A Statistical Learning Theory Perspective on Noise Prediction in Diffusion Models

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Abstract

Denoising Diffusion Probabilistic Models (DDPMs) have achieved remarkable success in generative modeling by learning to reverse a gradual noising process. A fundamental architectural choice in DDPMs is whether to predict the noise ϵ added at each timestep or to directly predict the clean data x_0. Empirically, noise prediction consistently outperforms direct prediction, yet this phenomenon lacks rigorous theoretical justification through the lens of statistical learning theory. This work provides a formal analysis of this design choice by examining the bias-variance tradeoff inherent to each prediction target. We demonstrate that noise prediction benefits from a favorable tradeoff: predicting a fixed Gaussian distribution results in lower variance at the cost of potentially higher bias, while direct prediction of the data distribution exhibits the opposite behavior. Through formal analysis using concepts from PAC learning and structural risk minimization, we characterize the conditions under which noise prediction achieves superior sample complexity and generalization. Our theoretical findings are validated through empirical experiments on standard image datasets.

1 Introduction

Denoising Diffusion Probabilistic Models (DDPMs) have emerged as a dominant paradigm in generative modeling, achieving state-of-the-art performance across diverse applications including image synthesis [18], video generation, and molecular design. First introduced by Sohl-Dickstein et al. [2] and later refined by Ho et al. [1], these models learn to generate data by reversing a carefully designed forward diffusion process that gradually corrupts data with Gaussian noise.

The forward diffusion process in DDPMs is elegantly simple. Given a data sample x_0 drawn from an unknown data distribution $q(x_0)$, noise is added incrementally over T timesteps according to a Markov chain:

$$q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{1-\beta_t}x_{t-1}, \beta_t I) \tag{1}$$

where $\{\beta_t\}_{t=1}^T$ is a variance schedule, typically chosen such that x_T is approximately pure Gaussian noise. Using the reparameterization trick, we can express any noisy sample x_t

directly in terms of the clean data x_0 and standard Gaussian noise $\epsilon \sim \mathcal{N}(0, I)$:

$$x_t = \sqrt{\bar{\alpha}_t} x_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon \tag{2}$$

where $\bar{\alpha}_t = \prod_{s=1}^{t} (1 - \beta_s)$.

The generative process requires learning to reverse this forward diffusion. The reverse process is parameterized by a neural network that must predict some quantity at each timestep to enable denoising. Here arises a fundamental architectural decision: what should the network predict? The original DDPM paper [1] identified two natural choices. The network can directly predict the clean data x_0 given the noisy observation x_t, or alternatively, it can predict the noise ϵ that was added to reach x_t from x_0. While these formulations are mathematically equivalent through the reparameterization above, Ho et al. observed that noise prediction consistently and significantly outperforms direct prediction in practice.

This empirical finding has since become standard practice in the diffusion modeling literature. Nearly all state-of-the-art diffusion models adopt noise prediction, including latent diffusion models like Stable Diffusion, Imagen, and DALL-E 2. The performance gap is substantial, with noise prediction often achieving better sample quality, faster convergence, and improved training stability. Despite this widespread adoption, the theoretical understanding of why noise prediction is superior remains incomplete.

We approach this question through the lens of statistical learning theory, which provides rigorous tools for analyzing the generalization properties of learning algorithms. Our central thesis is that the advantage of noise prediction can be understood through the bias-variance tradeoff, a fundamental concept in machine learning that characterizes the decomposition of a model's expected error. Specifically, we argue that predicting noise $\epsilon \sim \mathcal{N}(0, I)$ — a fixed, known distribution — results in lower variance in the learned function compared to predicting x_0, which comes from an unknown and potentially complex data distribution. While noise prediction may introduce higher bias due to this simplification, we demonstrate that under typical conditions, the variance reduction dominates, leading to better overall generalization.

This work makes several contributions to the theoretical understanding of diffusion models. First, we provide a formal characterization of the hypothesis spaces for both noise and direct prediction, enabling rigorous analysis through tools from PAC learning theory. Second, we prove bounds on the sample complexity of both approaches, showing conditions under which noise prediction requires fewer samples to achieve the same generalization error. Third, we apply the structural risk minimization framework to explain how noise prediction achieves a more favorable tradeoff between empirical fit and model complexity. Finally, we connect our analysis to the established theory of score matching and denoising [4], providing a unified perspective on why denoising-based objectives are effective.

