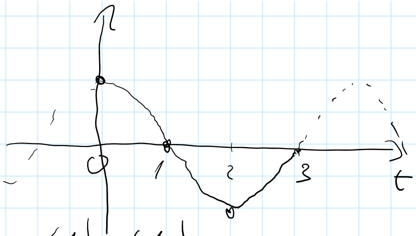
P 12

Donnerstag, 8. Dezember 2022 08:33

c) $c = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

diskreter Cosinus



$$\hat{c}(0) = \frac{\langle c \mid s_0 \rangle}{\| s_0 \|^2} = \frac{\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}{4} = \frac{1 \cdot 1 + 0 \cdot 1 - 1 \cdot 1 + 0 \cdot 1}{4} = 0$$

$\langle s_0 \mid s_0 \rangle = \| s_0 \|^2$

$$\hat{c}(1) = \frac{\langle c \mid s_1 \rangle}{\| s_1 \|^2} = \frac{\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix}}{4} = \frac{1 \cdot 1 + 0(-i) + (-1)(-1) + 0 \cdot i}{4} = \frac{2}{4}$$

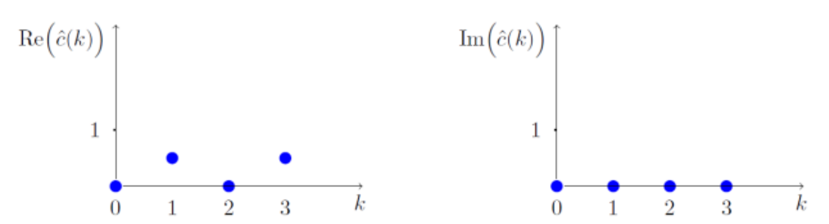
Achtung: Komplex konjugierter s_1

Skalarprodukt im Komplexen!

$$\hat{c}(2) = \frac{\langle c \mid s_2 \rangle}{\| s_2 \|^2} = \frac{\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}}{4} = \frac{1 + 0 - 1 + 0}{4} = 0$$

$$\hat{c}(3) = \frac{\langle c \mid s_3 \rangle}{\| s_3 \|^2} = \frac{\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix}}{4} = \frac{1 + 0 + 1 + 0}{4} = \frac{2}{4}$$

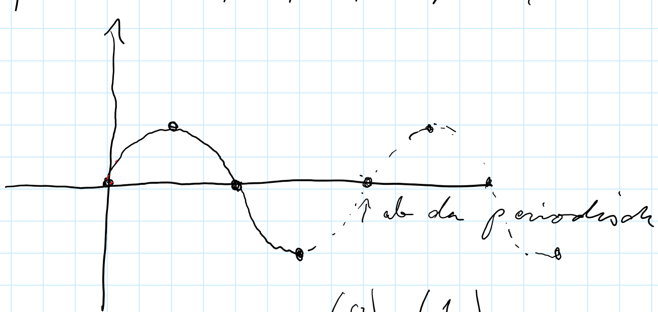
s_3 - Komplex konjugiert



(a) Realteile von $\hat{c}(k)$ über k.

(b) Imaginärteile von $\hat{c}(k)$ über k.

d) $s = (0, 1, 0, -1)^T$ der diskrete Sinus



Periode $= 4$

$f = \frac{1}{4}$

$$\hat{s}(0) = \frac{\langle s \mid s_0 \rangle}{\| s_0 \|^2} = \frac{\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \circ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}{4} = \frac{0 + 1 + 0 - 1}{4} = 0$$

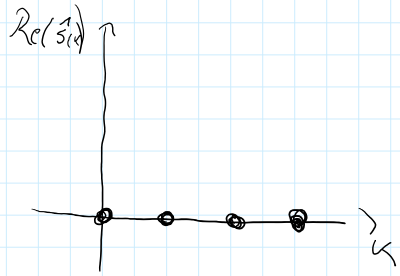
$$\hat{s}(0) = \frac{\ldots}{\|s_0\|^2} = \frac{\ldots}{4} = \frac{\ldots}{4} = 0$$

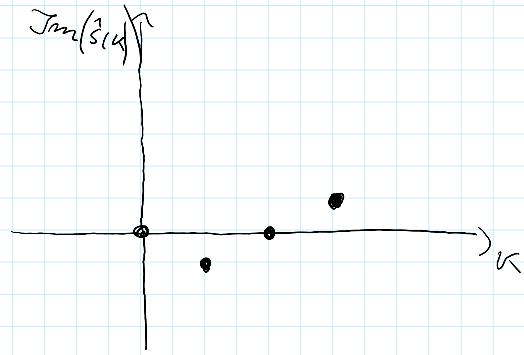
$$\hat{s}(1) = \frac{\langle s | s_1 \rangle}{\|s_1\|^2} = \frac{\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix}}{4} = \frac{0 - i + 0 - i}{4} = -\frac{2i}{4} = -\frac{i}{2}$$

$$= -\frac{1}{2} i$$

$$\hat{s}(2) = \frac{\langle s | s_2 \rangle}{\|s_2\|^2} = \frac{\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}}{4} = \frac{0 - 1 + 0 + 1}{4} = 0$$

$$\hat{s}(3) = \frac{\langle s | s_3 \rangle}{\|s_3\|^2} = \frac{\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix}}{4} = \frac{0 + i + 0 + i}{4} = \frac{2i}{4} = \frac{i}{2} = \frac{1}{2} i$$





c) Signalrekonstruktion aus Summe der Projektionen

$$\hat{c}(0)\, s_0 + \hat{c}(1)\, s_1 + \hat{c}(2)\, s_2 + \hat{c}(3)\, s_3$$

$$= 0 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix} + 0 \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} i \\ -\frac{1}{2} \\ -\frac{1}{2} i \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} i \\ -\frac{1}{2} \\ \frac{1}{2} i \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

f)

$$\hat{s}(0)\, s_0 + \hat{s}(1)\, s_1 + \hat{s}(2)\, s_2 + \hat{s}(3)$$

$$= 0 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{2} i \begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix} + 0 \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \frac{1}{2} i \begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} i \\ -\frac{1}{2} i^2 \\ \frac{1}{2} i \\ \frac{1}{2} i^2 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} i \\ \frac{1}{2} (-) i^2 \\ -\frac{1}{2} i \\ \frac{1}{2} i^2 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{2} + \frac{1}{2} \\ 0 \\ -\frac{1}{2} - \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$