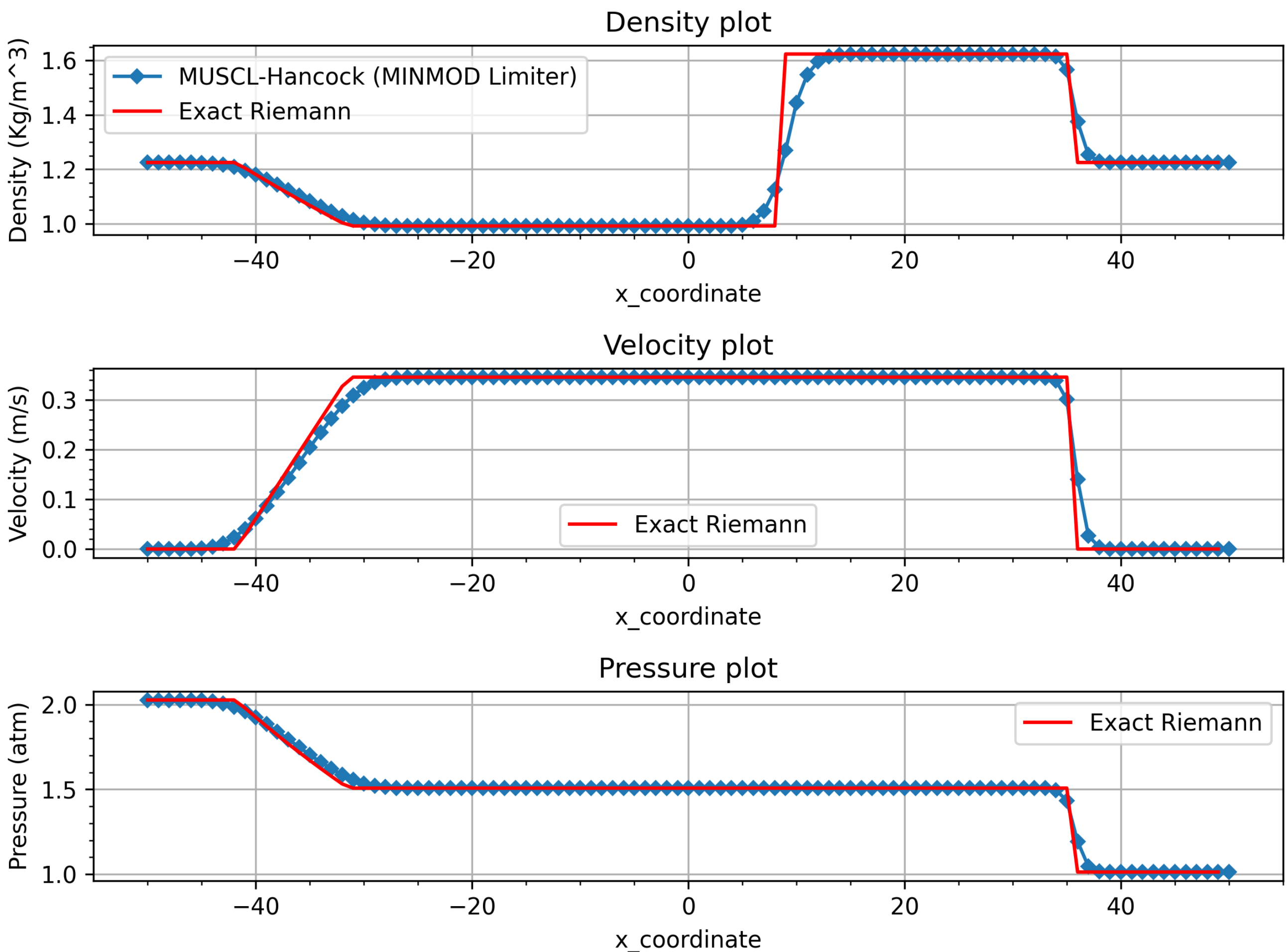


Figure 4: Density, Velocity and Pressure comparison plot with MINMOD limiter

**$\beta$ Limiter**

Fig. 5 shows the MUSCL-Hancock scheme with the $\beta$ limiter. As the $\beta$ value is 1.5, which is higher than that of MINMOD limiter, the anti-dissipation is higher and hence the overall numerical dissipation in this limiter reduces as compared to MINMOD limiter.

