Final Project: Shell and tube heat exchanger design optimization

Name: Swadesh Suman
Email: swadesh.suman@mail.mcgill.ca

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1 Introduction

Shell and tube heat exchangers are crucial in cooling hydraulic fluids, demanding a balance between heat transfer efficiency and cost-effectiveness. There are several applications of this type of heat exchanger in the industry. Optimization involves various components like shell, tube, and baffle to maximize conductivity while maintaining affordability. This project aims to enhance the heat conductivity of a shell and tube heat exchanger using Multi-disciplinary design Optimization.

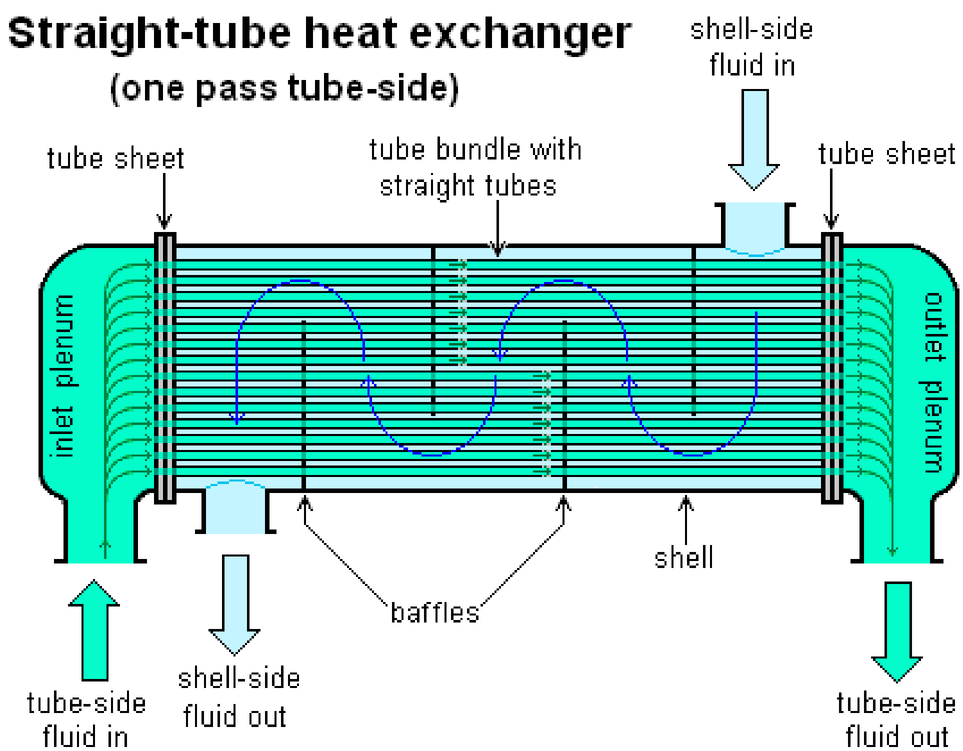

Figure 1: Shell and tube heat exchanger

Fig. 1 shows the diagram of a shell and tube heat exchanger. It is a one-pass tube side. The fluid to be cooled is water flowing in the tube, whereas the shell contains the hydraulic fluid used to cool the water. In this project, I have considered air as the cooling fluid. The entry temperature of the air is lower than the temperature of water.

2 Discipline

The following disciplines are involved in a Shell and Tube Heat Exchanger:

1. Structure Design
2. Thermal Engineering

3 Objective function

3.1 Structure Design Discipline

For efficient water cooling, a larger shell with more cooling fluid seems ideal, but beyond a point, increasing fluid volume doesn't enhance the cooling rate. Thus, determining the minimum shell size essential for required cooling efficiency is crucial for economic viability. This defines the objective function of the shell subsystem, f:

$$\text{minimize } F_s = 2.66175 \times L_t \times (D_{ot})^2 \times N_t$$

3.2 Thermal Engineering Discipline

Tubes play a vital role in preventing fluid contamination and facilitating efficient heat transfer. Increasing tube count directly enhances overall heat transfer. Objective: Minimize f to maximize the overall heat transfer coefficient U.