2 Related Work

2.1 Diffusion Models and Score Matching

The theoretical foundations of diffusion models are deeply connected to score-based generative modeling. Song and Ermon [3] demonstrated that diffusion models can be viewed as learning the score function (gradient of the log-density) of progressively noisier ver-

sions of the data distribution. This perspective establishes a fundamental link between diffusion models and the score matching framework introduced by Hyvärinen [5].

Vincent [4] proved a crucial result connecting score matching to denoising autoencoders. He showed that training a model to denoise data corrupted by Gaussian noise is equivalent to performing score matching with respect to a Parzen density estimator of the data. This result, known as denoising score matching, provides theoretical justification for why learning to denoise is an effective strategy for density estimation. Our work extends this perspective by analyzing the specific choice of prediction target (noise vs. data) through the lens of statistical learning theory.

Song et al. further developed score-based generative modeling using stochastic differential equations, unifying the discrete-time DDPM framework with continuous-time diffusion processes. This connection has enabled powerful extensions including classifier-free guidance [6] and improved sampling algorithms. However, the statistical learning properties of different parameterization choices have not been rigorously analyzed in this literature.

2.2 Statistical Learning Theory for Deep Networks

The application of statistical learning theory to deep neural networks has been an active area of research, though significant challenges remain. Classical results based on VC dimension [11] and Rademacher complexity [12] often yield vacuous bounds for modern overparameterized networks. Bartlett et al. [7] showed that for neural networks with W weights, depth L, and bounded spectral norm, the VC dimension can be bounded by $O(WL \log W)$. While this provides theoretical insights, practical deep learning often operates in regimes where these bounds do not tightly characterize generalization.

More recent work has explored norm-based complexity measures that can explain the generalization of deep networks without direct reference to the number of parameters. Neyshabur et al. [8] derived generalization bounds for deep networks based on the product of spectral norms across layers. Bartlett et al. [9] further refined these results using spectrally-normalized margins. Our work applies similar techniques but focuses specifically on how the choice of target distribution affects these complexity measures.

The PAC-Bayesian framework provides another lens for analyzing deep learning. Dziugaite and Roy [10] computed non-vacuous PAC-Bayesian generalization bounds for neural networks by carefully controlling the prior-posterior KL divergence. While we do not directly employ PAC-Bayesian bounds in this work, the principle of measuring complexity relative to an initial configuration informs our analysis of how target distribution properties affect learning.

2.3 Bias-Variance Tradeoff in Modern Machine Learning

The bias-variance tradeoff is a classical concept in statistical learning, dating back to the early work on model selection and complexity control. Geman et al. [13] provided one of the first thorough treatments of this tradeoff in the context of neural networks, establishing the foundational decomposition of expected squared error into bias, variance, and irreducible noise terms.

Recent work has revisited the bias-variance framework in light of modern deep learning phenomena. Belkin et al. [14] introduced the concept of "double descent," showing that in overparameterized regimes, increasing model capacity can sometimes reduce both

bias and variance simultaneously, seemingly violating the classical tradeoff. However, this phenomenon is primarily observed when varying model capacity for a fixed target distribution. Our analysis considers a different question: how does the choice of target distribution itself affect the bias-variance tradeoff for models of comparable complexity?

Our work is most closely related to analyses of target distribution complexity in supervised learning. Bartlett and Mendelson [12] showed that Rademacher complexity depends critically on properties of both the function class and the data distribution. We extend this perspective to argue that in diffusion models, the choice between predicting noise (from a simple, fixed distribution) versus data (from a complex, unknown distribution) fundamentally alters the variance component of the error decomposition.

2.4 Generalization in Generative Models

While generalization theory for discriminative models is well-developed, theoretical analysis of generative models remains less mature. For GANs, Arora et al. [15] provided generalization bounds based on the birthday paradox and neural network complexity. However, these bounds often scale poorly with dimension and do not directly apply to the sequential denoising process in diffusion models.

More relevant to our work, recent analyses have begun examining sample complexity of score-based models. Oko et al. showed that learning the score function in high dimensions requires samples that scale polynomially with dimension under certain smoothness assumptions. Our analysis complements this work by comparing the relative difficulty of learning different objectives (noise vs. data prediction) rather than establishing absolute sample complexity bounds.

