Sahyadri Classes

Mathematics

Time : 1.00 hr XII - A Div **Marks : 50**

Vector Algebra

1) The volume of the parallelopiped, whose edges are represented by $-12\hat{i}+\alpha\hat{k}, 3\hat{j}-\hat{k}, 2\hat{i}+\hat{j}-15\hat{k}$, is 546, then α is :

A) 3 **B**) 2 **C**) – 3 **D**) – 2.

2) $(\vec{a}\times\vec{b})^2$ is equal to :

A) $\vec{a}^2\vec{b}^2-(\vec{a}\cdot\vec{b})^2$ **B**) $\vec{a}^2\vec{b}^2-(\vec{a}\cdot\vec{b})$

C) $(\vec{a}\cdot\vec{b})^2$ **D**) a^2b^2

3) If $\vec{a}=4\hat{i}+6\hat{j}$ and $\vec{b}=3\hat{j}+4\hat{k}$, then the vector form of the component of $\vec{a}$ along $\vec{b}$ is :

A) $\frac{18}{10\sqrt{13}}(3\hat{j}+4\hat{k})$ **B**) $\frac{18}{25}(3\hat{j}+4\hat{k})$

C) $\frac{18}{\sqrt{113}}(3\hat{j}+4\hat{k})$ **D**) $3\hat{j}+4\hat{k}$

4) The vectors $\hat{i}+2\hat{j}+3\hat{k}$, $\lambda\hat{i}+4\hat{j}+7\hat{k}$, $-3\hat{i}-2\hat{j}-5\hat{k}$ are collinear if λ is :

A) 3 **B**) 4 **C**) 5 **D**) 6.

5) If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}\cdot\vec{b}$ = 0 and $\vec{a}\times\vec{b}$ = 0, then :

A) $\vec{a}$ is parallel to $\vec{b}$

B) $\vec{a}\perp\vec{b}$

C) either $\vec{a}$ or $\vec{b}$ is a null vector

D) None of these.

6) The projection of the vector $\hat{i}-2\hat{j}+\hat{k}$ on the vector $4\hat{i}-4\hat{j}+7\hat{k}$ is:

A) $\frac{5}{19}\sqrt{5}$ **B**) $2\frac{1}{9}$ **C**) $\frac{9}{19}$ **D**) $\frac{1}{19}\sqrt{6}$

7) If the vectors $2\hat{i}-\hat{j}+\lambda\hat{k}$, $\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$ are coplanar, then the value of λ is :

A) – 1 **B**) – 2 **C**) – 3 **D**) – 4.

8) $\vec{a}\cdot(\vec{a}\times\vec{b})=$

A) $\vec{a}\cdot\vec{b}$ **B**) a^2b **C**) 0 **D**) a^2+ab

9) The points with position vectors $60\hat{i}+3\hat{j}$, $40\hat{i}-8\hat{j}$ and $a\hat{i}-52\hat{j}$ are collinear if :

A) a = – 40 **B**) a = 40

C) a = 20 **D**) None of these.

10) Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-coplanar vectors and $\vec{p}$, $\vec{q}$, $\vec{r}$ be vectors defined by the relations:
$\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$**, then the value of the expression :**
$(\vec{a}+\vec{b})\cdot\vec{p}+(\vec{b}+\vec{c})\cdot\vec{q}+(\vec{c}+\vec{a})\cdot\vec{r}$ **is equal to:**

A) 0 **B**) 1 **C**) 2 **D**) 3.

11) The scalar $\vec{A}\cdot(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})$ is :

A) 0

B) $[\vec{A}\,\vec{B}\,\vec{C}]+[\vec{B}\,\vec{C}\,\vec{A}]$

C) $2[\vec{A}\,\vec{B}\,\vec{C}]$

D) $[\vec{A}\,\vec{B}\,\vec{C}]$.

12) If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ (a $\neq$ 1, b $\neq$ 1, c $\neq$ 1) are coplanar, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is:

A) 1 **B**) 2

C) 0 **D**) None of these.

13) If the sum of two unit vectors is a unit vector, then the angle between them is equal to :

A) π /6 **B**) π /3 **C**) π /2 **D**) 2π /3.

14) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product : [$2\vec{a}-\vec{b}$, $2\vec{b}-\vec{c}$, $2\vec{c}-\vec{a}$]=

A) 0 **B**) 1 **C**) $-\sqrt{3}$ **D**) $\sqrt{3}$

15) Given two vectors $\hat{i}-\hat{j}$ and $\hat{i}+2\hat{j}$, the unit vector coplanar with the two vectors and perpendicular to first is:

A) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ **B**) $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$

C) $\pm\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$ **D**) None of these.

16) If $\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ a vector such that $\vec{c}=\vec{a}\times\vec{b}$, then $|\vec{a}|:|\vec{b}|:|\vec{c}|=$

A) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ **B**) $\sqrt{34}:\sqrt{45}:39$

C) 34:39:45 **D**) 39:35 :34.

17) Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-\hat{j}+2\hat{k}$ and $\vec{c} = x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals:

A) 1 **B**) –4 **C**) –2 **D**) 0.

18) Given two vectors $\vec{a} = 2\hat{i}-3\hat{j}+6\hat{k}$, $\vec{b} = -2\hat{i}+2\hat{j}-\hat{k}$ and $p = \dfrac{\text{projection of } \vec{b} \text{ on } \vec{a}}{\text{projection of } \vec{a} \text{ on } \vec{b}}$, then the value of p is:

A) 3/7 **B**) 7/3 **C**) 3 **D**) 7.

