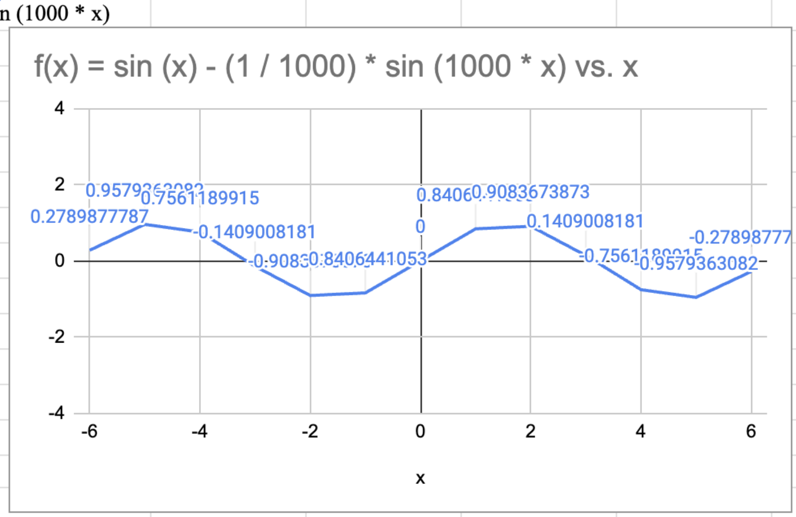
Swagat Neupane
19698
Calculus
Assignment-4

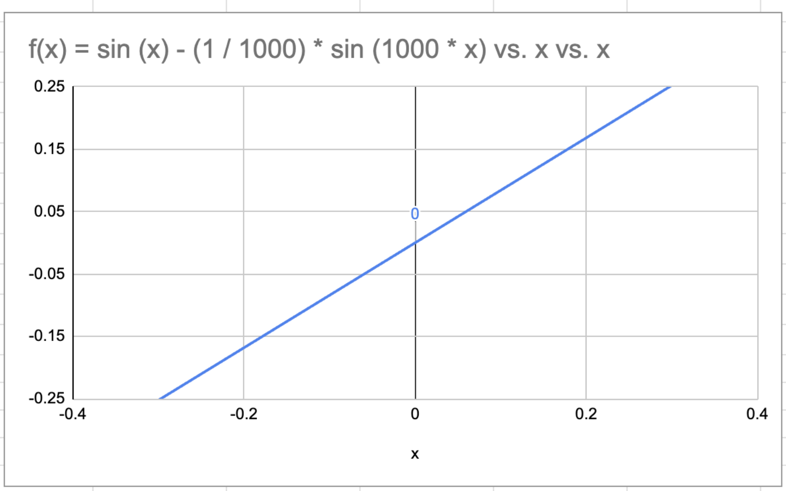
#Solution-1:

a)We can see that, the slope of the graph appears to have at the origin is: -0.0509 x – 0.070.
slope=y⇒-0.0509x - 0.070⇒R^2=0.0421

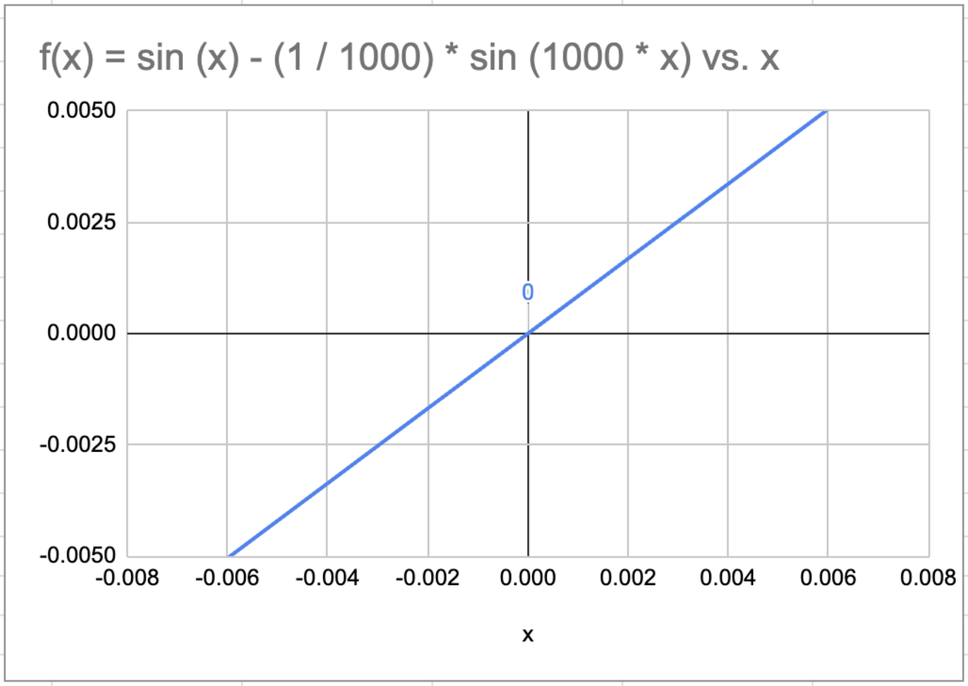
	#Solution-1
x	f(x) = sin (x) - (1 / 1000) * sin (1000 * x)
-6	0.2789877787
-5	0.9579363082
-4	0.7561189915
-3	-0.1409008181
-2	-0.9083673873
-1	-0.8406441053
0	0
1	0.8406441053
2	0.9083673873
3	0.1409008181
4	-0.7561189915
5	-0.9579363082
6	-0.2789877787



