Graphs

What is a graph?#

A **graph** is a set of nodes that are connected to each other in the form of a network. Let's define the two basic components of a graph.

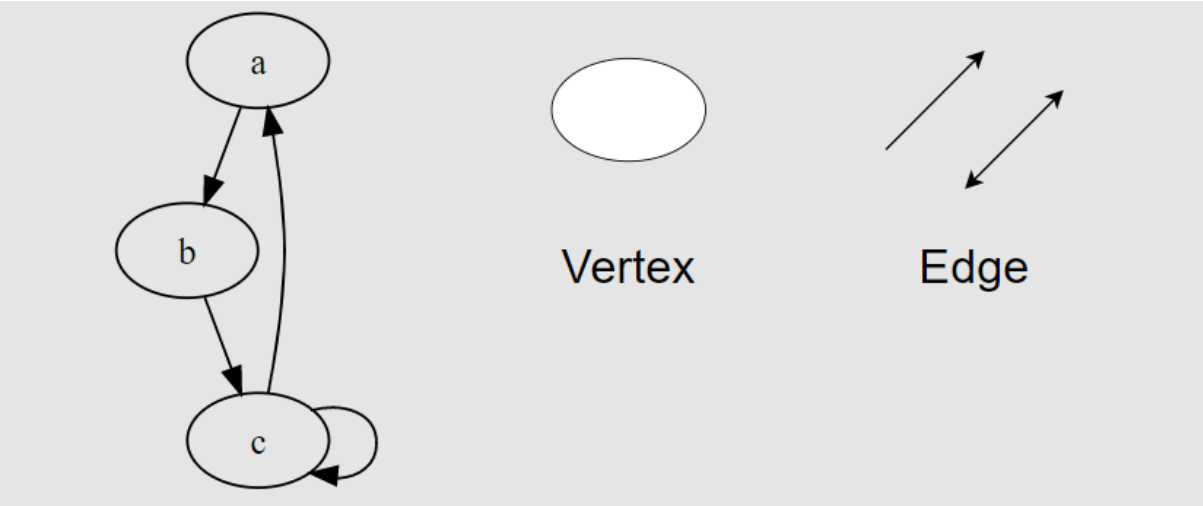
Vertex#

A **vertex** (or node) is an essential part of a graph. A collection of vertices forms a graph. In that sense, vertices are similar to linked list nodes.

Edge#

An **edge** is a link between two vertices. It can be directed or undirected. An undirected edge (u, v)(u,v) has no sense of direction and can be traversed in either direction - uu to vv or vv to uu. A directed edge (u, v)(u,v), on the other hand can only be traversed from uu to vv. Visually, a directed edge is drawn with an arrow and an undirected edge is drawn without an arrow.

> An edge can exist from one vertex to itself. This is called a self loop



Path#

A path between nodes uu and vv in a graph is an alternating sequence of nodes and edges, such that it:

- starts with u

- ends with *v*

- all nodes and edges in the path are present in the graph

Cycle#

A path that starts and ends at the same node in a graph, is called a cycle.