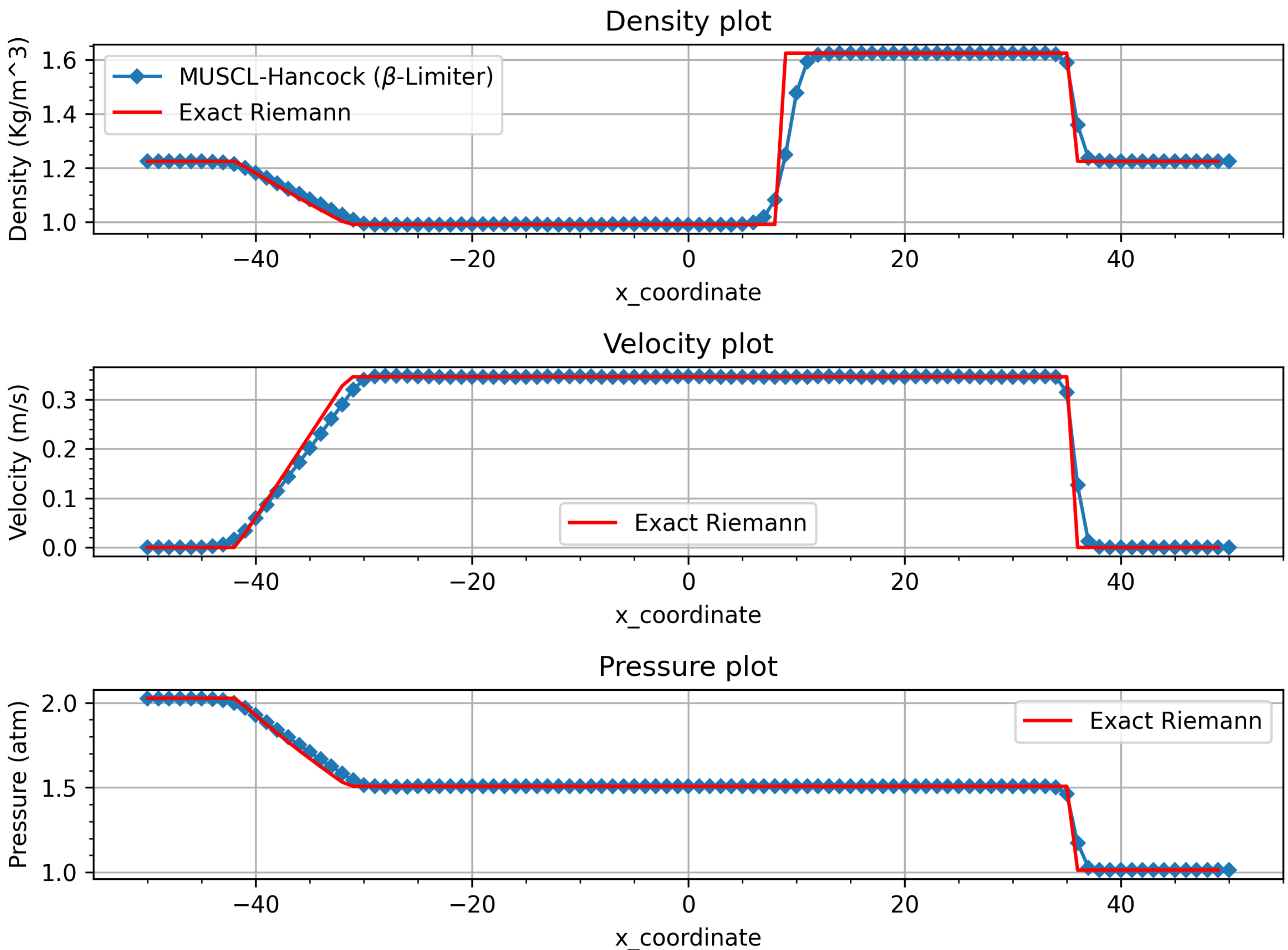


Figure 5: Density, Velocity and Pressure comparison plot with $\beta$ limiter

**SUPER-BEE Limiter**

Fig. 4 shows the MUSCL-Hancock scheme with the SUPER-BEE limiter. As the $\beta$ value is 2, the overall anti-dissipation is relatively higher than $\beta$ limiter. The result shows rectangular behavior better predicting the shock waves and other discontinuities.

