Question no-1:
Based on the experience of customs clearance inspections at the airport, the probability of luggage containing prohibited items is 10^{-4} . Given that the X-ray inspection machine has a probability of 1/10 of falsely identifying a regular luggage as containing prohibited items, and the probability of falsely identifying prohibited items as regular luggage is 10^{-6} , find the probability that it actually does contain prohibited items for a luggage belonging to someone determined by the X-ray inspection machine to contain prohibited items.

Solution:
Let's consider A and B ,the luggage contains prohibited items and the X-ray inspection machine identifies the luggage as containing prohibited items, respectively.
We can use Bayes' theorem to calculate the probability that a luggage actually contains prohibited items given that the X-ray inspection machine has identified it as containing prohibited items.

We are given that,

P(A) = 10^-4,

P(B|A) = 1 - 10^-6, and

P(B|~A) = 1/10.

We need to find P(A|B).

Using Bayes' theorem, we can write:

P(A|B) = $\frac{P(B|A)\cdot P(A)}{P(B)}$

We can calculate the denominator using the law of total probability:

P(B) = P(B | A) · P(A)+P(B | A') · P(A')

Substituting the given values, we get:

P(B) = (1 - 10^-6) * (10^-4) + (1/10) * (1 - 10^-4)

P(B) = 0.0001999

Now we can substitute this value and the given values into the Bayes' theorem equation:

P(A|B) = $\frac{P(B|A)\cdot P(A)}{P(B|\sim A)\cdot P(\sim A)+ P(B|A)\cdot P(A)}$

P(A|B) = (1 - 10^-6) * (10^-4) / 0.0001999

$P(A|B) = 0.499975$

Question-2:
There are two basketball teams A and B in the final for a 7-game series, where each team needs to win 4 out of 7 games. The probability of A winning in each game is 0.6 and B winning is 0.4 respectively. If defining the total game as N, $N \in \{4,5,6,7\}$, find the probability distribution of N and the expected value of N.

Solution:
While finding the probability distribution of N and the expected value of N, we can consider the possible outcomes for each value of N (4, 5, 6, 7),

Let's consider P(N) as the probability of N games played.
When,
N = 4 => P(4) = (0.6)^4 * (0.4)^0
=0.18522

N = 5 => P(5) = C(4,3) * (0.6)^3 * (0.4)^1
=0.3552
N = 6 => P(6) = C(5,3) * (0.6)^3 * (0.4)^2
=0.3025
N = 7 => P(7) = C(6,3) * (0.6)^3 * (0.4)^3
=0.1152
Here, C(n, k) is the combination function, which represents the number of ways to choose k elements from a set of n elements.

Now, to find the expected value of N (E[N]),
we can use the formula:

$E[N]=\sum_i P(N_i)\times N_i$

$$E[N]=P(4)\times 4+P(5)\times 5+P(6)\times 6+P(7)\times 7$$
$$E[N]=(0.18522\times 4)+(0.3552\times 5)+(0.3025\times 6)+(0.1152\times 7)$$
$$E[N]=5.04$$

Question-3:
The values of the discrete random variable X are 0, 1, 2, 3 and the probability P(X) is as follows. Find the expected value of X.

Solution:

$E(X)=\Sigma_x x \cdot P(X = x)$

Given the probability distribution for X:

P(X=0)=0.2

P(X=1)=0.1 · (k+1)

P(X=2)=0.3 · (k+1)

P(X=3)=0.2

Now, let's Substitute these probabilities into the formula:
E(X)=0 · 0.2+1 · (0.1 · (k+1))+2 · (0.3 · (k+1))+3 · 0.2
=0+0.1 · (k+1)+0.6 · (k+1)+0.6
E(X)=0.7 · (k+1)+0.6

Question-4:
Assuming that there are 30 male students and 20 female students in a class, the five students will be randomly selected to attend the speech contests organized by the student association. If the random variable X is the number of female students in the selected group, find the probability distribution of X.

Solution:
To find the probability distribution of the random variable X, which represents the number of female students in a randomly selected group of five students, we can use the binomial probability distribution formula.

The probability mass function (PMF) of a binomial distribution is given by:

P(X=k)=$\binom{n}{k} \cdot p^n \cdot (1 - p)^{n-k}$

Here, n and k is the number of trials (students selected) and number of successes (female students), respectively.
p is the probability of success on each trial, and $\binom{n}{k}$is the binomial coefficient, representing the number of ways to choose k successes out of n trials.
here, n=5 (selecting 5 students), p = $\frac{20}{50}$=$\frac{2}{5}$ (the probability of selecting a female student) and we can take the value of K from 0 to 5.