$$\text{minimize } F_t = \frac{1}{\mathrm{h}_i A_i (T_i - T_a)} + \left(\frac{1}{2\pi k L_t (T_o - T_i)}\right) \ln\left(\frac{D_{ot}}{D_{it}}\right) + \frac{1}{h_o A_o (T_{eW} - T_o)}$$

3.3 Combined objective function

$$\text{minimize } F = w_1(F_s) - (1 - w_1)F_t$$

where, w_1 is the weight associated with the objective function. In this case, it will be varied from 0.1 to 0.9 in the step of 0.2.

4 Optimization problem formulation

$$
\begin{aligned}
\text{minimize} \quad & F = w_1(F_s(\mathbf{X_s}, \mathbf{Z}) - (1 - w_1)F_t(\mathbf{X_t}, \mathbf{Z}, T_o(\mathbf{X_t}, \mathbf{Z}, A)) \\
\text{with respect to} \quad & \mathbf{X_s} \in R^3, \mathbf{X_t} \in R^3, \mathbf{Z} \in R^2 \\
\text{subject to Governing equation} \quad & \hat{R}_k(\mathbf{X_t}, A, \mathbf{Z}, T_o(A, \mathbf{X_t}, \mathbf{Z})) = 0 \\
\text{subject to constraint} \quad & \hat{C}_1(\mathbf{X_t}, \hat{R}_k, \mathbf{Z}, \mathrm{HTR}(\mathbf{X_t}, \hat{R}_k, \mathbf{Z})) > 0 \\
\text{subject to constraint} \quad & \hat{C}_2(\mathbf{X_t}, \mathbf{Z}) > 0 \\
\text{subject to constraint} \quad & \hat{C}_3(\mathbf{X_t}, \mathbf{Z}) > 0 \\
\text{subject to constraint} \quad & \hat{C}_4(\mathbf{X_t}) > 0
\end{aligned}
$$

T_o will be solved using the heat conduction governing equation given in the constraint section. HTR will be solved using the formulae given in the Design variable section.

5 Design variables

5.1 Global Variables (Z)

L_t = length of a tube
D_{ot} = Outer tube diameter

5.2 Local Variables

Structure Design Discipline ($\mathbf{X_s}$)

D_s = Diameter of the shell - related to D_{ot} and L_t (discussed in section 5.4)

Thermal Engineering Discipline ($\mathbf{X_t}$)

D_{it} = Inner tube diameter
T_i = Temperature of the inner surface of the tube
T_o = Temperature of the outer surface of the tube
V_w = Velocity of the water A = Thermal constant (present in the governing equation)

5.3 Constant Variables and other parameters

Both disciplines

P_t - Pitch of the tube = 1.5*D_{ot}
N_t = Number of tubes - 5
L_s - Length of the shell = 1.1*L_t
K - Thermal conductivity of the tube

V_a - Velocity of the cooling fluid
T_a - Temperature of the cooling fluid
T_{eW} - Entry temperature of the water
T_{iW} - Exit temperature of the water
h_i - Heat transfer coefficient - inside pipe
h_o - Heat transfer coefficient - outside pipe
Re_i - Reynolds number - inside pipe
Re_o - Reynolds number - outside pipe
$\dot{M}$ - Mass flow rate of water
Q - Energy supplied from the water

5.4 Formulae relating variables

$$D_s = 0.660538 \left(\frac{A_{\text{tube}} \left(\frac{P_t}{D_t}\right)^2 \cdot D_t}{L_t} \right)^{1/2}$$

$$h_i = \frac{\left(\frac{f}{2}\right)(\mathrm{Re}_i - 1000)\,\mathrm{Pr}_i}{1 + 12.7\left((\mathrm{Pr}^{2/3} - 1)(f/2)^{0.5}\right)} \frac{k}{D_{it}}$$

$$h_o = 0.0237 \cdot \mathrm{Re}_o^{0.618} \cdot \mathrm{Pr}_o^{1/3} \cdot \frac{k}{D_{ot}}$$

