**Q.No.1**

**Solution:**

**A)** ~ (𝑝 ⋃~𝑞) ⋃ (~𝑝⋂ ~𝑞) ≡~p

| p | q | ~p | ~q | pv~q | ~p^~q | ~(pv~q) | ~(pv~q)v(~P^~q) |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |

Hence, ~(pv~q)v(~P^~q) == ~p. Proved

B) ~ ((~𝑝 ⋂𝑞) ⋃ (~𝑝⋂ ~𝑞)) ⋃(𝑝⋂𝑞) ≡ 𝑝

| p | q | ~p | ~q | p ^ q | ~p^q | ~p^~q | ((~p^q)v(~p^~q)) | ~((~p^q)v(~p^~q)) | ~((~p^q)v(~p^~q))v(p^q) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |

Hence, ~((~p^q)v(~p^~q))v(p^q) == p. proved

**C)** (𝑝 ⋂(~(~𝑝⋂𝑞))) ⋃ (𝑝⋂ 𝑞) ≡ 𝑝

| p | q | ~p | ~p^q | ~(~p^q) | (p^(~(~p^q))) | p^q | (p^(~(~p^q)))v(p^q) |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |

Hence, (p^(~(~p^q)))v(p^q)==p. Proved

**Q.No.2 —- Solutions:**

**a. (**𝑝⊕𝑝**)** ⊕𝑝

| **p** | **p** ⊕**p** | **(p** ⊕ **p)** ⊕**p** |
| --- | --- | --- |
| 1 | 0 | 1 |
| 0 | 0 | 0 |

**B. Is (**𝑝⊕𝑞**)** ⊕𝑟**≡**𝑝⊕ **(**𝑞⊕𝑟**)?**

| **p** | **q** | **r** | **p** ⊕ **q** | **q** ⊕ **r** | **(p**⊕ **q)** ⊕ **r** | **p** ⊕ **(q** ⊕ **r)** |
| --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 |

**C. Is (**𝑝⊕𝑞**)**⋂𝑟 **≡ (**𝑝**⋂**𝑟**)** ⊕ **(**𝑞**⋂**𝑟**)?**

| **p** | **q** | **r** | **p** ⊕ **q** | **(p** ⊕ **q)** ∧ **r** | **p** ∧ **r** | **q** ∧ **r** | **(p** ∧ **r)** ⊕ **(q** ∧ **r)** |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |

**Q.No.3 —— Solutions**

**Here,** p→q is false, p is true and q is false. Hence, ~p is false, so p→q is true.

1. **~p→q**

| p | q | ~p | p→q | ~p→q |
| --- | --- | --- | --- | --- |
| 1 | 0 | 0 | 0 | 1 |

**B. p V q**

| p | q | p V q |
| --- | --- | --- |
| 1 | 0 | 1 |

**C. q→p**

| p | q | q→p |
| --- | --- | --- |
| 1 | 0 | 1 |

**Q.No.4 —— Solutions:**

**Ans: A)** *p→(*𝑞*V*𝑟*) ≡ (*𝑝∧*~*𝑞*) →* 𝑟

| p | q | r | ~q | qvr | p^~q | p-->(qvr) | p^~q-->r | (p-->(qvr))<-->((p^~q)-->r) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | TRUE |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | TRUE |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | TRUE |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | TRUE |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | TRUE |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | TRUE |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | TRUE |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | TRUE |

Hence, the given statement is Tautology because all the value of the given statement is true.

**B)** *p→(*𝑞*V*𝑟*) ≡ (*𝑝∧*~*𝑞*) →* 𝑟

| p | q | r | qvr | p^(qvr) | p^q | p^r | (p^q)v(p^r) | p^(qvr)<-->(p^q)v(p^r) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | TRUE |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | TRUE |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | TRUE |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | TRUE |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | TRUE |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | TRUE |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | TRUE |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | TRUE |

Hence, the given statement is Tautology because all the value of the given statement is true

**C)** *p→(*𝑞*→*𝑟*)≡(*𝑝∧𝑞*)→*𝑟

| p | q | r | q-->r | p-->(q-->r) | p^q | (p^q)-->r | p-->(q-->r)<-->(p^q)-->r |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | TRUE |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | TRUE |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | TRUE |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | TRUE |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | TRUE |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | TRUE |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | TRUE |
| 0 | 0 | 0 | 1 | 1 | 0 | 1 | TRUE |

Hence, the given statement is Tautology because all the value of the given statement is true

Q.no.5,

Answer,

1. Suppose the cook was in the kitchen at the time of the murder.

2. The butler killed Lord Hazelton with strychnine.

3. We have a contradiction: Lord Hazelton was killed by strychnine and a blow on the head.

4. The supposition that the cook was in the kitchen is false.

5. The cook was not in the kitchen at the time of the murder.

6. Sara was not in the dining room when the murder was committed.

7. Lady Hazelton was in the dining room when the murder was committed.

8. The chauffeur killed Lord Hazelton.