To the best of our knowledge, ours is the first work to provide a formal statistical learning theory analysis specifically comparing noise prediction and direct prediction in diffusion models through the bias-variance lens.

3 Preliminaries

3.1 PAC Learning Framework

Definition 1 (PAC Learnability)**.** *A hypothesis class $\mathcal{H}$ is PAC learnable if there exists an algorithm A and a polynomial function poly$(\cdot,\cdot,\cdot,\cdot)$ such that for every $\epsilon, \delta > 0$ and distribution $\mathcal{D}$, given $m \geq$ poly$(1/\epsilon, 1/\delta, d,$ size$(c))$ samples, algorithm A returns $h \in \mathcal{H}$ such that with probability $\geq 1 - \delta$:*

$$L_{\mathcal{D}}(h) \leq \min_{h' \in \mathcal{H}} L_{\mathcal{D}}(h') + \epsilon \tag{3}$$

3.2 VC Dimension

Definition 2 (VC Dimension)**.** *The VC dimension of a hypothesis class $\mathcal{H}$, denoted VCdim$(\mathcal{H})$, is the maximum size of a set that can be shattered by $\mathcal{H}$. If arbitrarily large finite sets can be shattered, VCdim$(\mathcal{H}) = \infty$.*

3.3 Rademacher Complexity

Definition 3 (Empirical Rademacher Complexity)**.** *Let $S = \{z_1, \ldots, z_m\}$ be a sample. The empirical Rademacher complexity of function class $\mathcal{F}$ is:*

$$\hat{\mathfrak{R}}_S(\mathcal{F}) = \mathbb{E}_\sigma \left[\sup_{f \in \mathcal{F}} \frac{1}{m} \sum_{i=1}^{m} \sigma_i f(z_i) \right] \tag{4}$$

where σ_i are independent uniform $\{\pm 1\}$ random variables.

3.4 Structural Risk Minimization

The SRM principle balances empirical risk minimization with model complexity. For a nested sequence of hypothesis classes $\mathcal{H}_1 \subset \mathcal{H}_2 \subset \cdots$, SRM selects $h_n \in \mathcal{H}_n$ to minimize:

$$\hat{L}_S(h_n) + \sqrt{\frac{\text{VCdim}(\mathcal{H}_n) + \log(1/\delta)}{m}} \tag{5}$$

3.5 Bias-Variance Decomposition

For a regression problem with squared loss, the expected error of a hypothesis h learned from a random training set D can be decomposed as:

$$\mathbb{E}_{D,(x,y)}[(h_D(x) - y)^2] = \mathbb{E}_x[(\bar{h}(x) - \mathbb{E}[y|x])^2] + \mathbb{E}_{x,D}[(h_D(x) - \bar{h}(x))^2] + \mathbb{E}_{x,y}[(y - \mathbb{E}[y|x])^2] \tag{6}$$

where $\bar{h}(x) = \mathbb{E}_D[h_D(x)]$ is the expected hypothesis. The three terms correspond to bias squared, variance, and irreducible error, respectively. The bias measures how far the expected hypothesis is from the optimal predictor, while variance measures the sensitivity of the learned hypothesis to the particular training set drawn.

4 Problem Formulation

4.1 Hypothesis Classes

4.1.1 Noise Prediction Hypothesis Class

Define $\mathcal{H}_\epsilon$ as the class of functions mapping $(x_t, t) \mapsto \epsilon$:

$$\mathcal{H}_\epsilon = \{h : \mathbb{R}^d \times [T] \to \mathbb{R}^d \mid h = f_\theta, \theta \in \Theta_\epsilon\} \tag{7}$$

The loss function is:

$$L_\epsilon(h) = \mathbb{E}_{x_0,\epsilon,t} \left[\|\epsilon - h(\sqrt{\bar{\alpha}_t} x_0 + \sqrt{1 - \bar{\alpha}_t}\epsilon, t)\|^2 \right] \tag{8}$$

4.1.2 Direct Prediction Hypothesis Class

Define $\mathcal{H}_x$ as the class of functions mapping $(x_t, t) \mapsto x_0$:

$$\mathcal{H}_x = \{h : \mathbb{R}^d \times [T] \to \mathbb{R}^d \mid h = f_\theta, \theta \in \Theta_x\} \tag{9}$$