b)The values of x-axis and y-axis on the graph to match the viewing window [-0.4, 0.4] by [-0.25, 0.25]. We can observe that, the slope is still the same, which agrees with an answer from part(a)



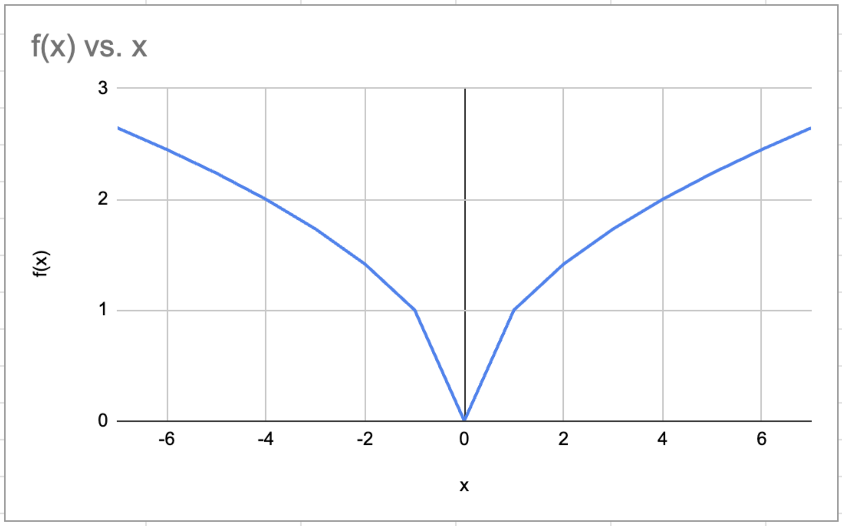
c)Now the graph gets much stronger. The slope of the curve at 0 which is f'(0) is - 0.0509. The slope remains the same.



#Solution-2:

Zooming at (-1,0) f is differentiable at (-1,0) because it is smooth at (-1,0) and there are no sudden changes.
Zooming at the origin f is NOT differentiable because it has a kink at the origin. There are sudden changes in the slope.

	#Solution-2:
x	f(x)
-7	2.645751311
-6	2.449489743
-5	2.236067977
-4	2
-3	1.732050808
-2	1.414213562
-1	1
0	0
1	1
2	1.414213562
3	1.732050808
4	2
5	2.236067977
6	2.449489743
7	2.645751311



#Solution-3:

a) Find $f'-(4)$ and $f'+(4)$ for the function

To find f'-(4). we substitute the value of 4 in the function.

We get,

f'(-4) = $\lim_{h \to 0^-} \frac{f(4+h)-f(4)}{h}$ [Here, if h<0, 4+h is also less than 4]

= $\lim_{h \to 0^-} \frac{5-(4+h)-1}{h}$ since, from definition, f(4)=1/(5-x)=1]

= $\lim_{h \to 0^-} \frac{-h}{h}$

= -1

To find f'+(4). we substitute the value of 4 in the function. We get,

f'(+4) = $\lim_{h \to 0^+} \frac{f(4+h)-f(4)}{h}$ [Here, if h>0, 4+h is greater than 4]

= $\lim_{h \to 0^+} \frac{\frac{1}{5(4+h)}-1}{h}$ [From the definition, if x is > or equal to 4, we use 1/ (5-x)]