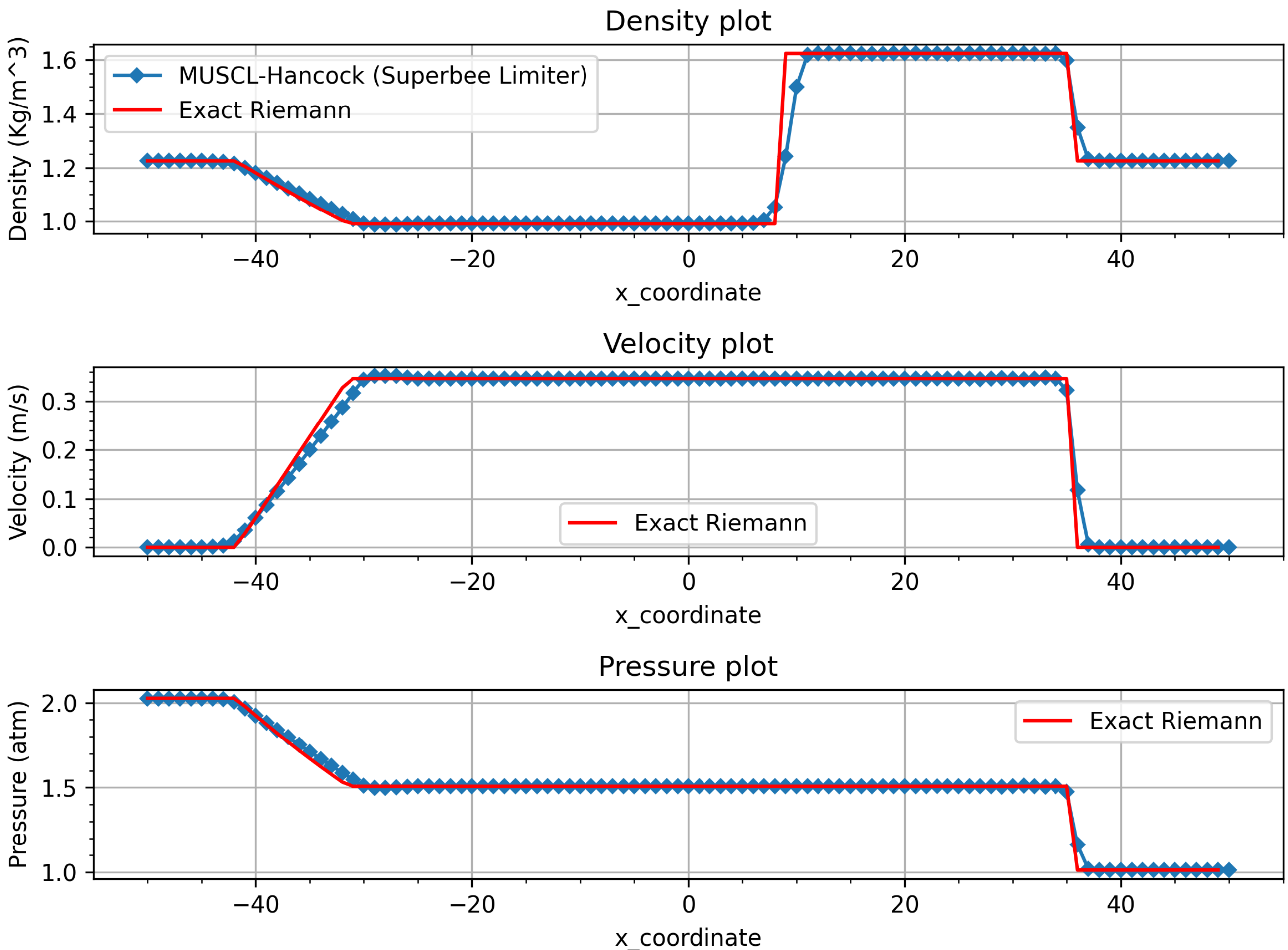


Figure 6: Density, Velocity and Pressure comparison plot with SUPER-BEE limiter

Table 1 shows the $L_2$ norm for all three limiters used above for all three primitive variables. SUPER-BEE limiter has the lowest $L_2$ norm and hence has been used for solving blast wave in the next question.