$$\mathrm{HTR} = \frac{k(T_{ot} - T_{it})2\pi D_{ot} L_t}{D_{ot} - D_{it}}$$

6 Constraints

Constraint	Definition
$\hat{R}_k : \frac{d}{dr}(r\frac{dT}{dr}) = 0$	Temperature across the thickness of tube must satisfy the differential heat conduction equation
$\hat{C}_1 : L_t/V_w - \left(\frac{Q(D_{ot}-D_{it})}{K(T_o-T_i)2\pi D_{ot} L_t}\right) > 0$	Energy supplied from water must satisfy required cooling time
$\hat{C}_2 : D_o - D_i - 0.0128 > 0$	Difference between inner and outer tube should be greater than 0.0128
$\hat{C}_3 : D_i - D_o + 0.0215 > 0$	Difference between inner and outer tube should be less than 0.0215
$\hat{C}_4 : T_o - T_i - 1 > 0$	The outer tube temperature should always be greater than the inner temperature by 1 K

Problem d)Write a code to solve the stated optimization problem using a direct or adjoint method to compute the derivatives. Show your derivation of the equations. Use a fixed step length (No line search is required) and select an appropriate point as your initial design point. Explain the choice of the initial design point.

Governing equation : $\frac{d}{dr}(r\frac{dT}{dr}) = 0$
The following analytical equations can be derived from the above governing equation:

Analytical equation: $T_o = A * \ln \frac{D_{ot}}{D_{it}} + T_i$

Method to find derivative
For finding the gradient and hessian of the objective function, automatic differentiation is used and this is calculated using the **Autograd** library. This gradient and hessian are then supplied to the scipy library for optimization.

Initial Point Selection
The design variable vector: $\mathbf{X}$ = [Inner tube dia, Outer tube dia, Inner tube temp, A (constant), Length of tube, Velocity of Water, Outer tube temp].
The initial point: $\mathbf{X}$ = [0.1, 0.21, 245, 30, 11, 0.11, 322].

Parameters	**Bounds**
D_{it}	0.025 to 0.416 ft
D_{ot}	0.0416 to 0.429 ft
T_i	243 to 322 K
A	0 to 41
L_t	5 to 12 ft
V_w	0.1 to 3.0 ft/s
T_o	273 to 322 K

Table 1: Bounds on the parameters for practical applications

For the length, the unit is taken in ft, temperature in K, and Velocity in ft/s. The inner and outer tube diameter is taken around 0.1 ft and 0.21 ft, respectively. The initial points are in the bounds for practical applications. The inner and outer temp is taken around 245 and 322 k, respectively. The initial velocity of water is taken in 0.11 ft/s, this has been selected as keeping in mind mass flow rate.

Weight (W_1) for multiobjective function
For the contour plots and function optimization $W_1 = 0.5$. However, for plotting the Pareto front W_1 is varying from 0.1 to 0.9 in the step of 0.2.

Constant variable values

Parameters	**Values**
Prandtl number (cooling fluid) Pr_l	4.34
Prandtl number (water) Pr_w	6.90
Velocity (cooling fluid)	5 ft/s
Temperature (cooling fluid)	242 K
Thermal conductivity (steel)	0.00257 $Btu/ft/s/F$
Entry temperature (Water)	323 K
Exit temperature (Water)	273 K
Mass flow rate (Water)	2.94 Kg/s

Table 2: Constant value of the parameters

Problem e) Show a contour plot of the function, f(x) and overplot the optimization path.