= $\lim_{h \to 0^+} \frac{\frac{1}{1-h}-1}{h}$

$$= \lim_{h \to 0^+} \frac{1-1+h}{h(1-h)}$$

$$= \lim_{h \to 0^+} \frac{1}{(1-h)}$$

$$= \lim_{h \to 0^+} \frac{1}{(1-0)}$$

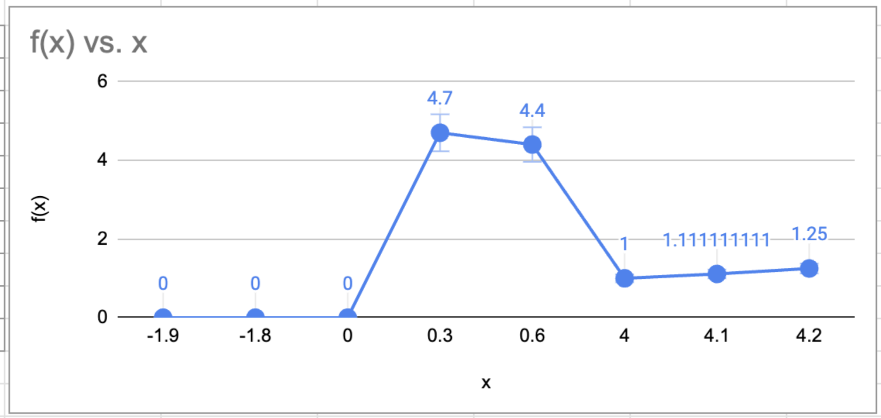
$$= \frac{1}{(1-0)}$$

$= 1$

b)

#Solution-3

x	f(x)
-2	0
-1.9	0
-1.8	0
0	0
0.3	4.7
0.6	4.4
4	1
4.1	1.111111111
4.2	1.25



c) Where is f discontinuous?

The graph of f is discontinuous at x=0 and x=5.

d) Where is f not differentiable?

The graph of f is not differentiable at x=0 and x=4 and x=5.

#Solution-4:

To show that $g'(x) = xf'(x) + f(x)$, we'll use the definition of the derivative. The derivative of $g(x)$ with respect to x is defined as:

$$g'(x) = \lim_{x \to a} \frac{g(x)-g(a)}{x-a}$$

Let's substitute the function $g(x) = xf(x)$ into the above expression:

$$g'(x) = \lim_{x \to a} \frac{xf(x)-af(a)}{x-a}$$

Now, let's simplify the expression:

$$g'(x) = \lim_{x \to a} \frac{xf(x) - af(x) + af(a) - af(a)}{x-a}$$

$$g'(x) = \lim_{x \to a} \frac{(x-a)f(x) + a[f(x) - f(a)]}{x-a}$$

Next, we can rewrite the expression as:

$$g'(x) = \lim_{x \to a} \frac{(x-a)f(x)}{x-a} + \lim_{x \to a} \frac{a[f(x) - f(a)]}{x-a}$$

The first term in the above expression simplifies to $f(x)$ and the second term simplifies to $af'(a)$. Therefore, we have:

$g'(x) = f(x) + af'(a)$

Since a is a constant, we can substitute a with x in the equation to get the desired result:

$g'(x) = xf'(x) + f(x)$

Thus, we have shown that $g'(x) = xf'(x) + f(x)$ using the definition of a derivative.

#Solution-5:

a) Boyle's Law can be expressed mathematically as: P . V = k

where k is a constant for a given amount of gas at a constant temperature.

To express V as a function of P, we rearrange the equation:

$$V = \frac{k}{P}$$

Given that the pressure P = 50*kPa* when the volume V = 0.106m^3, we can find k :

k= P . V = 50kPa . 0.106m^3 = 5.3kPa . m^3

So, the function V(P) is:

$$V(P) = \frac{5.3}{P}$$

(b) To find $\frac{dV}{dP}$, we take the derivative of V(P) :

$$V(P) = \frac{5.3}{P}$$

The derivative is:

$$\frac{dV}{dP} = -\frac{5.3}{P^2}$$

When P = 50kPa:

$$\frac{dV}{dP} = -\frac{5.3}{(50)^2} \Rightarrow -\frac{5.3}{2500} \Rightarrow \text{-0.00212m^3 /kPa}$$

The derivative $\frac{dV}{dP}$ represents the rate at which the volume of the gas changes for pressure at constant temperature. Its value indicates that for every 1 kPa increase in pressure, the volume decreases by approximately 0.00212m^3. The units of the derivative are m^3 /kPa.

#Solution-6:

a) Use a calculator to model tire life with a quadratic function of the pressure.

→ Here, the input to the function is the pressure and the output is the tire life.

The quadratic function will be: $L(P) = a * P ^2 + b * P + c$

Now we get,

$L(P) = -0.2754P^2 + 19.7485P - 273.5523$

b)Use the model to estimate dL/dP when P = 30 and P = 40. What is the meaning of the derivative? What are the units? What is the significance of the signs of the derivatives?

$dL/dP = -0.5508P + 19.7485$

so,

$dL/dP(P = 30)$

$= -0.5508 (30) + 19.7485$

≈ 3.22 lb/in²

$dL/dP(P=40)$

$= -0.5508 (40) + 19.7485$

≈ -2.28 lb/in²

Here, the derivative gives the rate of change of tire life as a function if the pressure. The units are thousands of miles/ (lb/in^2). (As with all derivatives, the units are units of output from the original function divided by units of input to the original function. At P = 30, the derivative is positive, so tire life is increasing, while at P = 40 the derivative is negative, so tire life is decreasing.