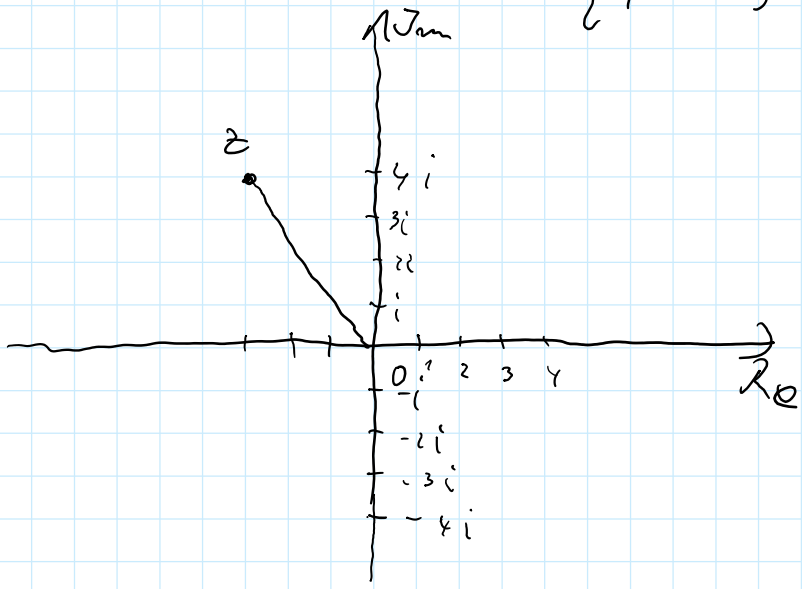
P9

$z = -3 + 4i$ $\quad \{z\} = \begin{Bmatrix} -3 & -4 \\ 4 & -3 \end{Bmatrix}$

a)



b) $|z| = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$z \cdot \overline{z} = (x+iy)(x-iy) = x^2 + y^2 = |z|^2$

$= 9 + 16 = 25$

c) $z^2 = (-3+4i)(-3+4i) = 9 - 12i - 12i + 16\underbrace{i^2}_{-1} = 9 - 16 - 24i$

$= -7 - 24i$

$\{z\} \cdot \{z\} = \begin{Bmatrix} -3 & -4 \\ 4 & -3 \end{Bmatrix} \begin{Bmatrix} -3 & -4 \\ 4 & -3 \end{Bmatrix}$

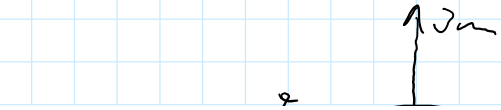
$= \begin{Bmatrix} 9-16 & 12+12 \\ -12-12 & -16+9 \end{Bmatrix} = \begin{Bmatrix} -7 & 24 \\ -24 & -7 \end{Bmatrix} = -7\mathbb{1} - 24\mathbb{i}$

d) $iz = i(-3+4i) = -3i + 4i^2 = -3i - 4 = -4 - 3i$

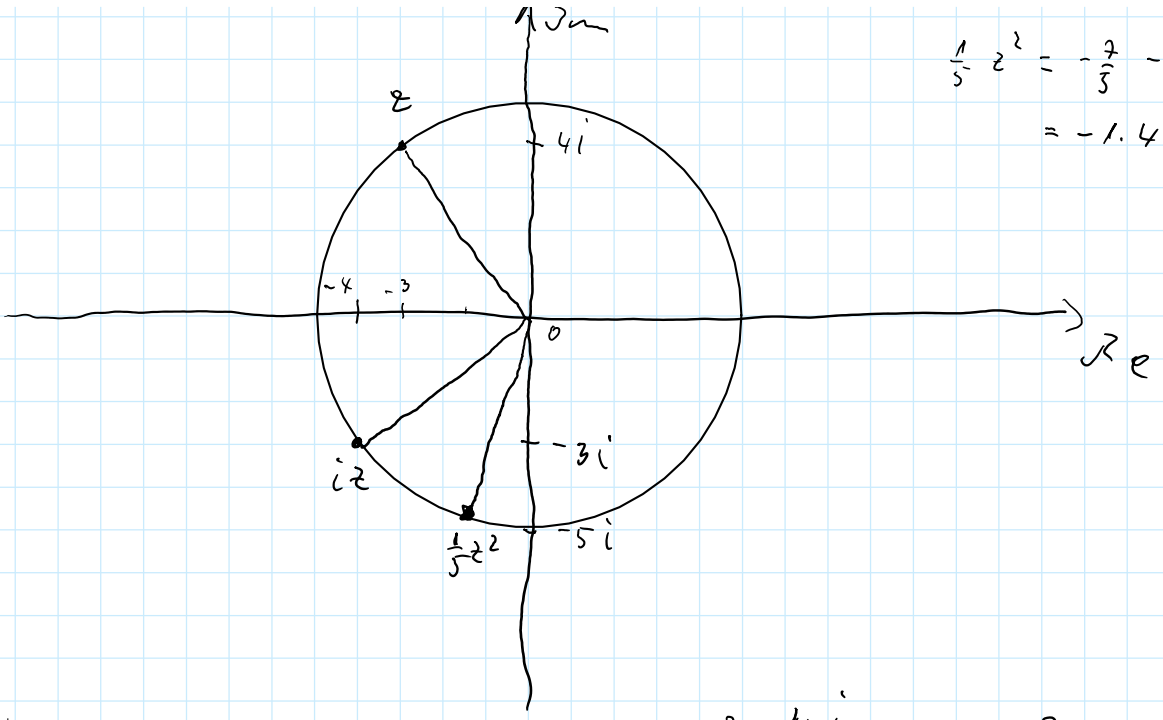
$\begin{Bmatrix} 0 & -1 \\ 1 & 0 \end{Bmatrix} \begin{Bmatrix} -3 & -4 \\ 4 & -3 \end{Bmatrix} = \begin{Bmatrix} -4 & 3 \\ -3 & -4 \end{Bmatrix} = -4\mathbb{1} - 3\mathbb{i}$

e) $|z^2| = |-7-24i| = \sqrt{49+576} = \sqrt{625} = 25$

$|iz| = |-4-3i| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$



$\frac{1}{5} z^2 = -\frac{7}{5} - \frac{24}{5} i$



$\frac{1}{5}z^2 = -\frac{7}{5} - \frac{24}{5}i$

$= -1.4 - 4.8i$

e) $l = z^{-1} = \frac{1}{|z|^2}\bar{z} = \frac{-3-4i}{25} = -\frac{3}{25} - \frac{4}{25}i$

$l \cdot z = \left(-\frac{3}{25} - \frac{4}{25}i\right)(-3+4i) = \frac{9}{25} - \frac{12}{25}i + \frac{12}{25}i - \frac{16}{25}i^2$ $\quad i^2 = -1$

$= \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$ ✓