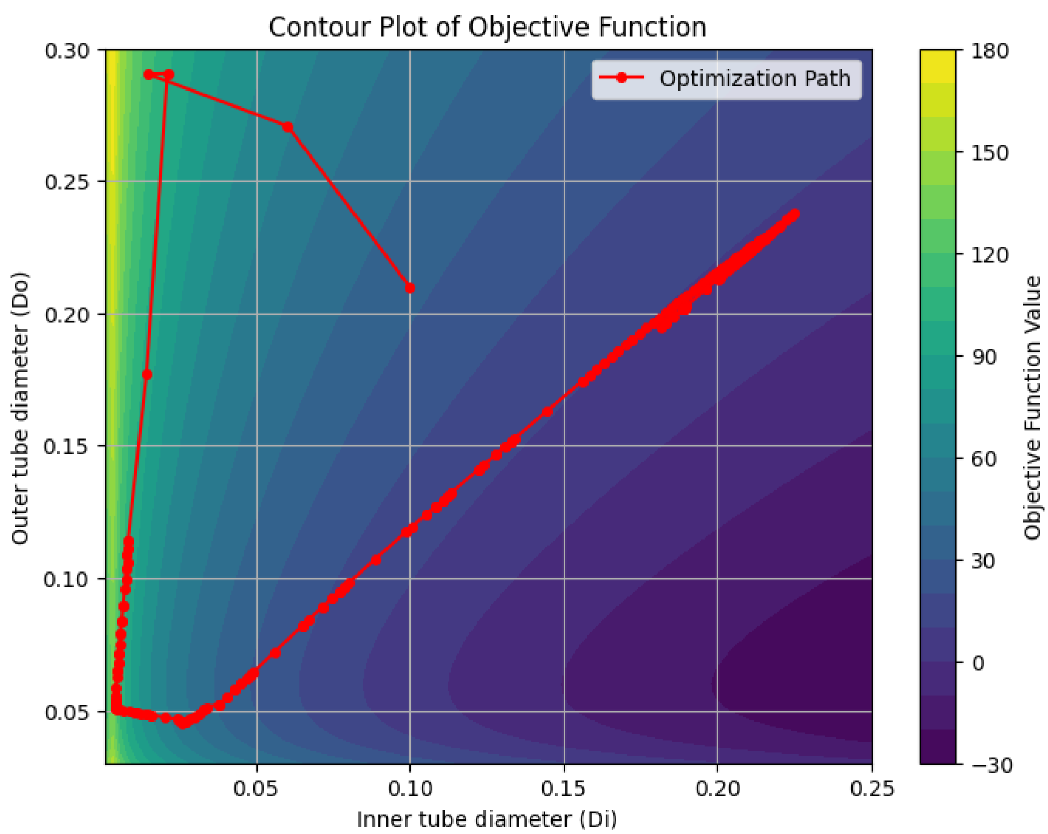

Figure 2: Contour plot and optimization path of algorithm

Fig. 2 shows the contour plot of the objective function based on two parameters inner and outer tube diameter. It also shows the optimization path for the algorithm. For plotting the objective function, these two parameters have been used. For this optimization weight is kept equal for both the objective functions from structure and thermal discipline. So, in this case, $W_1 = 0.5$.

The "**trust-constr**" method is used in the **Scipy** library for optimization. This method used the **finite difference method** to calculate the gradient of the lagrangian function and the constraint. The tolerance value of the convergence is set at the default value of 1E-08.

The final optimized point: $\mathbf{X}$ = [0.188, 0.201, 272, 15.234, 11.999, 0.249, 273].

Problem f) Provide convergence plots of the log of the norm of the gradient and the function value versus the design iterations.