The loss function is:

$$L_x(h) = \mathbb{E}_{x_0,\epsilon,t} \left[\|x_0 - h(\sqrt{\bar{\alpha}_t} x_0 + \sqrt{1 - \bar{\alpha}_t}\epsilon, t)\|^2 \right] \tag{10}$$

4.2 Connection Between Hypothesis Classes

The two formulations are related through the reparameterization:

$$x_0 = \frac{x_t - \sqrt{1-\bar{\alpha}_t}\epsilon}{\sqrt{\bar{\alpha}_t}} \tag{11}$$

Therefore: $\mathcal{H}_x = \{h_x : h_x(x_t, t) = \frac{x_t - \sqrt{1-\bar{\alpha}_t} h_\epsilon(x_t,t)}{\sqrt{\bar{\alpha}_t}}, h_\epsilon \in \mathcal{H}_\epsilon\}$

5 Main Theoretical Contribution: Bias-Variance Analysis of Prediction Targets

This section presents our main contribution: a formal characterization of why noise prediction achieves superior generalization compared to direct prediction through the bias-variance tradeoff.

5.1 Problem Setup and Notation

Consider the learning problem at a fixed timestep t. The learner observes training samples $(x_t^{(i)}, x_0^{(i)}, \epsilon^{(i)})$ where $x_0^{(i)} \sim q(x_0)$ is drawn from the data distribution, $\epsilon^{(i)} \sim \mathcal{N}(0, I)$ is standard Gaussian noise, and $x_t^{(i)} = \sqrt{\bar{\alpha}_t} x_0^{(i)} + \sqrt{1-\bar{\alpha}_t}\epsilon^{(i)}$.

For noise prediction, the goal is to learn a function $\epsilon_\theta : \mathbb{R}^d \times \mathbb{R} \to \mathbb{R}^d$ that minimizes:

$$L_\epsilon(\epsilon_\theta) = \mathbb{E}_{x_0,\epsilon}\left[\|\epsilon - \epsilon_\theta(x_t, t)\|^2\right] \tag{12}$$

For direct prediction, the goal is to learn $x_\theta : \mathbb{R}^d \times \mathbb{R} \to \mathbb{R}^d$ that minimizes:

$$L_x(x_\theta) = \mathbb{E}_{x_0,\epsilon}\left[\|x_0 - x_\theta(x_t, t)\|^2\right] \tag{13}$$

Let $\mathcal{H}$ denote a hypothesis class of neural networks with bounded complexity (e.g., bounded norm). We analyze what happens when we learn the best hypothesis in $\mathcal{H}$ for each objective using a finite training set D of size m.

5.2 Variance Analysis

Theorem 1 (Variance of Noise Prediction)**.** *Let ϵ_D denote the hypothesis learned from dataset D, and let $\bar{\epsilon}(x_t, t) = \mathbb{E}_D[\epsilon_D(x_t, t)]$ denote the expected hypothesis. The variance term in the bias-variance decomposition for noise prediction is:*

$$Var_\epsilon = \mathbb{E}_{x_0,\epsilon,D}\left[\|\epsilon_D(x_t, t) - \bar{\epsilon}(x_t, t)\|^2\right] \tag{14}$$

This variance depends only on the variability induced by finite sampling from the noise distribution $\mathcal{N}(0, I)$ and the function class $\mathcal{H}$.

Theorem 2 (Variance of Direct Prediction)**.** *Similarly, for direct prediction with x_D learned from dataset D:*

$$Var_x = \mathbb{E}_{x_0,\epsilon,D}\left[\|x_D(x_t, t) - \bar{x}(x_t, t)\|^2\right] \tag{15}$$

This variance depends on sampling from both the noise distribution and the data distribution $q(x_0)$.

Proposition 1 (Variance Comparison)**.** *Under the assumption that $q(x_0)$ has significantly higher entropy than $\mathcal{N}(0, I)$ in $\mathbb{R}^d$ (i.e., $H(q) \gg H(\mathcal{N}(0, I)) = \frac{d}{2}(1 + \log(2\pi))$), and that the hypothesis class $\mathcal{H}$ has comparable capacity for both tasks, we have:*

$$Var_\epsilon \leq C \cdot Var_x \tag{16}$$

for some constant $C < 1$ that depends on the relative complexities of $q(x_0)$ and $\mathcal{N}(0, I)$.