19) If the vectors $\vec{a} = \hat{i}-\hat{j}+2\hat{k}$, $\vec{b} = 2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c} = \lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

A) $(-3, 2)$ **B**) $(2, -3)$ **C**) $(-2, 3)$ **D**) $(3, -2)$.

20) If $\vec{a} = \dfrac{1}{\sqrt{10}}(3\hat{i}+\hat{k})$ and $\vec{b} = \dfrac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})$, then the value of $\left(2\vec{a}-\vec{b}\right)\cdot\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right]$ is:

A) –5 **B**) –3 **C**) 5 **D**) 3

21) If the vectors $\vec{AB} = 3\hat{i}+4\hat{k}$ and $\vec{AC} = 5\hat{i}-2\hat{j}+4\hat{k}$ the sides of a triangle ABC, then the length of the median through A is :

A) $\sqrt{72}$ **B**) $\sqrt{33}$ **C**) $\sqrt{45}$ **D**) $\sqrt{18}$

22) $\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right] = \lambda\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$, then λ is equal to :

A) 3 **B**) 0 **C**) 1 **D**) 2

23) If one side of a square be represented by the vector $3i+4j+5k$, then the area of the square is

A) 12 **B**) 13 **C**) 25 **D**) 50

24) If C is the middle point of AB and P is any point outside AB, then

A) $\vec{PA}+\vec{PB} = \vec{PC}$

B) $\vec{PA}+\vec{PB} = 2\vec{PC}$

C) $\vec{PA}+\vec{PB}+\vec{PC} = 0$

D) $\vec{PA}+\vec{PB}+2\vec{PC} = 0$

25) The value of b such that scalar product of the vectors $(\mathbf{i}+\mathbf{j}+\mathbf{k})$ with the unit vector parallel to the sum of the vectors $(2\mathbf{i}+4\mathbf{j}-5\mathbf{k})$ and $(b\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ is 1, is

A) –2 **B**) –1 **C**) 0 **D**) 1

26) A vector whose modulus is $\sqrt{51}$ and makes the same angle with $\mathbf{a} = \dfrac{\mathbf{i}-2\mathbf{j}+2\mathbf{k}}{3}$, $\mathbf{b} = \dfrac{-4\mathbf{i}-3\mathbf{k}}{5}$ and $\mathbf{c} = \mathbf{j}$, will be

A) $5\mathbf{i}+5\mathbf{j}+\mathbf{k}$ **B**) $5\mathbf{i}+\mathbf{j}-5\mathbf{k}$

C) $5\mathbf{i}+\mathbf{j}+5\mathbf{k}$ **D**) $\pm(5\mathbf{i}-\mathbf{j}-5\mathbf{k})$

27) If in a right angled triangle ABC, the hypotenuse $AB = p$, then $\vec{AB}\,.\,\vec{AC}+\vec{BC}\,.\,\vec{BA}+\vec{CA}\,.\,\vec{CB}$ is equal to

A) $2p^2$ **B**) $\dfrac{p^2}{2}$

C) p^2 **D**) None of these

28) A, B, C, D are any four points, then $\vec{AB}\,.\,\vec{CD} + \vec{BC}\,.\,\vec{AD} + \vec{CA}\,.\,\vec{BD} =$

A) $2\,\vec{AB}\,.\,\vec{BC}\,.\,\vec{CD}$ **B**) $\vec{AB} + \vec{BC} + \vec{CD}$

C) $5\sqrt{3}$ **D**) 0

29) If $|\mathbf{a}| = 3,\ |\mathbf{b}| = 4$ then a value of λ for which $\mathbf{a}+\lambda\mathbf{b}$ is perpendicular to $\mathbf{a}-\lambda\mathbf{b}$ is

A) $\dfrac{9}{16}$ **B**) $\dfrac{3}{4}$ **C**) $\dfrac{3}{2}$ **D**) $\dfrac{4}{3}$

30) If a and b are unit vectors and $\mathbf{a}-\mathbf{b}$ is also a unit vector, then the angle between a and b is

A) $\dfrac{\pi}{4}$ **B**) $\dfrac{\pi}{3}$ **C**) $\dfrac{\pi}{2}$ **D**) $\dfrac{2\pi}{3}$

31) If a, b and c are unit vectors such that $\mathbf{a}+\mathbf{b}-\mathbf{c} = 0$, then the angle between a and b is

A) $\pi/6$ **B**) $\pi/3$ **C**) $\pi/2$ **D**) $2\pi/3$

32) If the sum of two unit vectors is a unit vector, then the magnitude of their difference is

A) $\sqrt{2}$ **B**) $\sqrt{3}$ **C**) $\dfrac{1}{\sqrt{3}}$ **D**) 1

33) If a and b are two unit vectors such that $a+2b$ and $5a-4b$ are perpendicular to each other, then the angle between a and b is

A) 45° **B**) 60°

C) $\cos^{-1}\left(\dfrac{1}{3}\right)$ **D**) $\cos^{-1}\left(\dfrac{2}{7}\right)$

34) If $a \neq 0,\ b \neq 0$ and $|a+b| = |a-b|$, then the vectors a and b are

A) Parallel to each other

B) Perpendicular to each other

C) Inclined at an angle of 60°

D) Neither perpendicular nor parallel

35) $(\mathbf{a}-\mathbf{b})\times(\mathbf{a}+\mathbf{b}) =$

A) $2(\mathbf{a}\times\mathbf{b})$ **B**) $\mathbf{a}\times\mathbf{b}$

C) a^2-b^2 **D**) None of these

36) $(2\mathbf{a}+3\mathbf{b})\times(5\mathbf{a}+7\mathbf{b}) =$

A) $\mathbf{a} \times \mathbf{b}$ B) $\mathbf{b} \times \mathbf{a}$

C) $\mathbf{a} + \mathbf{b}$ D) $7\mathbf{a} + 10\mathbf{b}$

37) If a and b are two vectors such that $a.b = 0$ **and** $a \times b = 0$**, then**