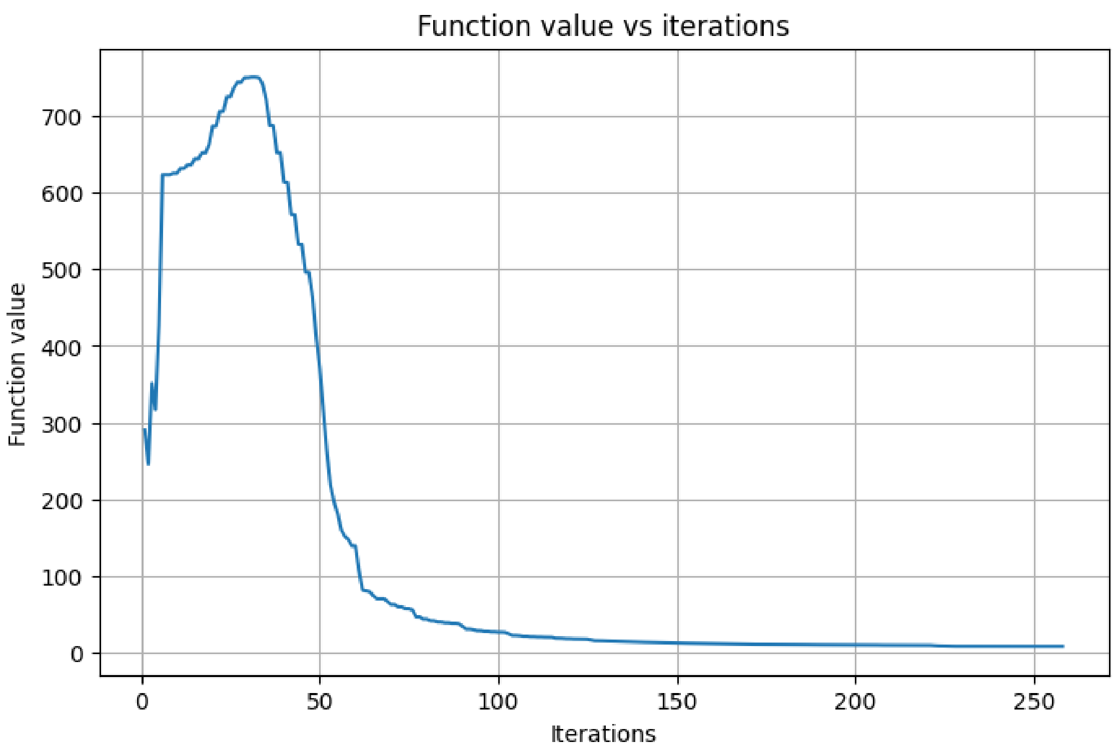

Figure 3: Convergence plot of the objective function vs iterations

Fig. 3 shows the minimization of the function value vs the number of iterations. Fig. 4 shows the convergence of the norm of the gradient vs the number of iterations. For the optimization problem, the gradient norm of the lagrangian function is reduced till 1E-08.