| **Limiters** | **Density ($L_2$ norm)** | **Velocity ($L_2$ norm)** | **Pressure ($L_2$ norm)** | **W ($L_2$ norm)** |
|---|---|---|---|---|
| MINMOD Limiter | 0.45414 | 0.226503 | 0.34729 | 0.61494 |
| $\boldsymbol{\beta}$ Limiter | 0.41422 | 0.23146 | 0.35243 | 0.59106 |
| SUPER-BEE Limiter | 0.40031 | 0.23906 | 0.36286 | **0.59081** |

Table 1: Performance of Limiters ($L_2$ norm)

**Task 2. Numerical simulation of a practical problem: blast wave interaction with an obstacle (wall).**

Using the best version of your second-order TVD scheme (based on Task 1), carry out the numerical modeling of blast wave interaction with a solid wall (assuming a one-dimensional flow model).

Computational grid = $[0, 1.5]$, $dx$ - 0.01, Time grid = $[0, 0.5]$, $dt$ - 0.001 s. SUPER-BEE limiter is used for the simulation of the blast wave.

**Exercise 1-1. Pressure and density instant distributions before and after blast wave reflection from the wall.**

Fig. 7 shows the pressure distribution along the x-coordinate for a few time moments. Time- 0.001 s shows the initial signal. Time - 0.025, 0.05, and 0.1 s shows the signal before reflection. Whereas, time - 0.2, 0.35, 0.45, and 0.5 s shows the reflected signal. Fig. 8 shows the density distribution along the x-coordinate for the same time moments. In these plots, along with shock, contact surface is also seen.