A) a is parallel to b

B) a is perpendicular to b

C) Either a or b is a null vector

D) None of these

38) $|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| = |\mathbf{a}|\ |\mathbf{b}|\ |\mathbf{c}|$**, if**

A) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = 0$

B) $\mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$

C) $\mathbf{c} \cdot \mathbf{a} = \mathbf{a} \cdot \mathbf{b} = 0$

D) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$

39) $|\mathbf{a} \times \mathbf{i}|^2 + |\mathbf{a} \times \mathbf{j}|^2 + |\mathbf{a} \times \mathbf{k}|^2 =$

A) $|\mathbf{a}|^2$ B) $2\ |\mathbf{a}|^2$ C) $3\ |\mathbf{a}|^2$ D) $4\ |\mathbf{a}|^2$

40) If $(\mathbf{a} \times \mathbf{b})^2 + (\mathbf{a}\ .\ \mathbf{b})^2 = 144$ **and** $|\mathbf{a}| = 4$**, then** $|\mathbf{b}| =$

A) 16 B) 8 C) 3 D) 12

41) A unit vector in the plane of the vectors $2\mathbf{i} + \mathbf{j} + \mathbf{k},\ \mathbf{i} - \mathbf{j} + \mathbf{k}$ **and orthogonal to** $5\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ **is**

A) $\dfrac{6\mathbf{i} - 5\mathbf{k}}{\sqrt{61}}$ B) $\dfrac{3\mathbf{j} - \mathbf{k}}{\sqrt{10}}$

C) $\dfrac{2\mathbf{i} - 5\mathbf{j}}{\sqrt{29}}$ D) $\dfrac{2\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{3}$

42) The area of the parallelogram whose diagonals are $\mathbf{a} = 3\,\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ **and** $\mathbf{b} = \mathbf{i} - 3\,\mathbf{j} + 4\,\mathbf{k}$ **is**

A) $10\sqrt{3}$ B) $5\sqrt{3}$ C) 8 D) 4

43) The moment of a force represented by $\vec{F} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ **about the point** $2\,\mathbf{i} - \mathbf{j} + \mathbf{k} =$

A) $5\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$ B) $5\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$

C) $-5\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$ D) $-5\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$

44) If the vectors $2\mathbf{i} - 3\mathbf{j},\ \mathbf{i} + \mathbf{j} - \mathbf{k}$ **and** $3\mathbf{i} - \mathbf{k}$ **form three concurrent edges of a parallelopiped, then the volume of the parallelopiped is**

A) 8 B) 10 C) 4 D) 14

45) If a and b be parallel vectors, then $[acb] =$

A) 0 B) 1

C) 2 D) None of these

46) If the vectors $2\mathbf{i} - \mathbf{j} + \mathbf{k},\ \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ **and** $3\mathbf{i} + \lambda\mathbf{j} + 5\mathbf{k}$ **be coplanar, then**

A) -1 B) -2 C) -3 D) -4

47) If a,b and c are unit coplanar vectors then the scalar triple product $[2a - b\ 2b - c\ 2c - a]$ **is equal to**

A) 0 B) 1 C) $-\sqrt{3}$ D) $\sqrt{3}$

48) The value of $[a - b\ \ b - c\ \ c - a]$**, where** $|a| = 1, |b| = 5$ **and** $|c| = 3$ **is**

A) 0 B) 1 C) 2 D) 4

49) If u, v and w are three non-coplanar vectors, then $(u + v - w) \cdot [(u - v) \times (v - w)]$ **equals**

A) 0 B) $u \cdot (v \times w)$

C) $u \cdot (w \times v)$ D) $3u \cdot (v \times w)$

50) $\mathbf{i} \times (\mathbf{j} \times \mathbf{k}) =$

A) 1 B) 0

C) -1 D) None of these

Answer Sheet

Mathematics : Vector Algebra

Total Questions : 50 **Total Marks : 50**

1	C	2	A	3	B	4	A	5	C	6	B	7	A	8	C	9	A	10	D	11	A	12	A
13	D	14	A	15	C	16	B	17	C	18	A	19	A	20	A	21	B	22	C	23	D	24	B
25	D	26	D	27	C	28	D	29	B	30	B	31	D	32	B	33	B	34	B	35	A	36	B
37	C	38	D	39	B	40	C	41	B	42	B	43	D	44	C	45	A	46	D	47	A	48	A
49	B	50	B																				

Solution Sheet

Mathematics : Vector Algebra

Total Questions : 50 **Total Marks : 50**

1) Volume $= \begin{vmatrix} -12 & 0 & \alpha \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix} = 546$

$\Rightarrow -12\,(-44) + 0 + \alpha\,(-6) = 546$
$\Rightarrow 528 - 6\alpha = 546$
$\Rightarrow 6\alpha = -546 + 528 = -18 \Rightarrow \alpha = -3.$

2) $(\vec{a} \times \vec{b})^2 = (ab \sin\theta)^2$
$= a^2b^2\left(1 - \cos^2\theta\right)$
$= a^2b^2 - a^2b^2\cos^2\theta = \vec{a}^2\vec{b}^2 - (\vec{a}\cdot\vec{b})^2.$

3) Vector form of the component of $\vec{a}$ along $\vec{b}$
$= \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$
$= (4\times 0 + 6\times 3 + 0\times 4)\,\frac{(3\hat{j}+4\hat{k})}{\left(\sqrt{9+16}\right)^2}$
$= \frac{18}{25}\,(3\hat{j}+4\hat{k})$

4) $\begin{vmatrix} 1 & 2 & 3 \\ \lambda & 4 & 7 \\ -3 & -2 & -5 \end{vmatrix} = 0$

$\Rightarrow 1.\,(-20+14) - 2\,(-5\lambda+21) + 3\,(-2\lambda+12) = 0$
$\Rightarrow -6 + 10\lambda - 42 - 6\lambda + 36 = 0$
$\Rightarrow 4\lambda = 12 \Rightarrow \lambda = 3.$

5) $\vec{a}\cdot\vec{b} = 0$ and $\vec{a}\times\vec{b} = 0$ means either $\vec{a}$ or $\vec{b}$ is a null vector.
$\because$ both cant be parallel and perp.