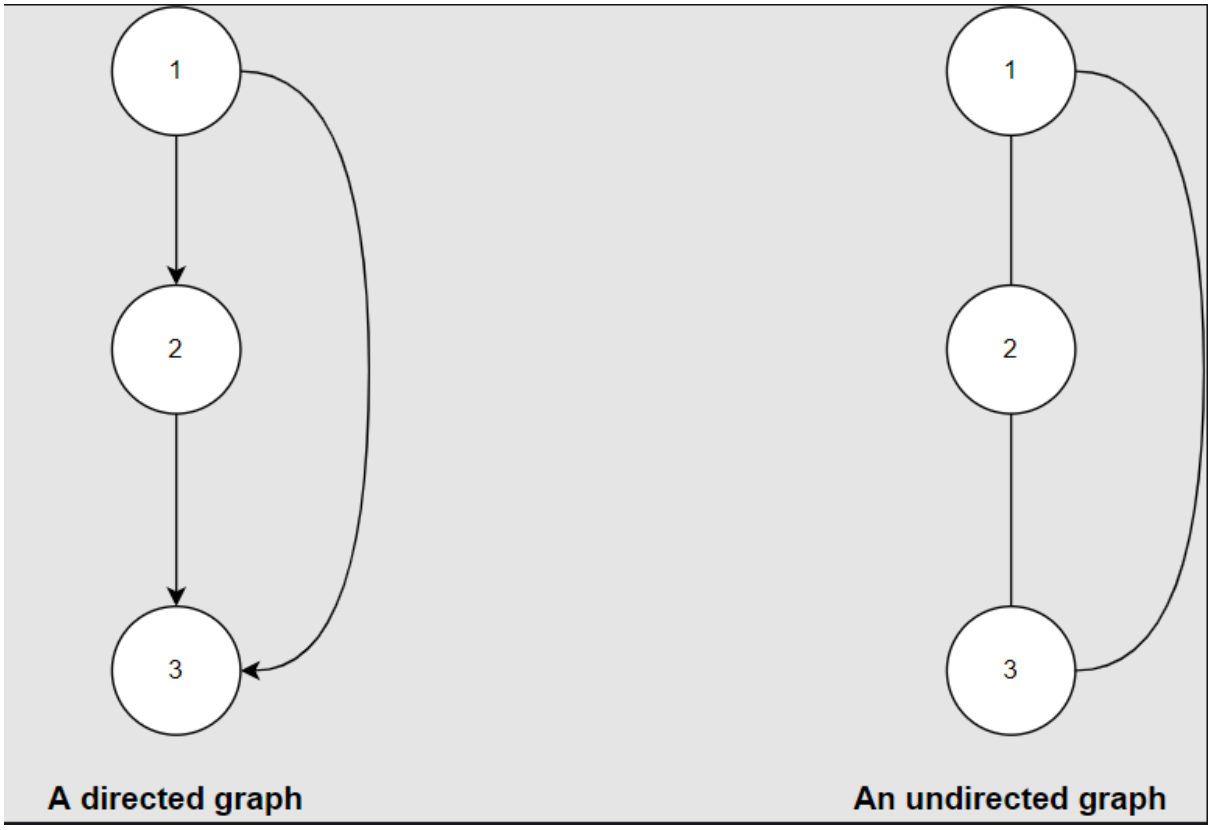
Types of graphs#

Graphs can be categorized across multiple criteria, such as:

- Undirected vs directed graph
- Weighted vs unweighted graph
- Connected graph

Undirected vs directed#

In an undirected graph, all the edges are undirected, whereas in aa directed graph, all the edges are directed.



Weighted vs unweighted graph#

In an unweighted graph, the edges have no weight associated with them. The above examples are both unweighted graphs. When representing a social network, for example, where an edge represents friendship between two people, no weight is needed on the edge.

In a weighted graph, each edge is associated with a weight. The weight may be used to represent certain properties of a problem. For instance, if an edge represents a road between two cities, the distance between the two cities may be represented as the edge weight.

Connected graph#

An undirected graph in which there is at least one path between any two pairs of nodes is called a connected graph.

> A tree is a connected graph in which there are no cycles

Shortest path#

When we talk about path in a graph, we are mostly interested in the *shortest* path between two vertices in a graph. A *shortest* path in a weighted graph is the one for which the sum of edge weights is minimum. In case of an unweighted graph, the shortest path is the one with the minimum number of edges.