Now, Calculating the probabilities for X(0-5):
Using the above formula:

For X=0, ⇒ P(X=0)=$\binom{5}{0} \cdot \left(\frac{2}{5}\right)^{0} \cdot \left(\frac{3}{5}\right)^{5}$

For X=1, ⇒ P(X=1)=$\binom{5}{1} \cdot \left(\frac{2}{5}\right)^{1} \cdot \left(\frac{3}{5}\right)^{4}$

For X=2, ⇒ P(X=2)=$\binom{5}{2} \cdot \left(\frac{2}{5}\right)^{2} \cdot \left(\frac{3}{5}\right)^{3}$

For X=3, ⇒ P(X=3)=$\binom{5}{3} \cdot \left(\frac{2}{5}\right)^{3} \cdot \left(\frac{3}{5}\right)^{2}$

For X=4, ⇒ P(X=4)=$\binom{5}{4} \cdot \left(\frac{2}{5}\right)^{4} \cdot \left(\frac{3}{5}\right)^{1}$

For X=5, ⇒ P(X=5)=$\binom{5}{5} \cdot \left(\frac{2}{5}\right)^{5} \cdot \left(\frac{3}{5}\right)^{0}$

Question-5:
In a batch of 12 TV sets, three of them are defective. Three TV sets will be randomly selected for inspection and the random variable X represents the number of good quality units in the inspection. If all three are good, the entire batch is accepted, otherwise, it is returned. Please answer the following questions

a. If the sampling is done without replacement, write the probability distribution of X, the mean, and the variance. Also, find the probability that the entire batch of TV sets can be accepted.

b. If the sampling is done with replacement, write the probability distribution of X, the mean, and the variance. find the probability that the entire batch of TV sets can be accepted.

C. If the sampling is done without replacement, calculate the probability that the third one is one defective.

Solution:
We know that there are 12 TV sets with 3 defective ones.
So, let us consider selecting 3 TV sets without replacement.
Probability Distribution of X can be given as:
Here , random variable X represents the number of good quality units which possibilities are

X=0 to 3

For X=0: P(X=0)==>$\frac{3}{12} \cdot \frac{2}{11} \cdot \frac{1}{10}$

For X=1: P(X=1)==>$\binom{3}{1} \cdot \frac{9}{12} \cdot \frac{2}{11} \cdot \frac{1}{10}$

For X=2: P(X=2)==>$\binom{3}{2} \cdot \frac{9}{12} \cdot \frac{8}{11} \cdot \frac{1}{10}$

For X=3: P(X=3)=$\frac{9}{12} \cdot \frac{8}{11} \cdot \frac{7}{10}$

Mean:

E(X)=$\Sigma_{x=0}^{3}$x·P(X=x)=0 x $\frac{1}{220}$x 1 x$\frac{54}{220}$x 2 x$\frac{216}{220}$x 3 x $\frac{168}{220}$=$\frac{36}{55}$

Variance:

E(X^2)=$\Sigma_{x=0}^{3}$x·P(X=x)=0^2 x $\frac{1}{220}$x 1^2 x$\frac{54}{220}$x 2^2 x$\frac{216}{220}$x 3^2 x $\frac{168}{220}$=$\frac{1128}{55}$

Var(X)=E(X^2)-$[E(X)]^2$=$\frac{1128}{55}$-$(\frac{36}{55})^2$

Probability that the Entire Batch of TV sets can be Accepted is the probability that all three selected TVs are good, which is given by: P(all three are good) = (9/12) * (8/11) * (7/10) = 0.3818

b. Sampling With Replacement:

Probability Distribution of X can be written as:

P(X=k)=$\binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k}$

Where, n is the number of trials and k is the number of successes, and p is the probability of success. Then,

P(X=0)=$\binom{3}{0} \cdot (\frac{3}{12})^0 \cdot (\frac{9}{12})^3 = \frac{27}{64}$⇒ 0.4218

P(X=1)=$\binom{3}{1} \cdot (\frac{3}{12})^1 \cdot (\frac{9}{12})^2 = \frac{27}{64}$⇒ 0.4218

P(X=2)=$\binom{3}{2} \cdot (\frac{3}{12})^2 \cdot (\frac{9}{12})^1 = \frac{9}{64}$⇒ 0.1406

P(X=3)=$\binom{3}{3} \cdot (\frac{3}{12})^3 \cdot (\frac{9}{12})^0 = \frac{1}{64}$⇒ 0.00156

Mean:

E(X)=n · p=3 $\cdot \frac{3}{12} = \frac{3}{4}$

Variance:

Var(X)=n · p · (1−p)=3 $\cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{9}{16}$

c. Sampling Without Replacement -

Probability of the Third One Being Defective:

the probability that the third TV set is defective is $\frac{3}{10}$,

Because the sampling is without replacement.