6) Projection $= \frac{(1)\,(4) + (-2)\,(-4) + (1)\,(7)}{\sqrt{16+16+49}}$
$= \frac{4+8+7}{\sqrt{81}} = \frac{19}{9} = 2\frac{1}{9}.$

7) Since the vectors are coplanar,
$\therefore \begin{vmatrix} 2 & -1 & \lambda \\ 1 & -2 & 2 \\ 3 & -2 & 1 \end{vmatrix} = 0$

$\Rightarrow 2\,(-1+4) + 1\,(1-6) + \lambda\,(-2+3) = 0$
$\Rightarrow 6 - 5 + \lambda = 0 \Rightarrow \lambda = -1.$

8) Since scalar triple product is zero if two vectors are identical.

9) Let $A(60\hat{i}+3\hat{j}),\ B(40\hat{i}-8\hat{j}),\ C(a\hat{i}-52\hat{j})$ be given points.
$\therefore \overrightarrow{AB} = 40\hat{i} - 8\hat{j} - 60\hat{i} - 3\hat{j} = -20\hat{i} - 11\hat{j}$
and $\overrightarrow{AC} = a\hat{i} - 52\hat{j} - 60\hat{i} - 3\hat{j}$
$= (a-60)\,\hat{i} - 55\hat{j}$
Since A, B, C are collinear,
$\therefore \overrightarrow{AB} = k\overrightarrow{AC}$, for some scalar k
$\Rightarrow -20\hat{i} - 11\hat{j} = k\left[(a-60)\,\hat{i} - 55\hat{j}\right]$
$\Rightarrow -11 = -55k \Rightarrow k = 1/5$
and $-20 = k\,(a-60) = \frac{1}{5}\,(a-60)$
$\Rightarrow -100 = a - 60$
$\Rightarrow a = -40.$

10) $(\vec{a}+\vec{b})\cdot\vec{p} = \frac{\vec{a}\cdot\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]} + \frac{\vec{b}\cdot(\vec{b}\times\vec{c})}{[\vec{a}\,\vec{b}\,\vec{c}]}$
$= \frac{[\vec{a}\,\vec{b}\,\vec{c}]}{[\vec{a}\,\vec{b}\,\vec{c}]} + 0 = 1.$
Similarly $(\vec{b}+\vec{c})\cdot\vec{q} = 1$ and $(\vec{c}+\vec{a})\cdot\vec{r} = 1$.
$\therefore$ Total value = 3.

11) $\vec{A}\cdot(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})$
$= \vec{A}\cdot(\vec{B}\times\vec{A} + \vec{B}\times\vec{C} + \vec{C}\times\vec{A} + \vec{C} + \vec{B})$
$= \vec{A}\cdot(\vec{B}\times\vec{A} + \vec{C}\times\vec{A})$
$= \vec{A}\cdot\vec{B}\times\vec{A} + \vec{A}\cdot\vec{C}\times\vec{A}$
$= [\vec{A}\,\vec{B}\,\vec{A}] + [\vec{A}\,\vec{C}\,\vec{A}]$
$= 0 + 0 = 0.$

12) Since given vectors are coplanar,
$\therefore [a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}, \hat{i}+\hat{j}+c\hat{k}] = 0$

$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$ i.e. $\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$

$[\text{Operating} C_2 \to C_2 - C_1, C_3 \to C_3 - C_1]$
$\Rightarrow a(b-1)(c-1) - (1-a)(c-1) - 1(1-a)(b-1) = 0$
Dividing throughout by $(1-a)(1-b)(1-c)$, we get:
$\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$
$\Rightarrow -1 + \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$
$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

13) Let $\vec{a}$, $\vec{b}$ be unit vectors. Then their sum $\vec{a}+\vec{b}=$
$\vec{c} \Rightarrow (\vec{a}+\vec{b})^2=(\vec{c})^2 \Rightarrow 1+1+2\vec{a}\cdot\vec{b}=1$
$\Rightarrow 2|\vec{a}||\vec{b}|\cos\theta=-1$
$\Rightarrow \cos\theta=-1/2=-\cos(\pi/3)=\cos\dfrac{2\pi}{3}$
$\Rightarrow \theta=2\pi/3.$

14) Since $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit coplanar vectors,
$\therefore 2\vec{a}-\vec{b}$, $2\vec{b}-\vec{c}$, $2\vec{c}-\vec{a}$ are also coplanar vectors,
$\therefore$ their scalar triple product is zero,

15) Let $\vec{a}=\hat{i}-\hat{j}$ and $\vec{b}=\hat{i}-2\hat{j}$
The reqd. vector is along the vector
$\vec{a}\times(\vec{a}\times\vec{b})=(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}$
$=-(\hat{i}-\hat{j})-2(\hat{i}+2\hat{j})=-3\hat{i}-3\hat{j}$
Hence reqd. vectors are given by :
$\pm\dfrac{(-3\hat{i}-3\hat{j})}{\sqrt{9+9}}=\pm\dfrac{1}{\sqrt{2}}(\hat{i}+\hat{j}).$

