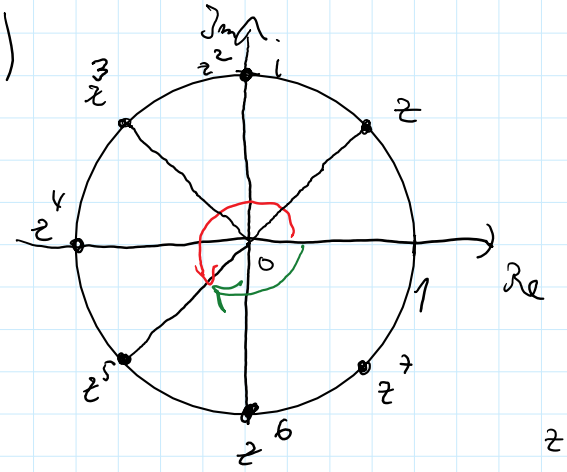
P 1.0

Donnerstag, 24. November 2022 09:02

a) $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$

$|z| = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{\frac{4}{4}} = 1$

b)



c) $z = r e^{i\varphi} = |z| e^{i\frac{\pi}{4}} = e^{i\frac{\pi}{4}}$

d) $z^2 = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)$

$= \frac{2}{4} + \frac{2}{4} i + \frac{2}{4} i + \frac{2}{4} i^2$

$= \frac{2}{4} + i - \frac{2}{4} = i$

$z^2 = e^{i\frac{\pi}{4}} \cdot e^{i\frac{\pi}{4}} = e^{i\frac{\pi}{2}}$

d $z^3 = z^2 \cdot z = e^{i\frac{\pi}{2}} e^{i\frac{\pi}{4}} = e^{i\frac{3\pi}{4}} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$ $\quad \frac{3\pi}{4} \hat{=} \frac{3 \cdot 180°}{4} = 135°$

$z^4 = e^{i\frac{3\pi}{4}} \cdot e^{i\frac{\pi}{4}} = e^{i\pi} = -1$

$z^5 = z^4 \cdot z = -1\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$

alternativ $z^5 = e^{i\pi} \cdot e^{i\frac{\pi}{4}} = e^{i\frac{5\pi}{4}} = e^{-i\frac{3\pi}{4}}$

$z^6 = z^5 \cdot z = e^{-i\frac{3\pi}{4}} \cdot e^{i\frac{\pi}{4}} = e^{i\left(-\frac{3\pi}{4} + \frac{\pi}{4}\right)} = e^{i\left(-\frac{2\pi}{4}\right)} = e^{-\frac{\pi}{2} i}$

$z^7 = z^6 \cdot z = e^{-\frac{\pi}{2} i} \cdot e^{i\frac{\pi}{4}} = e^{i\left(-\frac{\pi}{2} + \frac{\pi}{4}\right)} = e^{i\left(-\frac{\pi}{4}\right)} = e^{-\frac{\pi}{4} i}$

$z^8 = z^7 \cdot z = e^{-\frac{\pi}{4} i} \cdot e^{i\frac{\pi}{4}} = e^0 = 1.$

$z^9 = z^8 \cdot z = 1 \cdot z = z$ ab jetzt Periodisch