Intern_Roster

March 19, 2019

1 Intern Roster

1.1 Introduction

We have 11 interns. Let each intern be i.

We have 13 rotations. Let each rotation be j. There are also three annual leave rotations. These shall be j values 14, 15, 16. Therefore the total is 16.

We have 54 weeks for the whole period of the roster. Let each week be k.

j	Rotation Label	Duration	Maximum Interns per week
1	CPD-G	8	2
2	CPD-V	4	1
3	AP	4	1
4	MIC	4	1
5	MCH	2	1
6	CPCa	3	1
7	CPM	3	no limit
8	CPK	2	no limit
9	IP	4	2
10	DISP	3	no limit
11	CPC	5	no limit
12	QUM	1	1
13	H	1	1
14	A/L_1	1	11
15	A/L_2.1	1	6
16	A/L_2.2	1	5

1.2 Decision Variables

x^i_{jk} where i is an intern, rostered in area j, on week k

1.3 Objective Function

$$\text{maximise} \sum_i \sum_j \sum_k x^i_{jk} \quad \forall i \; \forall j \; \forall k$$

1.4 Constraints

Intern Physical Person Constraint

That one person can only be in one place at once:

$$\sum_j x^i_{jk} \leq 1 \quad \forall i \; \forall k$$

Intern Rotation Completion Constraint

Let $x^i_{jk} = 1$ if person i is doing rotation j for week k.

$$\sum_k x^i_{jk} \geq 1 \quad \forall i, \quad \forall j$$

This constraint my need to change to come in line with our duration requirements.

Intern Rotation Capacity Constraint

The limit of how many interns can work in an area at once

$$\sum_i x^i_{1,k} \leq 2 \quad \forall k$$

$$\sum_i x^i_{2,k} \leq 1 \quad \forall k$$

$$\sum_i x^i_{3,k} \leq 1 \quad \forall k$$

$$\sum_i x^i_{4,k} \leq 1 \quad \forall k$$

$$\sum_i x^i_{5,k} \leq 1 \quad \forall k$$

$$\sum_i x^i_{6,k} \leq 1 \quad \forall k$$

$$\sum_i x^i_{7,k} \geq 0 \quad \forall k$$

$$\sum_i x^i_{8,k} \geq 0 \quad \forall k$$

$$\sum_i x^i_{9,k} \leq 2 \quad \forall k$$

$$\sum_i x^i_{10,k} \geq 0 \quad \forall k$$

$$\sum_i x^i_{11,k} \geq 0 \quad \forall k$$

$$\sum_i x^i_{12,k} \leq 1 \quad \forall k$$

$$\sum_i x^i_{13,k} \leq 1 \quad \forall k$$

The constraints below are currently incorrect as they do not accurately express what is desired: That all interns should have 1 week's leave all together and then a second week's leave in two groups.

$$\sum_i x^i_{14,k} = 11 \quad \forall k$$

$$\sum_{i=1}^{6} x^i_{15,k} = 6 \quad \forall k$$

$$\sum_{i=7}^{11} x^i_{16,k} = 5 \quad \forall k$$

Intern Rotation Duration Constraint

This is the major point at which our current model falls apart. We require interns to spend a period of time in each rotation in **consecutive** blocks. Mathematically, the below system works, however it becomes flawed when trying to use it in CPLEX. This was an issue for two reasons: 1. The inability to input decision variables into "if" statements 2. Our initial use of the k values as strings.

To combat the 2. issue, we attempted to change k to an integer value, however this resulted in its inability to be used in the same fashion in x^i_{jk}.