16) Here $|\vec{a}|=\sqrt{9+25+0}=\sqrt{34}$
$|\vec{b}|=\sqrt{9+25+0}=\sqrt{45}$
Now $|\vec{c}|=|\vec{a}\times\vec{b}|$
$=|(3\hat{i}-5\hat{j})\times(6\hat{i}+3\hat{j})|$
$=|9\hat{k}+30\hat{k}|=|39\hat{k}|=39.$
Hence $|\vec{a}|:|\vec{b}|:|\vec{c}|=\sqrt{34}:\sqrt{45}:39$

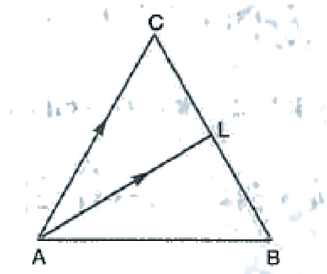
17) Here $[\vec{a}\ \vec{b}\ \vec{c}]=0$
$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix}=0$$
$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & -2 & 3 \\ x & -2 & 1-x \end{vmatrix}=0$$
$[\text{Operating} C_2 \to C_2-C_1$ and $C_3 \to C_3-C_2]$
$\Rightarrow 1.(-2+2x+6)=0 \Rightarrow 2x=-4 \Rightarrow x=-2.$

18) $$\frac{\dfrac{(\vec{b}\cdot\vec{a})}{|\vec{a}|}}{\dfrac{(\vec{a}\cdot\vec{b})}{|\vec{b}|}}=\frac{|\vec{b}|}{|\vec{a}|}=\frac{3}{7}.$$
$[\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}]$

19) $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually orthogonal
$\Rightarrow \vec{b}\cdot\vec{c}=0$ and $\vec{a}\cdot\vec{c}=0$
$\Rightarrow (2\hat{i}+4\hat{j}+\hat{k})\cdot(\lambda\hat{i}+\hat{j}+\mu\hat{k})=0$
$\Rightarrow 2\lambda+4+\mu=0 \Rightarrow 2\lambda+\mu=-4$ ...(1)
and $\Rightarrow (\hat{i}-\hat{j}+2\hat{k})\cdot(\lambda\hat{i}+\hat{j}+\mu\hat{k})=0$
$\Rightarrow \lambda-1+2\mu=0 \Rightarrow \lambda+2\mu=1$ ...(2)
Solving (1) and (2), $\lambda=-3$ and $\mu=2$

20) $\left(2\vec{a}-\vec{b}\right)\cdot\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right]$
$=\left(2\vec{a}-\vec{b}\right)\cdot\left[\left(\vec{a}\times\vec{b}\right)\times\vec{a}+2\left(\vec{a}\times\vec{b}\right)\times\vec{b}\right]$
$=\left(2\vec{a}-\vec{b}\right)\cdot$
$\left[(\vec{a}\cdot\vec{a})\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{a}+2\left(\vec{a}\cdot\vec{b}\right)\vec{b}-2\left(\vec{b}\cdot\vec{b}\right)\vec{a}\right]$
But $\vec{a}\cdot\vec{a}=\dfrac{1}{\sqrt{10}}(3\hat{i}+\hat{k})\cdot\dfrac{1}{\sqrt{10}}(3\hat{i}+\hat{k})$
$=\dfrac{1}{10}[(3)(3)+(1)(1)]=\dfrac{10}{10}=1$
$\vec{a}\cdot\vec{b}=\dfrac{1}{\sqrt{10}}(3\hat{i}+\hat{k})\cdot\dfrac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})$
$=\dfrac{1}{7\sqrt{10}}[(3)(2)+0+(1)(-6)]=0$
and $\vec{b}\cdot\vec{b}=\dfrac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})\cdot\dfrac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})$
$=\dfrac{1}{49}[(2)(2)+(3)(3)+(-6)(-6)]$
$=\dfrac{1}{49}[4+9+36]=1$
$\therefore$ From (1), Given = $2\left(\vec{a}-\vec{b}\right)$
$[(1)\vec{b}-0+0-2(1)\vec{a}]$
$=\left(2\vec{a}-\vec{b}\right)\cdot\left(\vec{b}-2\vec{a}\right)=2\vec{a}\cdot\vec{b}-4\vec{a}\cdot\vec{a}$
$-\vec{b}\cdot\vec{b}+2\vec{b}\cdot\vec{a}$
$=2(0)-4(1)-(1)+2(0)=-4-1=-5.$

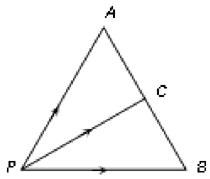
21) Here $\vec{AL}=\vec{AB}+\dfrac{\vec{AC}}{2}$
$=\dfrac{(3\hat{i}+4\hat{k})+(5\hat{i}-2\hat{j}+4\hat{k})}{2}=4\hat{i}-\hat{j}+4\hat{k}$
$\therefore \left|\vec{AL}\right|=\sqrt{16+1+16}=\sqrt{33}$



22) As usual, we will have :
$\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\lambda\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2$
Given : $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\lambda\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2$
Hence λ = 1.

23) $|a|=\sqrt{9+16+25}=5\sqrt{2}$
Area $=|a|^2=25\times 2=50$.

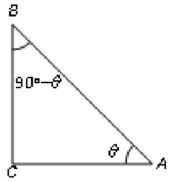
24) $\overrightarrow{PA}+\overrightarrow{PB}=(\overrightarrow{PA}+\overrightarrow{AC})+(\overrightarrow{PB}+\overrightarrow{BC})-(\overrightarrow{AC}+\overrightarrow{BC})$
$=\overrightarrow{PC}+\overrightarrow{PC}-(\overrightarrow{AC}-\overrightarrow{CB})=2\overrightarrow{PC}-0, (\because\ \overrightarrow{AC}=\overrightarrow{CB})$
$\therefore \overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$.