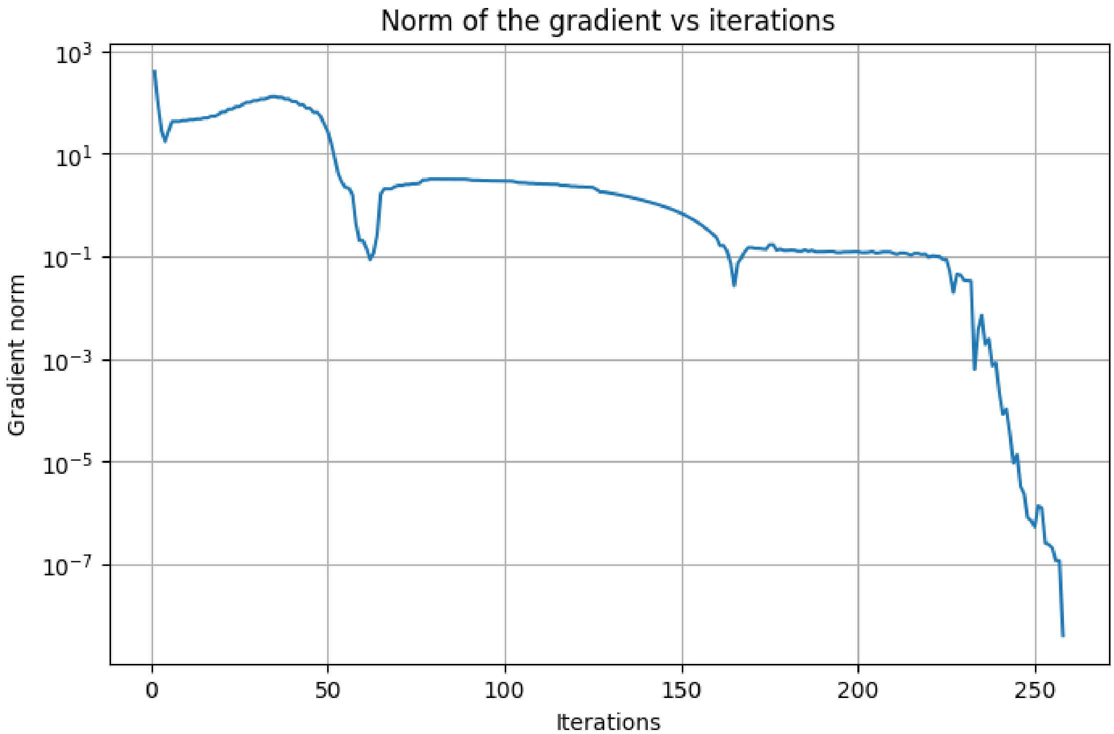

Figure 4: Convergence plot of the gradient norm vs iterations

Problem g) Demonstrate that at every design iteration, the governing equations are fully satisfied.

Governing equation : $\frac{d}{dr}(r\frac{dT}{dr}) = 0$
The following analytical equations can be derived from the above governing equation:

Analytical equation: $T_o = A * \ln \frac{D_{ot}}{D_{it}} + T_i$

The 1D heat conduction equation is used as the equality constraint in the above optimization problem. Fig. 5 shows the optimized values satisfying the governing equations at each iteration. For plotting the graph, the optimized point value at each iteration was put in the governing equation and the obtained governing equation value was plotted at each iteration.

The initial fluctuations in the values are because of the initial random point selection which does not satisfy the governing equation. But even if the initial points do not satisfy the governing equations, the new optimized points obtained in the next iterations satisfy the governing equations.

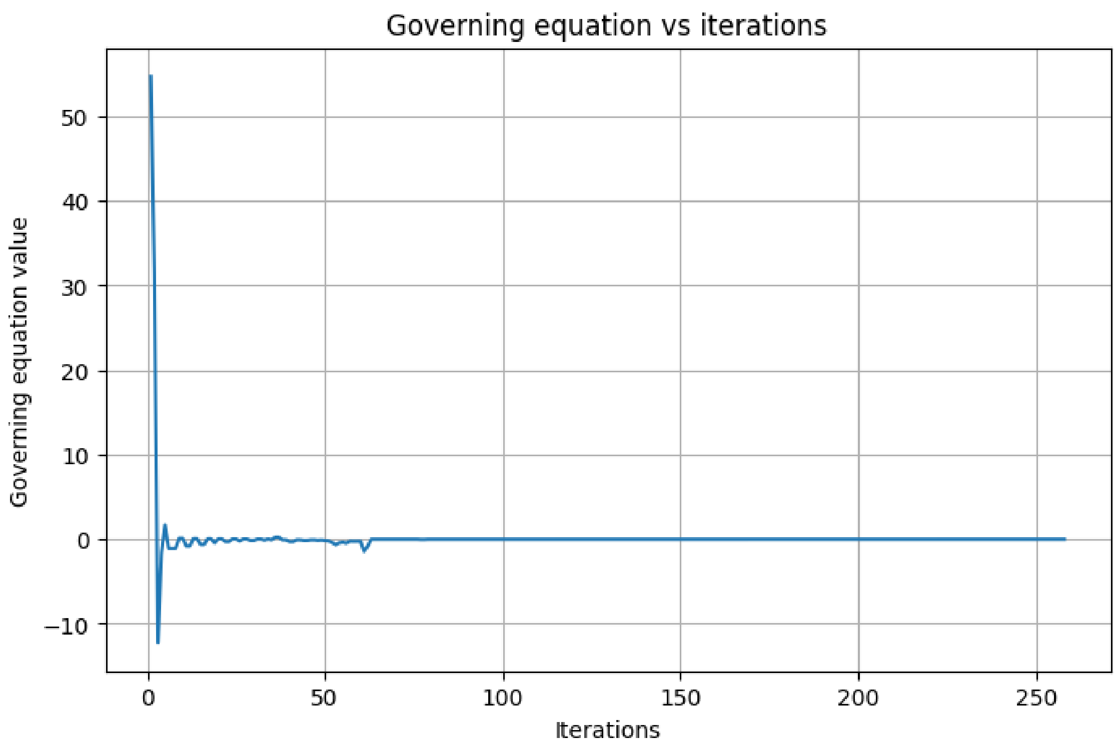

Figure 5: Plot showing the optimized values satisfying the governing equations at each iteration

