MA2202S Cheat Sheet

1 Cyclic groups

1. If a group G is cyclic, then
   (a) it is Abelian.
2. Equivalent statements:
   (a) G is cyclic.
   (b) Any subgroup $H \leq G$ is cyclic (and Abelian).

2 Abelian groups

1. Finite, simple Abelian $\iff$ C_p cyclic groups of prime order.
2. Equivalent statements:
   (a) G is Abelian.
   (b) All Sylow p-subgroups of G are normal Abelian.
   (c) $G/Z(G)$ is cyclic.

3 P-groups and P-subgroups

1. If G is a p-group, then
   (a) the center $Z(G)$ is non-trivial.
   (b) There is a normal subgroup of order p^k for all $p^k \leq G^a = |G|$.
   (c)
2. If G is a p-group, the center $Z(G)$ is non-trivial.
3. Given G, if a Sylow p-subgroup is normal $\iff$ characteristic in G $\iff$ unique for given p.
4. Equivalent statements:
   (a) For any prime $p \mid |G|$, Sylow p-subgroups are normal.
   (b) G is a direct product of all its Sylow p-subgroups .

4 Direct products and Semi-direct products