# Signature Assignment: Calculus-I

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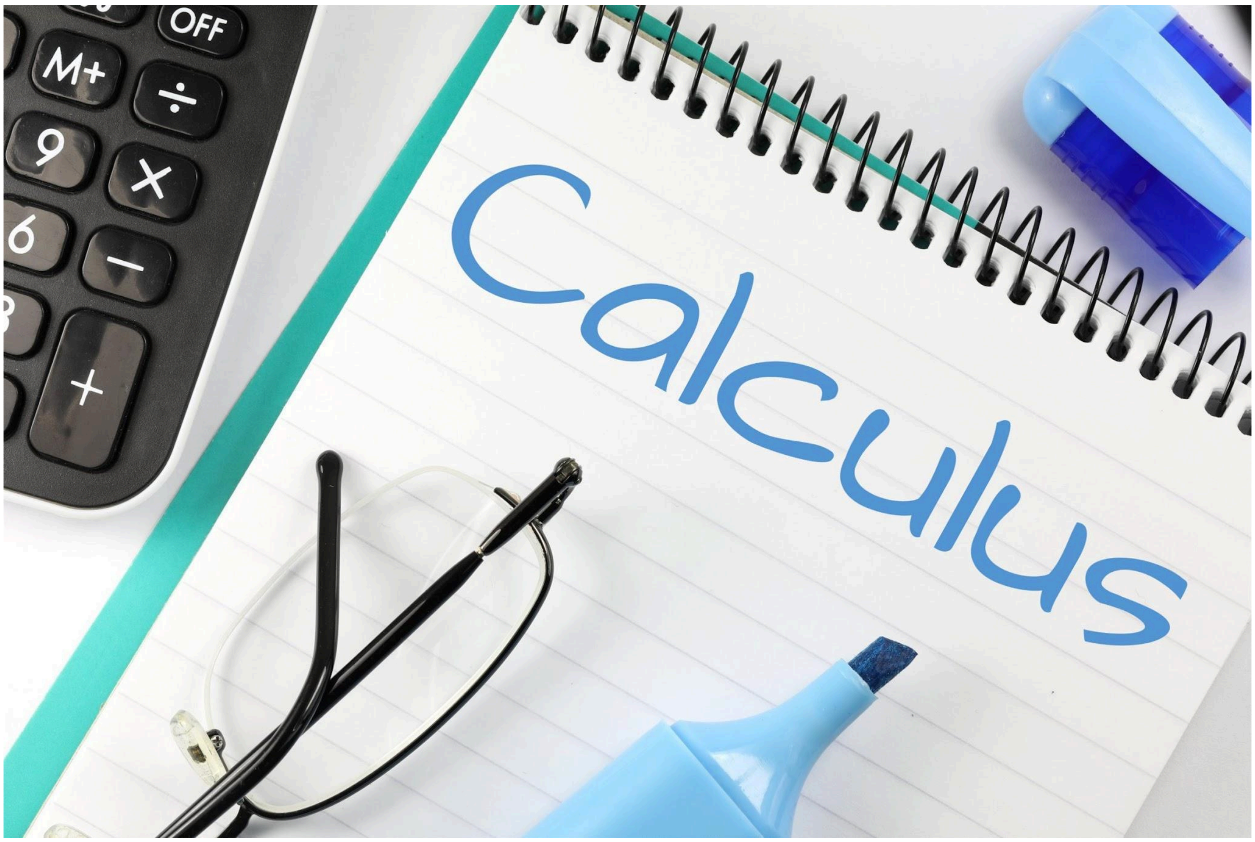



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**Abstract:**
This report explores the application of Newton's Method, a foundational numerical algorithm, to approximate the negative root of the equation $e^{x} = 4 - x^{2}$. Newton's Method iteratively refines root approximations by leveraging the concept of tangent lines to approach a precise solution. The equation was reformulated as $f(x) = e^{x} - 4 + x^{2}$, and iterative computations resulted in convergence to a root accurate to six decimal places. Python programming was employed to automate the calculations, verify results, and generate a plot that visualizes the root-finding process. The report provides an in-depth discussion of the methodology, including derivations, iterative steps, and graphical analysis. The critical analysis section evaluates the strengths and limitations of Newton's Method, offering insights into its practical applications and challenges.

**Introduction:**
Newton's Method, also known as the Newton-Raphson method, is a powerful numerical algorithm used to approximate the roots of a given equation. Developed by Sir Isaac Newton and Joseph Raphson in the 17th century, this method has become a cornerstone of numerical analysis and finds extensive applications in various science, engineering, and mathematics fields. This report aims to provide a comprehensive understanding of Newton's Method and its definition, along with a step-by-step explanation of the process involved. Approximating the negative root of the equation $e^{x} = 4 - x^{2}$ using Newton's method involves several steps.

In this report, we will provide a clear and detailed explanation of the methodology and a verification in Excel. We will also discuss the importance of plotting the graphs, reformulating the equation, computing the derivative, and applying Newton's method iteration. Additionally, we will highlight the need for a program in a computing language to generate data for the Excel plot. Finally, we will conclude by emphasizing the effectiveness of Newton's method for approximating the negative root and providing critical thinking, conclusion, and references for further reading.

**Definition of Newton's Method:**
Newton's Method is an iterative process to find the root of a given equation f(x) = 0. It starts with an initial guess, denoted as $x_0$, and then repeatedly refines this guess to converge towards the actual root of the equation. The method utilizes the concept of tangent lines to approximate the root by iteratively updating the guess using the formula:

$$x_{i+1} = x_{i} - \frac{f(x_{i})}{f'(x_{i})}$$

where $x_i$ is the current guess, $f(x_{i})$ is the value of the equation at $x_i$, and $f'(x_{i})$ is the derivative of the equation evaluated at $x_{i}$.

