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Calculus-I
Assignment-6

#Solution-1:

a)$\int_{4}^{9} 3\sqrt{x}dx$

Here,

Rewriting $\sqrt{x}$ as $x^{\frac{1}{2}}$ and,

Integrating

$\int 3x^{\frac{1}{2}}dx \Rightarrow 3.\frac{2}{3}x^{\frac{3}{2}}$

$\Rightarrow 2x^{\frac{3}{2}}$

Now, evaluating from 4 to 9:

$[2x^{\frac{3}{2}}]_4^9 \Rightarrow 2(9^{\frac{3}{2}})$ - $2(4^{\frac{3}{2}})$

$\Rightarrow$ 2(27) - 2(8)

$\Rightarrow$ 54-16

$\Rightarrow$ 38.

b)$\int_{1}^{e} ln(x)dx$

Using integration by parts. Let u= $ln(x)$, dv=dx.

$du = \frac{1}{x}dx,\ v = x$

Using Integration by part formula:

$\int u\, dv = uv - \int u\, du$

$\int ln(x)\, dx = xln(x) - \int x.\frac{1}{x}dx$

$\Rightarrow xln(x) - x$

Evaluating from 1 to e:

$[x\, ln(x) - x\,]_1^e = [eln(e) - e] - [1\,.\,ln(1) - 1]$

$= [e - e] - [0 - 1]$

= 0 + 1

=1

c) $\int_{0}^{1} cos^{-1}(x)\, dx$

Here, the indefinite integral of the given equation becomes,

$$\int_{0}^{1} cos^{-1}(x)\, dx = xcos^{-1}(x) - \sqrt{1 - x^2} + C$$

⇒Applying the definite integral limits:

$$\int_{0}^{1} cos^{-1}(x)\, dx = \left[xcos^{-1}(x) - \sqrt{1 - x^2}\right]_0^1$$

Evaluating at X=1:

$$xcos^{-1}(x) - \sqrt{1 - x^2} = 1.cos^{-1}(1) - \sqrt{1 - 1^2}$$

$$=1.0 - \sqrt{0}$$

$$=0$$

Evaluating at X = 0:

$$xcos^{-1}(x) - \sqrt{1 - x^2} = 0.cos^{-1}(0) - \sqrt{1 - 0^2}$$

$$=0.\frac{\pi}{2} - \sqrt{1}$$

$$= -1$$

Combining these results,

$$\int_{0}^{1} cos^{-1}(x)\, dx = [0] - [-1] = 1$$

Thus, the value of definite integral is $\int_{0}^{1} cos^{-1}(x)\, dx = 1$.

d) $\int_{-1}^{1} \pi cos(\frac{\pi x}{2})\ dx$

The constant π can be factored out:

$$\pi \int_{-1}^{1} \pi cos(\frac{\pi x}{2})\ dx$$

The antiderivative of $cos(kx)$ is:

$$\int cos(kx)\, dx = \frac{sin(kx)}{k}$$

Here, $k = \frac{\pi}{2}$ so:

$\int cos(\frac{\pi x}{2})dx = \frac{2}{\pi}sin(\frac{\pi x}{2})$

Now let's evaluate the definite integral:

$\pi\left[\frac{2}{\pi}sin\left(\frac{\pi x}{2}\right)\right]_{-1}^{1} = 2\left[sin\left(\frac{\pi}{2}\right) - sin\left(-\frac{\pi}{2}\right)\right]$

Simplifying this;

$sin\left(\frac{\pi}{2}\right) = 1, sin\left(-\frac{\pi}{2}\right) = -1$

$2(1 - (-1)) = 2(2) = 4$

Therefore, $\pi \int_{-1}^{1} \pi cos(\frac{\pi x}{2})\ dx = 4$

Question-2:

a) $\int x^2 cos(x^3)\ dx$

Let $u = x^3$, so $du = 3x^2\ dx,\ or\ \frac{1}{3}du = x^2 dx.$

The integral becomes:

$$\int x^2 cos(x^3)\ dx = \frac{1}{3}\int cos(u)\ du$$

The antiderivative of $cos(u)$ is $sin(u)$,so:

$$\frac{1}{3}sin(u) + C$$

Substituting back $u = x^3$:

$$\frac{sin(x^3)}{3} + C$$

Thus, $\int x^2 cos(x^3)\ dx = \frac{sin(x^3)}{3} + C$

b) $\int \frac{cos(3t)}{1+sin(3t)}dt$

Let $u = 1 + sin(3t),\ so\ du = 3cos(3t),\ or\ \frac{1}{3}du = cos(3t)dt.$

The integral becomes:

$$\frac{1}{3}\int \frac{1}{u}du$$

The antiderivative of $\frac{1}{u}\ is\ ln|u|$, so:

$$\frac{1}{3}ln|u| + C$$

Substituting back $u = 1 + sin(3t)$:

$$\frac{ln(1+sin(3t))}{3} + C$$

Thus, $\int \frac{cos(3t)}{1+sin(3t)} dt = \frac{ln(1+sin(3t))}{3} + C$

Question-3:

→ The function is $f(x) = x^3 - 5x^2 + 30$, and we will e aluate over $x\epsilon[0, 4]$.

The average value of a function is:

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\, dx$$

Substituting $f(x) = x^3 - 5x^2 + 30$ and $a = 0,\ b = 4$:

$$f_{avg} = \frac{1}{4-0}\int_0^4 (x^3 - 5x^2 + 30)\, dx$$

Evaluating the integral:

$$\int (x^3 - 5x^2 + 30)dx = \frac{x^4}{4} - \frac{5x^3}{3} + 30x$$

Evaluating from 0 to 4:

$$\left[\frac{x^4}{4} - \frac{5x^3}{3} + 30x\right]_0^4 = \left(\frac{4^3}{4} - \frac{5(4)^3}{3} + 30(4)\right) - (0)$$

Simplifying:

$\frac{4^3}{4} = 64$, $\frac{5.4^3}{3} = \frac{320}{3}$, $30(4) = 120$

$64 - \frac{320}{3} + 120 = \frac{192}{3} - \frac{320}{3} + \frac{360}{3} = \frac{232}{3}$

Thus, the average is,

$$f_{avg} = \frac{1}{4}.\frac{232}{3}$$

$$= \frac{58}{3} \approx 19.33$$

#Solution-3:

x	f(x)=x^3-5x^2+30
0	30
0.5	28.875
1	26
1.5	22.125
2	18
2.5	14.375
3	12
3.5	11.625
4	14