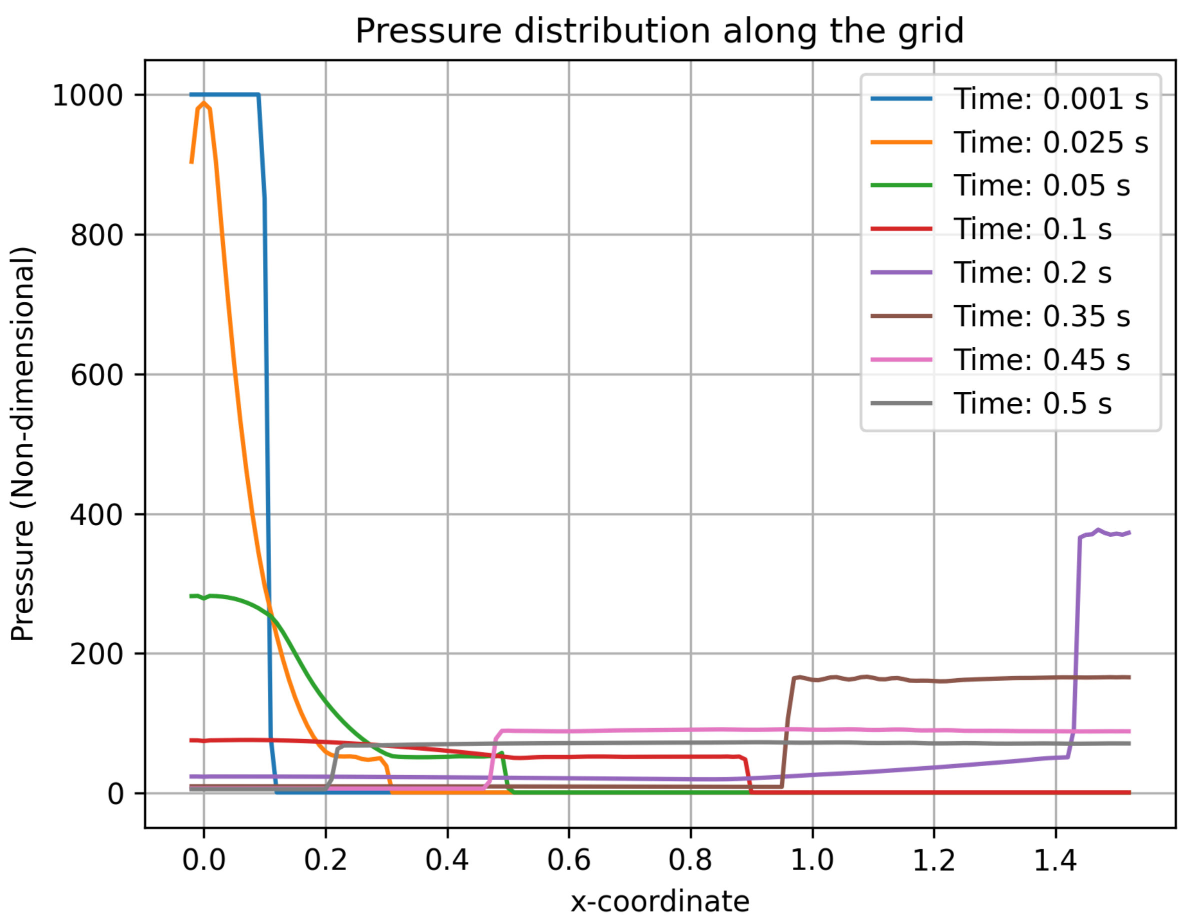


Figure 7: Pressure distributions along the grid at different time steps

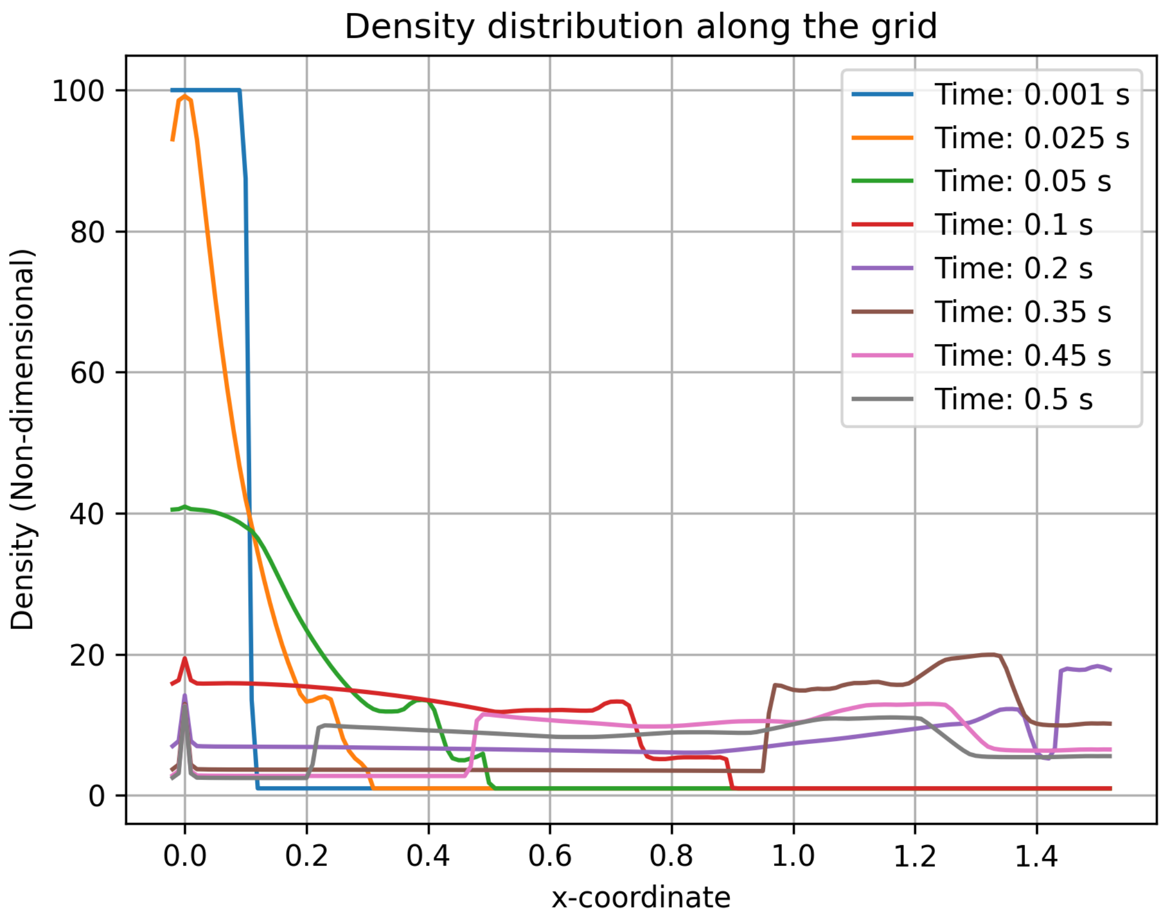


Figure 8: Density distributions along the grid at different time steps

**Exercise 1-2. Pressure history at $x = 1.3$. Obtain the same pressure history for a few successively refined meshes and make a judgment on what mesh size is needed to determine the peak (maximum) pressures at this point with a prescribed accuracy.**