**Methodologies:**
We have used Newton's formula to calculate the value up to six decimal points as shown

below. The equation $e^{x} = 4 - x^{2}$ represents the intersection of two functions: $y = e^{x}$ and $y = 4 - x^{2}$.

Our objective is to find the negative root of this equation, which is the value of $x$ where the two functions intersect in the negative x-axis.

The equation is given as:

$e^{x} = 4 - x^{2}$

We Equate it to 0

$4 - x^{2} = 0$

So, we have:

$x^{2} = 4$

Taking square roots of both sides

X = ±2

So, the negative root is:

X= -2

Here, $e^{x} - 4 - x^{2}$ becomes

$f(x) = e^{x} - 4 + x^{2}$

Differentiating it, we get

$f(x) = e^{x} + 2x$

Using Newton's method of approximation, we have:

$$x_{n+1} = x_{n} - \frac{f(x_{n})}{f'(x_{n})}$$

When x = -2, we have:

$f'(-2) = e^{(-2)} + 2(-2) = -3.864667$

$f(-2) = e^{(-2)} - 4 + (-2)^{2} = 0.13533$

So, we have:

$$x_{1} = -2 - \frac{0.13533528323}{-3.8646671676}$$

$$x_{1} = -2 + \frac{0.13533528323}{3.8646671676}$$

$$x_{1} = -1.96498136$$

Repeating the above process for repeated x values.

We have:

$x_{2} = -1.96463563$

$x_{1} = -1.96463560$

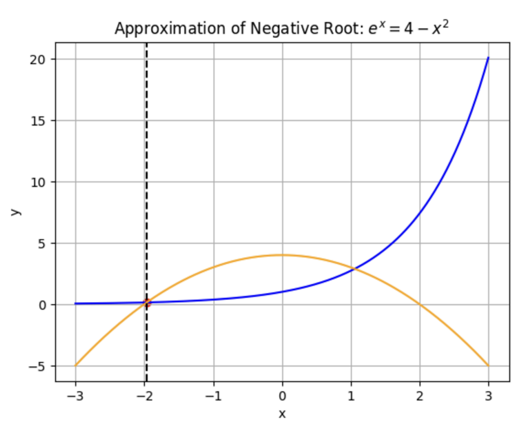
Since, up till the 6th decimal place,

X2=X3

Therefore, the approximated value of $e^{x} = 4 - x^{2}$ to 6 decimal places is -1.964636

**Using python program to generate Data for visualization:**

To visualize the convergence of Newton's method, a python program is written to automate the iteration process, calculate intermediate values, and generate data points for visualization. The program confirmed convergence to the same result, ensuring accuracy.



To further verify the solution, we plotted the two functions $y = e^x$ and $y = 4 - x^2$ using Python-generated data. The plot demonstrates the intersection of these functions at the approximated negative root, $x = -1.964636$. Key elements of the plot include:

- The **blue curve** represents $e^x$, showing exponential growth.
- The **orange curve** represents $4 - x^2$, forming a downward-opening parabola.
- A **red dot** marks the approximated root obtained through Newton's Method, highlighting its position at the intersection.
- A **dashed vertical line** emphasizes the root's position along the x-axis.

This visualization confirms the method's accuracy and demonstrates the convergence of the iterations toward the root. It also highlights the value of graphical tools in analyzing numerical methods.

## Applications of Newton's Method

Newton's Method is a cornerstone in numerical analysis, with applications extending across various fields of science, engineering, finance, data science, and astronomy. Its versatility lies in its ability to iteratively solve non-linear equations efficiently, making it indispensable for addressing complex problems that lack closed-form solutions. Below are the its applications in different disciplines:

- **Physics and Engineering:** Solving equations for motion, heat transfer, and structural analysis.
- **Finance:** Calculating interest rates, portfolio optimization, and option pricing.
- **Data Science and Machine Learning:** Optimizing cost functions in training algorithms.
- **Astronomy:** Determining orbital trajectories of celestial bodies.

Its versatility underscores its importance in scientific and technological advancements.

### Critical Thinking:

Newton's Method is highly effective for solving equations with smooth and differentiable functions. Its rapid convergence makes it one of the most efficient methods for root approximation. However, the algorithm has notable limitations:

- **Dependence on Initial Guess:** If the initial guess is too far from the actual root, the method may converge to a different root or fail to converge entirely.
- **Derivative Issues:** The method requires $f'(x) \neq 0$. At points where $f'(x)$ is close to zero, the iterations may become unstable.
- **Non-Convergence:** For equations with discontinuities or non-differentiable points, Newton's Method may not work effectively.

Despite these challenges, Newton's Method remains a powerful tool when applied correctly, especially in cases where graphical analysis and careful guess selection are possible.

**Conclusion:**

In conclusion, Newton's Method successfully approximated the negative root of the equation $e^x = 4 - x^2$ to six decimal places. The reformulation of the equation, derivation of the derivative, and iterative computations demonstrate the power and precision of the method. Visualizing the solution through Python-generated plots further validated the accuracy of the approximation. While the method is efficient and widely applicable, caution is necessary when selecting the initial guess to ensure convergence to the desired root. Overall, Newton's Method remains a cornerstone of numerical analysis, offering an elegant and practical solution for root-finding problems.

**References:**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