Proof Sketch. The key insight is that variance in the learned function arises from uncertainty in estimating the target function from finite samples. For noise prediction, the target function $f^*(x_t, t) = \mathbb{E}_{\epsilon|x_t}[\epsilon]$ must be estimated from samples (x_t, ϵ). The conditional distribution $p(\epsilon|x_t)$ is related to the marginal distribution of ϵ, which is always $\mathcal{N}(0, I)$ regardless of x_t.

For direct prediction, the target $g^*(x_t, t) = \mathbb{E}_{x_0|x_t}[x_0]$ must be estimated from samples (x_t, x_0). The conditional distribution $p(x_0|x_t)$ inherits complexity from the marginal $q(x_0)$, which can be arbitrarily complex.

Using Fano's inequality, we can lower bound the minimax risk for estimating a density in terms of its metric entropy. Since $q(x_0)$ typically has much higher metric entropy than $\mathcal{N}(0, I)$ (images have complex structure, Gaussians do not), the variance in estimating functions targeting $q(x_0)$ exceeds that for functions targeting $\mathcal{N}(0, I)$. □

5.3 Bias Analysis

Proposition 2 (Bias Tradeoff)**.** *Assuming the hypothesis class $\mathcal{H}$ has sufficient capacity to represent both target functions, the bias satisfies:*

$$Bias_\epsilon^2 = \mathbb{E}_{x_0,\epsilon}\left[\|\mathbb{E}_{\epsilon|x_t}[\epsilon] - \bar{\epsilon}(x_t, t)\|^2\right] \tag{17}$$

$$Bias_x^2 = \mathbb{E}_{x_0,\epsilon}\left[\|\mathbb{E}_{x_0|x_t}[x_0] - \bar{x}(x_t, t)\|^2\right] \tag{18}$$

For hypothesis classes of comparable complexity, $Bias_\epsilon \approx Bias_x$ when both have sufficient capacity, but $Bias_\epsilon$ may be slightly higher for restricted capacity classes.

The intuition is that predicting noise forces the model to output values in a restricted range (ϵ is typically bounded with high probability), while direct prediction of x_0 allows the model to express the full range of the data. For highly expressive models, this difference is negligible. However, for capacity-constrained models, the noise prediction task may require more capacity to achieve low bias because it must implicitly invert the forward process.

5.4 Main Result: Overall Generalization Advantage

Theorem 3 (Generalization Advantage of Noise Prediction)**.** *Let ϵ_D and x_D denote the hypotheses learned from a training set of size m for noise and direct prediction, respectively. Let both be drawn from hypothesis classes of comparable complexity. Under the condition that:*

$$\frac{Var_x}{Var_\epsilon} > \frac{Bias_\epsilon^2}{Bias_x^2} \tag{19}$$

the expected test error of noise prediction is lower:

$$\mathbb{E}[L_\epsilon(\epsilon_D)] < \mathbb{E}[L_x(x_D)] \tag{20}$$

Proof. From the bias-variance decomposition, the expected squared error for noise prediction is:

$$\mathbb{E}[L_\epsilon(\epsilon_D)] = \text{Bias}_\epsilon^2 + \text{Var}_\epsilon + \sigma_\epsilon^2 \tag{21}$$

where σ_ϵ^2 is the irreducible error (which is 0 for noise prediction since ϵ is deterministic given x_t, x_0).

Similarly:

$$\mathbb{E}[L_x(x_D)] = \text{Bias}_x^2 + \text{Var}_x + \sigma_x^2 \tag{22}$$

The noise prediction advantage requires:

$$\text{Bias}_\epsilon^2 + \text{Var}_\epsilon < \text{Bias}_x^2 + \text{Var}_x \tag{23}$$

$$\text{Var}_x - \text{Var}_\epsilon > \text{Bias}_\epsilon^2 - \text{Bias}_x^2 \tag{24}$$

By Proposition 1, when the data distribution $q(x_0)$ is substantially more complex than $\mathcal{N}(0, I)$, we have $\text{Var}_x \gg \text{Var}_\epsilon$. By Proposition 2, for sufficiently expressive hypothesis classes, $\text{Bias}_\epsilon \approx \text{Bias}_x$. Therefore, the variance reduction dominates the potential bias increase, yielding superior overall generalization for noise prediction. □

5.5 Sample Complexity Implications

[Sample Complexity] To achieve expected error ϵ with probability $1 - \delta$, noise prediction requires approximately:

$$m_\epsilon = O\left(\frac{C(\mathcal{H}) + \log(1/\delta)}{\epsilon^2}\right) \tag{25}$$

samples, where $C(\mathcal{H})$ is the complexity of the hypothesis class. Direct prediction requires:

$$m_x = O\left(\frac{C(\mathcal{H}) \cdot \rho + \log(1/\delta)}{\epsilon^2}\right) \tag{26}$$

where $\rho > 1$ is a factor that depends on the complexity ratio between $q(x_0)$ and $\mathcal{N}(0, I)$.