Pareto front

Fig. 6 shows the Pareto front for the multiobjective function. As mentioned in the beginning, the objective function is the combination of two functions namely: the structure function and the thermal design function. To plot this Pareto front, weight is varied from 0.1 to 0.9 in the step of 0.2, and the function value is obtained at all the optimized points for multiobjective functions at each weight.

Weight	D_i	D_o	T_i	A	L_t	V_w	T_o
0.1	0.325	0.338	272	25.39	11.99	0.419	273
0.3	0.233	0.246	272	18.72	11.99	0.304	273
0.5	0.188	0.201	272	15.23	11.99	0.249	273
0.7	0.151	0.163	271.9	13.06	11.99	0.215	273
0.9	0.0968	0.1096	271.1	14.9	11.99	0.251	273

Table 3: Optimized parameter value at each weight value

Table 3 shows the optimized parameters value at each of the weights.

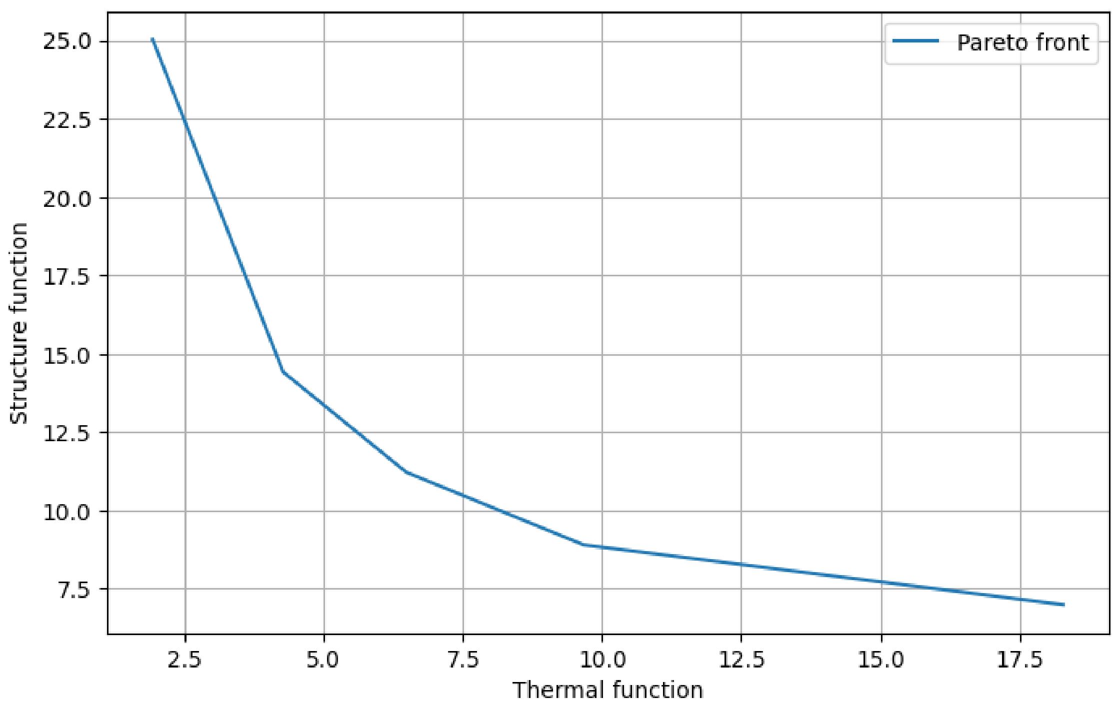

Figure 6: Plot showing the Pareto front for the multiobjective function