25) Parallel vector $=(2+b)\mathbf{i}+6\mathbf{j}-2\mathbf{k}$
Unit vector $=\dfrac{(2+b)\mathbf{i}+6\mathbf{j}-2\mathbf{k}}{\sqrt{b^2+4b+44}}$
According to the condition, $1=\dfrac{(2+b)+6-2}{\sqrt{b^2+4b+44}}$
$\Rightarrow b^2+4b+44=b^2+12b+36\Rightarrow 8b=8\Rightarrow b=1$.

26) Let the required vector be $\alpha=d_1\mathbf{i}+d_2\mathbf{j}+d_3\mathbf{k}$, where $d_1^2+d_2^2+d_3^2=51$, (given) .....(i)
Now, each of the given vectors $\mathbf{a},\ \mathbf{b},\ \mathbf{c}$ is a unit vector $\cos\theta=\dfrac{\mathbf{d}\,.\,\mathbf{a}}{|\mathbf{d}|\,|\mathbf{a}|}=\dfrac{\mathbf{d}\,.\,\mathbf{b}}{|\mathbf{d}|\,|\mathbf{b}|}=\dfrac{\mathbf{d}\,.\,\mathbf{c}}{|\mathbf{d}|\,|\mathbf{c}|}$
or $\mathbf{d}\,.\,\mathbf{a}=\mathbf{d}\,.\,\mathbf{b}=\mathbf{d}\,.\,\mathbf{c}$
$|\mathbf{d}|=\sqrt{51}$ cancels out and $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=1$
Hence,
$\frac{1}{3}(d_1-2d_2+2d_3)=\frac{1}{5}(-4d_1+0d_2-3d_3)=d_2$
$\Rightarrow d_1-5d_2+2d_3=0$ and $4d_1+5d_2+3d_3=0$
On solving, $\dfrac{d_1}{5}=\dfrac{d_2}{-1}=\dfrac{d_3}{-5}=\lambda$ (say)
Putting $d_1,\ d_2$ and d_3 in (i), we get $\lambda=\pm 1$
Hence the required vectors are $\pm(5\mathbf{i}-\mathbf{j}-5\mathbf{k})$. .
Trick : Check it with the options.

27) We have $\overrightarrow{AB}\,.\,\overrightarrow{AC}+\overrightarrow{BC}\,.\,\overrightarrow{BA}+\overrightarrow{CA}\,.\,\overrightarrow{CB}$
$(AB)\,(AC)\cos\theta+(BC)\,(BA)\cos(90^o-\theta)+0$
$=AB(AC\cos\theta+BC\sin\theta)=$
$AB\left(\dfrac{(AC)^2}{AB}+\dfrac{(BC)^2}{AB}\right)$
$=AC^2+BC^2=AB^2=p^2$.



28) $\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}=\mathbf{a}+\mathbf{b}+\mathbf{c}$
$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=\mathbf{a}+\mathbf{b}$ or $\overrightarrow{CA}=-(\mathbf{a}+\mathbf{b})$
$\overrightarrow{BD}=\overrightarrow{BC}+\overrightarrow{CD}=\mathbf{b}+\mathbf{c}$
Therefore $\overrightarrow{AB}\,.\,\overrightarrow{CD}+\overrightarrow{BC}\,.\,\overrightarrow{AD}+\overrightarrow{CA}\,.\,\overrightarrow{BD}$
$=\mathbf{a}\,.\,\mathbf{c}+\mathbf{b}\,.(\mathbf{a}+\mathbf{b}+\mathbf{c})+(-\mathbf{a}-\mathbf{b})\,.\,(\mathbf{b}+\mathbf{c})$
$=$
$\mathbf{a}\,.\,\mathbf{c}+\mathbf{b}\,.\,\mathbf{a}+\mathbf{b}\,.\,\mathbf{b}+\mathbf{b}\,.\,\mathbf{c}-\mathbf{a}\,.\,\mathbf{b}-\mathbf{a}\,.\,\mathbf{c}-\mathbf{b}\,.\,\mathbf{b}-\mathbf{b}\,.\,\mathbf{c}=0$

29) Since $\mathbf{a}+\lambda\mathbf{b}$ is perpendicular to $\mathbf{a}-\lambda\mathbf{b}$,then their product will be zero.
So, $(\mathbf{a}+\lambda\mathbf{b}).(\mathbf{a}-\lambda\mathbf{b})=0\Rightarrow|\mathbf{a}|^2-\lambda^2|\mathbf{b}|^2=0$
or $\lambda^2=\dfrac{|\mathbf{a}|^2}{|\mathbf{b}|^2}\Rightarrow\lambda^2=\dfrac{9}{16}$ or
$\lambda=\pm\dfrac{3}{4}, [\because\ |\mathbf{a}|=3, |\mathbf{b}|=4]$

30) $(\mathbf{a}-q\mathbf{b})^2=1=2-2\cos\Rightarrow\ \theta=60^\circ$.