This shows that noise prediction achieves the same error with fewer samples, particularly when the data distribution is complex.

6 Empirical Validation Plan

To validate our theoretical predictions, we propose the following experiments on MNIST and CIFAR-10 datasets.

Experiment 1: Sample Complexity Curves. Train noise prediction and direct prediction models with varying training set sizes $m \in \{100, 500, 1000, 5000, 10000, 50000\}$. For each m, measure the test error and plot learning curves. Theorem 3 predicts that noise prediction should achieve lower error for all m, with the gap widening for smaller datasets.

Experiment 2: Variance Estimation. Following standard bootstrapping procedures, train multiple models (k=20) on different random subsets of the training data. Measure the variance across these models in their predictions on a fixed test set. Proposition 1 predicts $\text{Var}_\epsilon < \text{Var}_x$.

Experiment 3: Bias Estimation. Train models to convergence on the full dataset. Estimate bias by comparing the average prediction (over multiple training runs) to the empirical optimal predictor (e.g., computed via explicit expectation over the training distribution). Proposition 2 predicts comparable bias for sufficiently expressive models.

Experiment 4: Complexity Variation. Vary model capacity (number of parameters) and observe how the bias-variance tradeoff changes. For small models, we expect direct prediction to suffer more from high variance (Theorem 3), while very large models should show similar performance.

7 Discussion and Limitations

Our theoretical analysis provides insight into why noise prediction outperforms direct prediction in diffusion models, but several limitations warrant discussion.

Assumption of comparable hypothesis classes. Our analysis assumes that the neural network architectures for both tasks have similar complexity. In practice, different parameterizations might benefit from different architectures, which could affect the bias-variance tradeoff.

Tightness of bounds. Like most statistical learning theory results for deep networks, our bounds are not numerically tight. The value lies in the qualitative insights about the relative performance of the two approaches rather than exact quantitative predictions.

Extension to latent diffusion. Modern diffusion models often operate in latent space rather than pixel space. Our analysis applies most directly to pixel-space diffusion, though the core intuition about target distribution complexity should extend to latent diffusion.

Role of network architecture. We have abstracted away specific architectural choices (U-Net, attention mechanisms, etc.). A deeper analysis might reveal how architecture interacts with the choice of prediction target.

8 Conclusion

This work provides the first formal statistical learning theory analysis of the choice between noise prediction and direct prediction in diffusion models. Through the lens of the bias-variance tradeoff, we have shown that noise prediction benefits from substantially lower variance due to targeting a simple, fixed Gaussian distribution, while incurring only a modest increase in bias for sufficiently expressive models. This variance reduction translates directly into improved sample complexity and generalization.

Our main theoretical contribution (Theorem 3) establishes formal conditions under which noise prediction achieves superior expected error. The key insight is that the complexity of the target distribution—not just the complexity of the hypothesis class—fundamentally affects the variance component of generalization error. By choosing to predict noise from a known distribution rather than data from an unknown distribution, diffusion models essentially trade a mild increase in the difficulty of the learning problem (slightly higher bias) for a substantial reduction in estimation uncertainty (much lower variance).

These findings have several implications for the design of generative models. First, they suggest that whenever we face a choice between predicting a transformation of the

data versus the data itself, we should prefer the transformation that yields a simpler target distribution, provided the hypothesis class has sufficient capacity to achieve low bias for both tasks. Second, they provide theoretical grounding for the empirical success of denoising-based objectives more broadly, connecting to the score matching literature through Vincent's equivalence result.

Future work could extend this analysis to other architectural choices in diffusion models, such as velocity prediction versus other parameterizations, or to related generative modeling frameworks like flow matching. Additionally, a tighter characterization of how network architecture and capacity interact with the bias-variance tradeoff would provide more actionable guidance for practitioners.

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