Fig. 9 shows the pressure at $x$ - 1.3 for different grid parameters. For plotting this, 75 points are selected as the initial number of points. Then it is increased in the multiple of two. During this refinement, dt is kept constant at 0.0003. The dt is kept low for keeping CFL below 1. As seen in the plot, the peak pressure is observed at approximately 0.275 s. This will be generated from the reflected shock.

The peak pressure is changing relatively less at different grids. Fig. 10 shows the pressure variation at $x$ - 1.3 m, for different grids. The peak pressure is observed for $dx$ - 0.02, which is around 336, and the lowest is observed for $dx$ - 0.01, which is around 317.5.

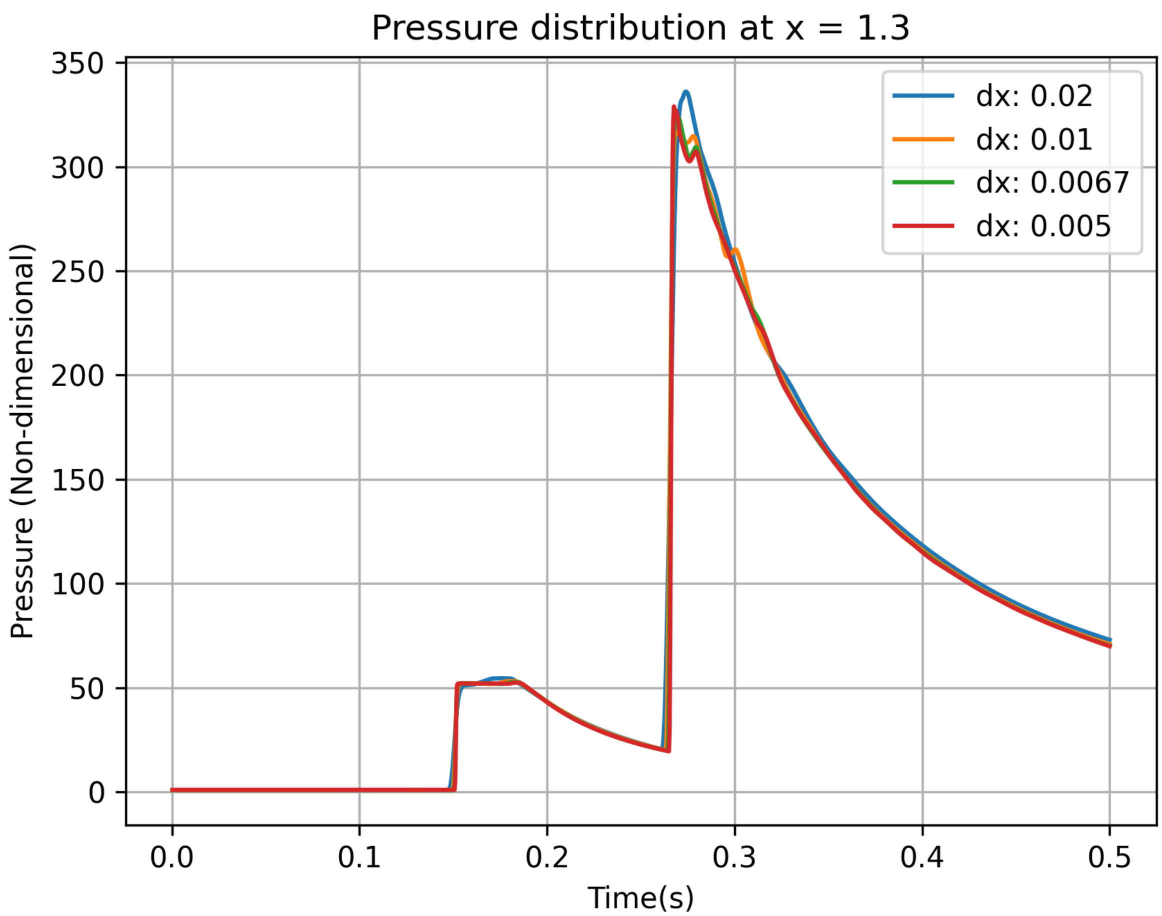


Figure 9: Pressure distributions at $x = 1.3$ with increasing grid refinement

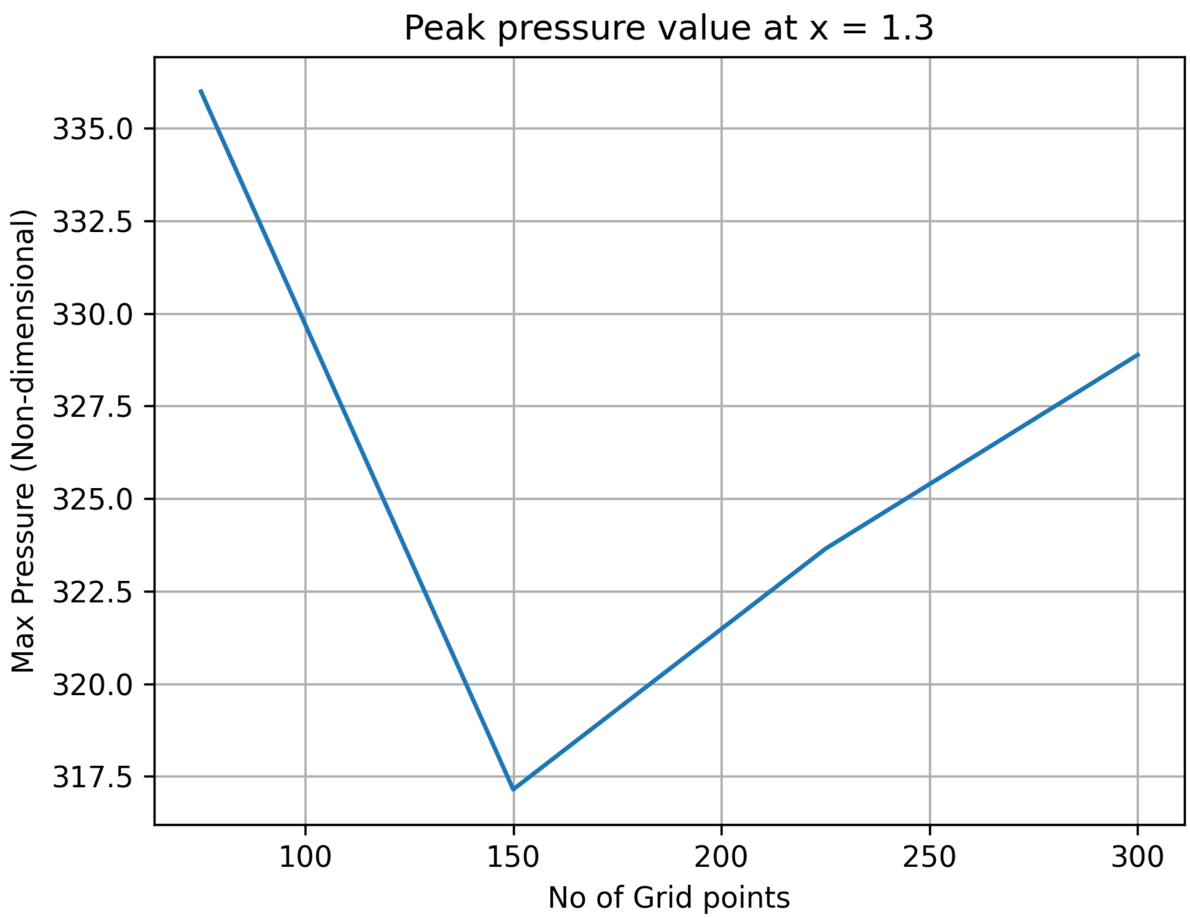


Figure 10: Peak pressure at $x = 1.3$ with increasing grid refinement