31) Given condition is $\mathbf{a}+\mathbf{b}=\mathbf{c}$.
Using dot product, $(\mathbf{a}+\mathbf{b}).(\mathbf{a}+\mathbf{b})=\mathbf{c}.\mathbf{c}$
$\Rightarrow \mathbf{a}.\mathbf{a}+\mathbf{b}.\mathbf{b}+2\mathbf{a}.\mathbf{b}=\mathbf{c}.\mathbf{c}$
$\Rightarrow\ |\mathbf{a}|.|\mathbf{a}|\cos 0^\circ+|\mathbf{b}|.|\mathbf{b}|\cos 0^\circ+2|\mathbf{a}|.|\mathbf{b}|\cos\alpha$
$=\ |\mathbf{c}|.|\mathbf{c}|\cos 0^\circ, (\because\ \ |\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=1)$
$\Rightarrow 1+1+2\cos\alpha=1\Rightarrow\cos\alpha=-\dfrac{1}{2}\Rightarrow\alpha=\dfrac{2\pi}{3}$.

32) Let $|\mathbf{a}|=1$ and $|\mathbf{b}|=1$
Also
$|\mathbf{a}+\mathbf{b}|^2=1^2\Rightarrow 1+1+2\cos\theta=1\Rightarrow\theta=120^\circ$
$\therefore\ \ |\mathbf{a}-\mathbf{b}|^2=1+1-2\cos\theta=3\Rightarrow\ |\mathbf{a}-\mathbf{b}|=\sqrt{3}$.

33) $(\mathbf{a}+2\mathbf{b})\,.\,(5\mathbf{a}-4\mathbf{b})=0$ or $5\mathbf{a}^2+6\mathbf{a}\,.\,\mathbf{b}-8\mathbf{b}^2=0$
or $6\,\mathbf{a}\,.\,\mathbf{b}=3, (\because \mathbf{a}^2=1,\ \mathbf{b}^2=1)$
$\therefore\ \mathbf{a}\,.\,\mathbf{b}=\dfrac{1}{2}$ or $|\mathbf{a}||\mathbf{b}|\ \cos\theta=\dfrac{1}{2}$
$\therefore\ \cos\theta=\dfrac{1}{2},\quad\therefore\theta=60^o$.

34) $|a+b|=|a-b|$; Squaring both sides, we get
$4a.b=0$
$\Rightarrow$ a is perpendicular to b.

35) $(\mathbf{a}-\mathbf{b})\times(\mathbf{a}+\mathbf{b})=\mathbf{a}\times\mathbf{a}-\mathbf{b}\times\mathbf{a}+\mathbf{a}\times\mathbf{b}-\mathbf{b}\times\mathbf{b}$
$=\mathbf{a}\times\mathbf{b}-\mathbf{b}\times\mathbf{a}=\mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{b}=2(\mathbf{a}\times\mathbf{b})$.

36) $14(\mathbf{a}\times\mathbf{b})+15(\mathbf{b}\times\mathbf{a})=\mathbf{b}\times\mathbf{a}$.

37) $a.b=0\Rightarrow a\perp b$ or $a=0$ or $b=0$
and $a\times b=0\Rightarrow a||b$ or $a=0$ or $b=0$
Hence, either a or b is a null vector.

38) We have $|(\mathbf{a}\times\mathbf{b}).\mathbf{c}|=|\mathbf{a}||\mathbf{b}||\mathbf{c}|$
$\Rightarrow ||\mathbf{a}||\mathbf{b}|\sin\theta\,\mathbf{n}.\mathbf{c}|=|\mathbf{a}||\mathbf{b}||\mathbf{c}|$
$\Rightarrow ||\mathbf{a}||\mathbf{b}||\mathbf{c}|\sin\theta\cos\alpha|=|\mathbf{a}||\mathbf{b}||\mathbf{c}|$
$\Rightarrow |\sin\theta||\cos\alpha|=1\Rightarrow\theta=\dfrac{\pi}{2}$ and $\alpha=0$
$\Rightarrow \mathbf{a}\perp\mathbf{b}$ and $\mathbf{c}||\mathbf{n}$
$\Rightarrow \mathbf{a}\perp\mathbf{b}$ and is perpendicular to both a and b
$\therefore\ \mathbf{a},\ \mathbf{b},\ \mathbf{c}$ are mutually perpendicular
Hence, $\mathbf{a}.\mathbf{b}=\mathbf{b}.\mathbf{c}=\mathbf{c}.\mathbf{a}=0$.

39) $|\mathbf{a}\times\mathbf{i}|^2=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\a_1&a_2&a_3\\1&0&0\end{vmatrix}^2,$
$(Since\ \ \mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k})$
$=|a_3\mathbf{j}-a_2\mathbf{k}|^2=a_3^2+a_2^2$
Similarly, $|\mathbf{a}\times\mathbf{j}|^2=a_1^2+a_3^2$ and $|\mathbf{a}\times\mathbf{k}|^2=a_1^2+a_2^2$
Hence the required result can be given as
$2(a_1^2+a_2^2+a_3^2)=2|\mathbf{a}|^2$.

40) We know that $(\mathbf{a}\times\mathbf{b})^2+(\mathbf{a}\,.\,\mathbf{b})^2=|\mathbf{a}|^2|\mathbf{b}|^2$
$\therefore\ \ 144=16|\mathbf{b}|^2\Rightarrow|\mathbf{b}|=3$.

