VectorAnalysis

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A brief introduction to vector analysis for physics students, lacking precise mathematical proof.

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1 Vector Algebra

1.1 Tri-vector mixed product

Tri-vector mixed product contains dot product(inner product) and cross product (outer product) among three vectors. The result is a scalar.

Define: for vectors $\vec{a}, \vec{b}$ and $\vec{c}$, the mixed product is

$$\vec{c} \cdot (\vec{a} \times \vec{b}) \tag{1}$$

The result of $\vec{c} \cdot (\vec{a} \times \vec{b})$ equals to the volume of a parallelepiped defined with these 3 vectors. Hence when the order of $\vec{a}, \vec{b}, \vec{c}$ doesn't change, the result is fixed. If the order of 2 vectors is changed, the result counter sign.

Property: for mixed product:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b}) = -\vec{a} \cdot (\vec{c} \times \vec{b}) \tag{2}$$