CPSC-406 Report

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Abstract

Contents

1 Introduction

2 Introduction

3 Week by Week

3.1 Week 1

HW1 – DFA Exercises

Exercise 1 We are given two DFAs A_1 and A_2.

Accepted Words Table

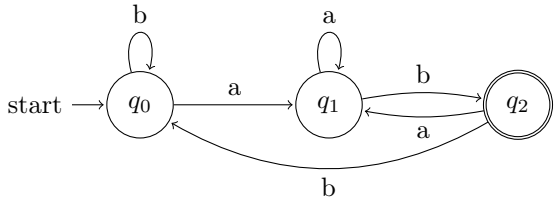
w	Accepted by A_1?	Accepted by A_2?
aaa	No	Yes
aab	Yes	No
aba	No	No
abb	No	No
baa	No	Yes
bab	No	No
bba	No	No
bbb	No	No

Language Descriptions

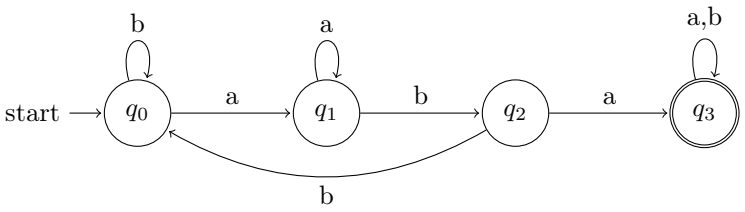
- $L(A_1)$: all strings over $\{a, b\}$ that start with a and end with an odd number of b's.
- $L(A_2)$: all strings over $\{a, b\}$ that end with at least two consecutive a's.

Exercise 2 – Designing DFAs

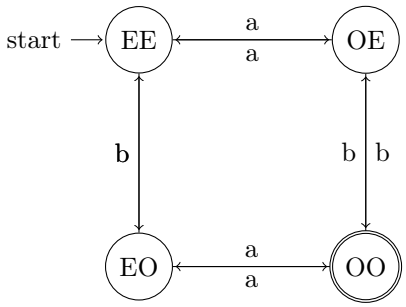
1. Words that end with ab



2. Words that contain aba

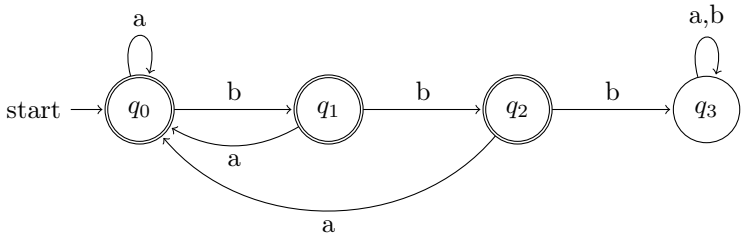


3. Odd number of a's and odd number of b's States represent parity: (a-parity, b-parity).

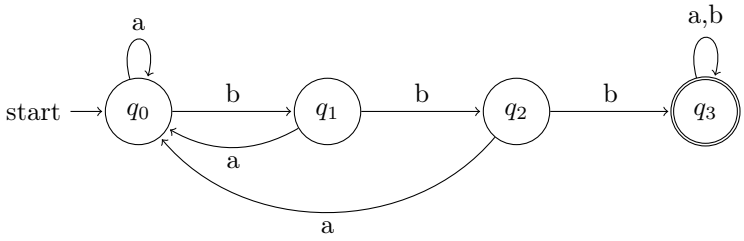


4. Even number of a's and odd number of b's Same automaton as above, but accepting state is EO.

5. Any three consecutive characters contain at least one a Equivalent to forbidding substring bbb.



6. Words that contain bbb



Observation

- Problems 3 and 4 use the same parity structure; only the accepting state changes.
- Problems 5 and 6 use the same "count consecutive b's" structure; one treats reaching three b's as rejection, the other as acceptance.
- Problems 1 and 2 track progress toward matching a pattern.

3.2 Week 2

HW2 – Operations on automata

Exercise 1 (Product automata). Throughout, $\Sigma = \{a, b\}$.

1. Description of $L(A^{(1)})$ and $L(A^{(2)})$.

$\underline{A^{(1)}}$:

The accepting states are $\{2, 4\}$ and state 3 is a non-accepting sink (it loops on both a and b). From state 2, reading a goes to the sink; from state 4, reading b goes to the sink. Thus, once the first symbol is read, the letters must alternate in order to avoid the sink.