41) Let a unit vector in the plane of $2\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{i}-\mathbf{j}+\mathbf{k}$
be $\hat{\mathbf{a}}=\alpha(2\mathbf{i}+\mathbf{j}+\mathbf{k})+\beta(\mathbf{i}-\mathbf{j}+\mathbf{k})$
$\hat{\mathbf{a}}=(2\alpha+\beta)\mathbf{i}+(\alpha-\beta)\mathbf{j}+(\alpha+\beta)\mathbf{k}$
As $\hat{\mathbf{a}}$ is unit vector, we have
$\Rightarrow(2\alpha+\beta)^2+(\alpha-\beta)^2+(\alpha+\beta)^2=1$
$\Rightarrow 6\alpha^2+4\alpha\beta+3\beta^2=1$ .....(i)
As $\hat{\mathbf{a}}$ is orthogonal to $5\mathbf{i}+2\mathbf{j}+6\mathbf{k}$, we get
$5(2\alpha+\beta)+2(\alpha-\beta)+6(\alpha+\beta)=0$
$\Rightarrow 18\alpha+9\beta=0\Rightarrow\beta=-2\alpha$
From (i), we get $6\alpha^2-8\alpha^2+12\alpha^2=1$
$\Rightarrow\alpha=\pm\dfrac{1}{\sqrt{10}}\Rightarrow\beta=\mp\dfrac{2}{\sqrt{10}}$. Thus
$\hat{\mathbf{a}}=\pm\left(\dfrac{3}{\sqrt{10}}\mathbf{j}-\dfrac{1}{\sqrt{10}}\mathbf{k}\right)$.

42) $\Delta=\dfrac{1}{2}|\mathbf{a}\times\mathbf{b}|$
But $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\3&1&-2\\1&-3&4\end{vmatrix}=-2\mathbf{i}-14\mathbf{j}-10\mathbf{k}.$
Hence $\Delta=\dfrac{1}{2}|\mathbf{a}\times\mathbf{b}|=\dfrac{1}{2}\sqrt{4+196+100}=5\sqrt{3}.$

43) $\mathbf{r}\times\mathbf{F}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&-1&1\\1&2&3\end{vmatrix}=-5\mathbf{i}-5\mathbf{j}+5\mathbf{k}.$

44) Here, $\overrightarrow{OA}=2\mathbf{i}-3\mathbf{j}=\mathbf{a}$ (say)
$\overrightarrow{OB}=\mathbf{i}+\mathbf{j}-\mathbf{k}=\mathbf{b}$ (say)
and $\overrightarrow{OC}=3\mathbf{i}-\mathbf{k}=\mathbf{c}$ (say)
Hence volume is
$[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]=\mathbf{a}.\,(\mathbf{b}\times\mathbf{c})=\begin{vmatrix}2&-3&0\\1&1&-1\\3&0&-1\end{vmatrix}=4$

46) If the given vectors are coplanar, then their scalar triple product is zero.
$\begin{vmatrix}2&-1&1\\1&2&-3\\3&\lambda&5\end{vmatrix}=0\Rightarrow\lambda=-4.$

47) Here $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]=0$
The given scalar triple product$=k[abc]=0$.

48) $[a-b\ \ b-c\ \ c-a]=\{(a-b)\times(b-c)\}\,.\,(c-a)$
$=(a\times b-a\times c-b\times b+b\times c)\,.\,(c-a)$
$=(a\times ab+ca\times a+b\times c)\,.\,(\mathbf{c}-\mathbf{a})$
$=(a\times b)\,.\,c-(a\times b)\,.\,a+(c\times a)\,.\,c-(c\times a)\,.\,a$
$=(\mathbf{a}\times\mathbf{b}).\mathbf{c}-(\mathbf{a}\times\mathbf{b}).\mathbf{a}+(\mathbf{c}\times\mathbf{a}).\mathbf{c}-(\mathbf{c}\times\mathbf{a}).\mathbf{a}$
$+(\mathbf{b}\times\mathbf{c}).\mathbf{c}-(\mathbf{b}\times\mathbf{c}).\mathbf{a}$
$=$
$[a\,b\,c]-[a\,b\,a]+[c\,a\,c]-[c\,a\,a]+[b\,c\,c]-[b\,c\,a]=0$

49) $(\mathbf{u}+\mathbf{v}-\mathbf{w})\,.\,(\mathbf{u}\times\mathbf{v}-\mathbf{u}\times\mathbf{w}-\mathbf{v}\times\mathbf{v}+\mathbf{v}\times\mathbf{w})$
$=(\mathbf{u}+\mathbf{v}-\mathbf{w})\,(\mathbf{u}\times\mathbf{v}-\mathbf{u}\times\mathbf{w}+\mathbf{v}\times\mathbf{w})$
$=\dfrac{\mathbf{u}\,.\,(\mathbf{u}\times\mathbf{v})}{0}-\dfrac{\mathbf{u}\,.\,(\mathbf{u}\times\mathbf{w})}{0}+\mathbf{u}\,.\,(\mathbf{v}\times\mathbf{w})+\dfrac{\mathbf{v}\,.\,(\mathbf{u}\times\mathbf{v})}{0}$
$-\mathbf{v}\,.\,(\mathbf{u}\times\mathbf{w})+\dfrac{\mathbf{v}\,.\,(\mathbf{v}\times\mathbf{w})}{0}-\mathbf{w}\,.\,(\mathbf{u}\times\mathbf{v})+\dfrac{\mathbf{w}\,.\,(\mathbf{u}\times\mathbf{w})}{0}$
$-\dfrac{\mathbf{w}\,.\,(\mathbf{u}\times\mathbf{w})}{0}=\mathbf{u}\,.\,(\mathbf{v}\times\mathbf{w})-\mathbf{v}\,.\,(\mathbf{u}\times\mathbf{w})-\mathbf{w}\,.\,(\mathbf{u}\times\mathbf{v})$
$=[\mathbf{u}\,\mathbf{v}\,\mathbf{w}]+[\mathbf{v}\,\mathbf{w}\,\mathbf{u}]-[\mathbf{w}\,\mathbf{u}\,\mathbf{v}]=\mathbf{u}\,.\,(\mathbf{v}\times\mathbf{w}).$

50) $\mathbf{i} \times \mathbf{j} \times \mathbf{k} = \mathbf{k} \times \mathbf{k} = 0$.