We are currently re-evaluating our options. One that was explored was that we might just ascribe weights to each Rotation (this being the duration), and use a binary variable instead of x_{ij}. However this limits us in the particular regard that we would not be able to specify specific weeks for availability - an aspect of flexibility integral as we refine the model. There are to be a number of final constraints still to be added which depend on the model having this quality.

$$\sum_{\alpha=0}^{7} y^i_{1,k+\alpha} = 8 \text{ if } x^i_{1,k} = 1$$

$$\sum_{\alpha=0}^{3} y^i_{2,k+\alpha} = 4 \text{ if } x^i_{2,k} = 1$$

$$\sum_{\alpha=0}^{3} y^i_{3,k+\alpha} = 4 \text{ if } x^i_{3,k} = 1$$

$$\sum_{\alpha=0}^{3} y^i_{4,k+\alpha} = 4 \text{ if } x^i_{4,k} = 1$$

$$\sum_{\alpha=0}^{1} y^i_{5,k+\alpha} = 2 \text{ if } x^i_{5,k} = 1$$

$$\sum_{\alpha=0}^{2} y^i_{6,k+\alpha} = 3 \text{ if } x^i_{6,k} = 1$$

$$\sum_{\alpha=0}^{2} y^i_{7,k+\alpha} = 3 \text{ if } x^i_{7,k} = 1$$

$$\sum_{\alpha=0}^{1} y^i_{8,k+\alpha} = 2 \text{ if } x^i_{8,k} = 1$$

$$\sum_{\alpha=0}^{3} y^i_{9,k+\alpha} = 4 \text{ if } x^i_{9,k} = 1$$

$$\sum_{\alpha=0}^{2} y^i_{10,k+\alpha} = 3 \text{ if } x^i_{10,k} = 1$$

$$\sum_{\alpha=0}^{4} y^i_{11,k+\alpha} = 5 \text{ if } x^i_{11,k} = 1$$

$$y^i_{12,k} = 1 \text{ if } x^i_{12,k} = 1$$

$$y^i_{13,k} = 1 \text{ if } x^i_{13,k} = 1$$

$$y^i_{14,k} = 1 \text{ if } x^i_{14,k} = 1$$

$$y^i_{15,k} = 1 \text{ if } x^i_{15,k} = 1$$

$$y^i_{16,k} = 1 \text{ if } x^i_{16,k} = 1$$

It should be noted that were the above constraints valid in the model, additional decision variables would be needed.

Expanding one of these...

$$\sum_{\alpha=0}^{7} 8 - x^i_{1,k+\alpha} \leq M \cdot y \quad \forall_i \ \forall_{k=1}^{47}$$

$$x^i_{1,k} \leq M \cdot (1 - y) \quad \forall_i \ \forall_{k=1}^{47}$$

$$8 - \left(x^1_{1,1} + x^1_{1,2} + \cdots + x^1_{1,8} \right) \leq M \cdot y$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$8 - \left(x^1_{1,47} + x^1_{1,48} + \cdots + x^1_{1,54} \right) \leq M \cdot y$$

$$8 - \left(x^2_{1,1} + x^2_{1,2} + \cdots + x^2_{1,8} \right) \leq M \cdot y$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$8 - \left(x^2_{1,47} + x^2_{1,48} + \cdots + x^2_{1,54} \right) \leq M \cdot y$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$8 - \left(x^{11}_{1,47} + x^{11}_{1,48} + \cdots + x^{11}_{1,54} \right) \leq M \cdot y$$

$$x^1_{1,1} \leq M \cdot (1 - y)$$

$$
\begin{aligned}
&\vdots \quad \vdots \quad \vdots \\
x^{1}_{1,47} &\leq M \cdot (1-y) \\
x^{2}_{1,1} &\leq M \cdot (1-y) \\
&\vdots \quad \vdots \quad \vdots \\
&\vdots \quad \vdots \quad \vdots \\
x^{11}_{1,47} &\leq M \cdot (1-y)
\end{aligned}
$$

Intern Leave Constraint

These contraints are currently flawed with our current model - particular as one is unable to use decision variables in 'if' statements and conditions

$$\sum_i x^i_{14,k} = 11 z_k \quad \text{if} \quad \sum_k z_k = 1$$

$$\sum_i x^i_{15,k} = 6 z_k \quad \text{if} \quad \sum_k z_k = 1$$

$$\sum_i x^i_{16,k} = 5 z_k \quad \text{if} \quad \sum_k z_k = 1$$

Additionally, after analysis we have deducted that if these constraints were to be working properly, there would be no need for those currently causing issue under the *Intern Rotation Capacity Constraint*. We believe that those can be manipulated with little trouble however.

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