Ways to represent a graph#

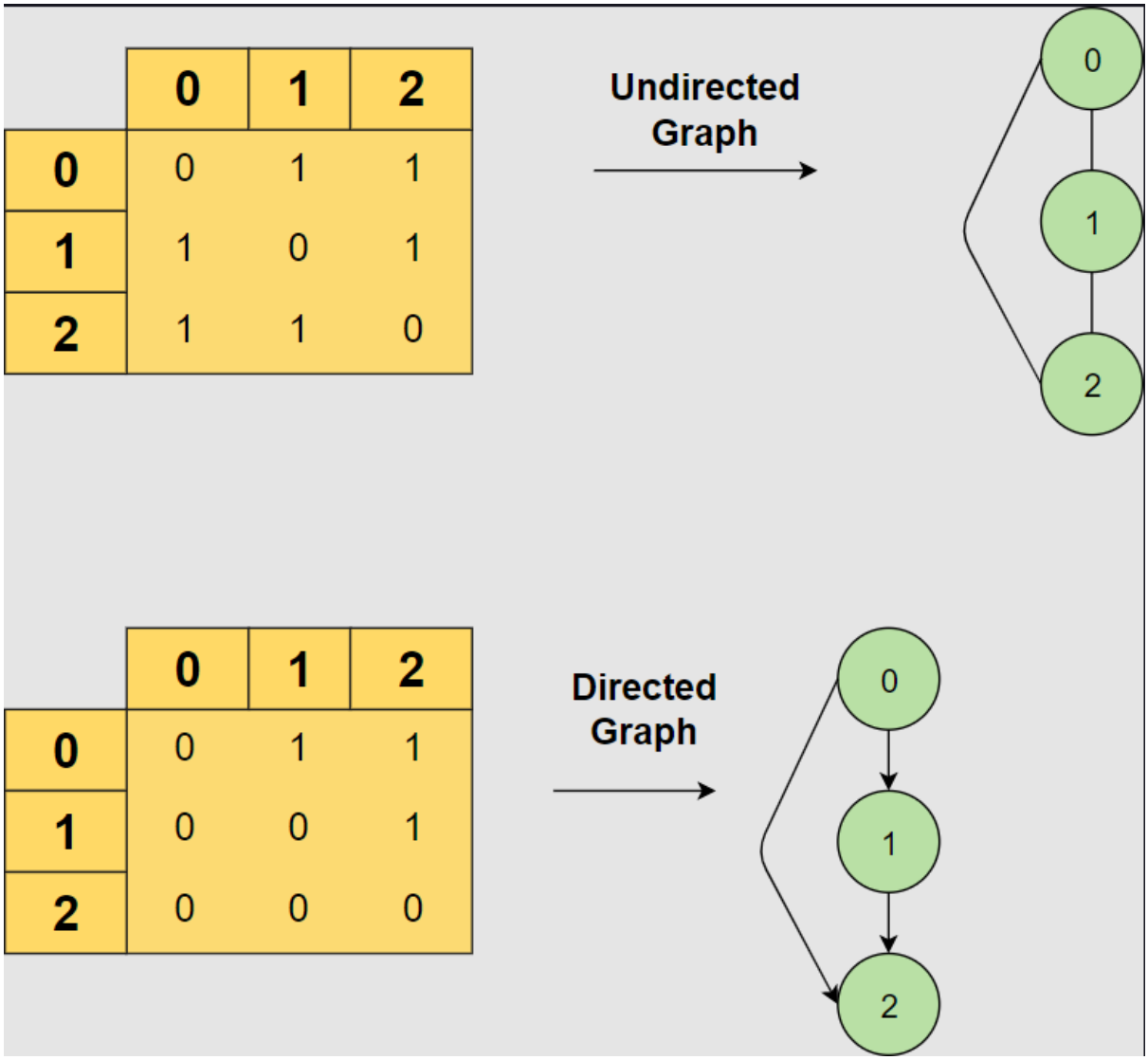
The two most common ways to represent a graph are:

- Adjacency Matrix
- Adjacency List

Adjacency matrix#

The **adjacency matrix** is a two-dimensional matrix where each cell can contain a **0** or a **1**. The row and column headings represent the vertices.

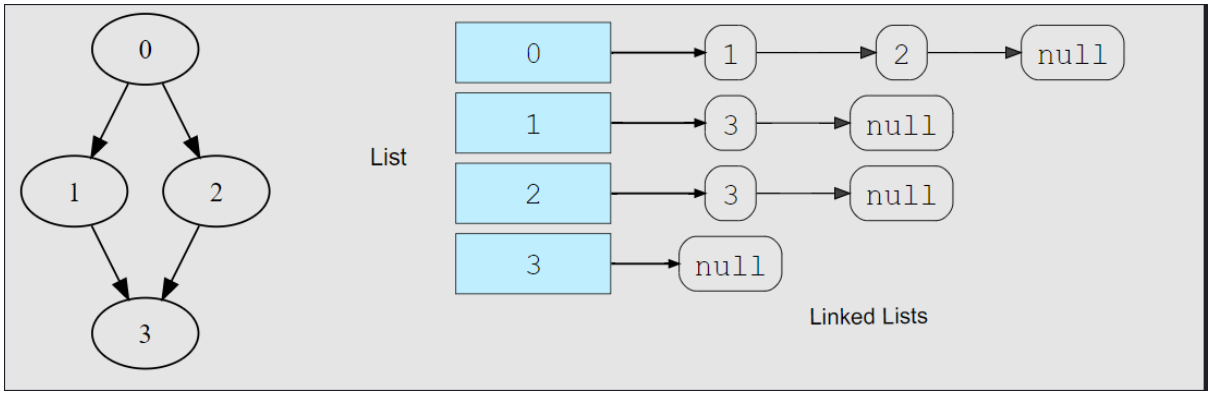
If a cell contains **1**, there exists an edge between the corresponding vertices e.g., Matrix[0][1]=1*Matrix*[0][1]=1 shows that an edge exists between vertex **0** and **1**.



Adjacency list#

An array of linked list is used to store all the edges in the graph. The size of the array is equal to the number of vertices.

The entry at index `i` of the array contains a linked list containing the vertices that are adjacent to vertex `i`.



The above examples represented unweighted graphs using adjacency matrix and list. To represent weighted graph, instead of 1, we need to place the weight of the corresponding edge.