Therefore,

$$L(A^{(1)}) = \{\, w \in \{a, b\}^* : |w| \geq 1 \text{ and } w \text{ contains no substring } aa \text{ or } bb \,\}.$$

Equivalently,

$$L(A^{(1)}) = a(ba)^*(\varepsilon \mid b) \ \cup \ b(ab)^*(\varepsilon \mid a).$$

$\underline{A^{(2)}}$:

The only accepting state is 2. From state 1, reading b leads to a sink. Reading a moves $1 \to 2$, and from 2 any symbol returns to 1.

Thus a word must:

- start with a,
- have odd length,
- have a in every odd position.

Hence,

$$L(A^{(2)}) = a\big((a \mid b)a\big)^*.$$

2. Intersection automaton $A = A^{(1)} \times A^{(2)}$.

- States: $Q = Q_1 \times Q_2$
- Start state: $(1, 1)$
- Accepting states:
$$F = F_1 \times F_2 = \{2, 4\} \times \{2\}$$
- Transition function:
$$\delta((p, q), x) = (\delta_1(p, x), \delta_2(q, x))$$

The reachable accepting state is $(2, 2)$.

3. Proof that $L(A) = L(A^{(1)}) \cap L(A^{(2)})$.

For any word w, by induction on prefixes, after reading w the product automaton is in state

$$(\delta_1(1, w), \delta_2(1, w)).$$

Thus A accepts w iff both component automata accept w. Therefore,

$$L(A) = L(A^{(1)}) \cap L(A^{(2)}).$$

4. Construction of union automaton A'.

Keep the same product states and transitions, but define the accepting set as

$$F' = (F_1 \times Q_2) \ \cup \ (Q_1 \times F_2).$$

Then A' accepts whenever at least one component accepts, so

$$L(A') = L(A^{(1)}) \cup L(A^{(2)}).$$

Exercise 2 (More automata). 1. Description of $L(B^{(1)})$ and $L(B^{(2)})$.

$\underline{B^{(1)}}$:

State p_0 is accepting. Reading a cycles

$$p_0 \to p_1 \to p_2 \to p_0,$$

while b loops at each state.

Thus $B^{(1)}$ accepts exactly when the number of a's is divisible by 3:

$$L(B^{(1)}) = \{\, w \in \{a,b\}^* : \#_a(w) \equiv 0 \pmod 3 \,\}.$$

$\underline{B^{(2)}}$:

State q_0 is accepting. The automaton goes to a sink q_2 if the substring aa occurs. Also, q_1 is non-accepting, so a word ending in a is rejected.

Hence

$$L(B^{(2)}) = \{\, w \in \{a,b\}^* : w \text{ has no substring } aa \text{ and does not end with } a \,\}.$$

Equivalently,

$$L(B^{(2)}) = (b \mid ab)^*.$$

2. Intersection automaton $B = B^{(1)} \times B^{(2)}$.

- Start state: (p_0, q_0)
- Accepting states:
$$F = \{(p_0, q_0)\}$$
- Transition function:
$$\delta((p_i, q_j), x) = (\delta_1(p_i, x), \delta_2(q_j, x))$$

3. Proof that $L(B) = L(B^{(1)}) \cap L(B^{(2)})$.

After reading any word w, the product automaton is in state

$$(\delta_1(p_0, w), \delta_2(q_0, w)).$$

Thus B accepts w iff both component automata accept w. Therefore,

$$L(B) = L(B^{(1)}) \cap L(B^{(2)}).$$

4. Construction of union automaton B'.

Using De Morgan's law:

$$L(B^{(1)}) \cup L(B^{(2)}) = \overline{\overline{L(B^{(1)})} \cap \overline{L(B^{(2)})}}.$$

1. Complement $B^{(1)}$ and $B^{(2)}$ by swapping accepting and non-accepting states.
2. Construct their product automaton for intersection.
3. Complement the resulting automaton.

This yields B' such that

$$L(B') = L(B^{(1)}) \cup L(B^{(2)}).$$

4 Synthesis

5 Evidence of Participation

6 Conclusion

References

[BLA] Nathan Carnnahan, CPSC-406 Report, Nathan Carnnahan, 2026.