# PART-B (50 MARKS)

## UNIT-I

1 (a) Solve the system of Equations using Gauss elimination method $3x+y+2z=3$, $2x-3y-z=-3$, $x+2y+z=4$.

Consider Augmented matrix

$$[A:B] = \left[\begin{array}{ccc:c} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{array}\right]$$

$$R_1 \leftrightarrow R_3, \quad \left[\begin{array}{ccc:c} 1 & 2 & 1 & 4 \\ 2 & -3 & -1 & -3 \\ 3 & 1 & 2 & 3 \end{array}\right]$$

$$\begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array} \sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & -5 & -1 & -9 \end{array}\right]$$

$$R_3 \to 7R_3 - 5R_2 \sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & 0 & 8 & -8 \end{array}\right]$$

By back substitution,

we have $8z = -8 \Rightarrow z = -1$

Substituting $z=-1$ in $\cancel{7}y + 3z = 11 \Rightarrow 7y = 14 \Rightarrow y = 2$

Substituting $z=-1$ & $y=2$ in $x+2y+z=4$, we get $x=1$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

1 (b) Find the inverse using Gauss-Jordan method $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

Consider, $\left[\begin{array}{ccc:ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$

$$\begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{array} \sim \left[\begin{array}{ccc:ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & -3 & -2 & 0 & 1 \end{array}\right]$$

$$R_1 \to R_1 - R_2, \quad R_3 \to R_2 - R_3 \sim \left[\begin{array}{ccc:ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 4 & 1 & 1 & -1 \end{array}\right]$$

$$R_3 \to \frac{R_3}{4} \sim \left[\begin{array}{ccc:ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/4 & 1/4 & -1/4 \end{array}\right]$$

$$R_1 \to R_1 - R_3, \quad R_2 \to R_2 - R_3 \sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & \frac{7}{4} & \frac{-5}{4} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{-5}{4} & \frac{3}{4} & 1/4 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & \frac{-1}{4} \end{array}\right]$$

$\therefore$ we get $A^{-1} = \begin{bmatrix} \frac{7}{4} & \frac{-5}{4} & \frac{1}{4} \\ \frac{-5}{4} & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{-1}{4} \end{bmatrix}$

(OR)

3 (a) Find the rank of the matrix using Normal form

$$A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

$$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1 \sim \begin{bmatrix} 1 & 3 & 6 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 2 & -2 & 4 \end{bmatrix}$$

$$C_2 \to C_2 - 3C_1, \quad C_3 \to C_3 - 6C_1, \quad C_4 \to C_4 + C_1 \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 2 & -2 & 4 \end{bmatrix}$$

$$R_3 \to R_3 - 2R_2 \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_3 \to C_3 + C_2, \quad C_4 \to C_4 - 2C_2 \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

i.e, $\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore \rho(A) = 2$

3 (b) Test the consistency, if so, solve the system of equations

$x+y+z=6,\ x+2y+3z=10,\ x+2y+3z=5$

Consider the Augmented matrix

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & 3 & : & 5 \end{bmatrix}$$

$$\begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array} \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & 2 & : & -1 \end{bmatrix}$$

$$R_3 \to R_3 - R_2 \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & 0 & : & -5 \end{bmatrix}$$

Here $\rho(A) = 2,\ \rho(A:B) = 3$

i.e, $\rho(A) \neq \rho(A:B)$

$\therefore$ The given system of equations are inconsistent.

---

UNIT – II

4 (a) Determine the Eigen values of $A^{-1}$ where $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$

i.e, $$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow (1-\lambda)[(2-\lambda)(3-\lambda)-2] - 1[2-4+2\lambda] = 0$

$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\Rightarrow (\lambda-1)(\lambda-2)(\lambda-3) = 0$

$\Rightarrow \lambda = 1, 2, 3$

$\therefore$ The Eigen values of A are 1, 2, 3 we know that, if $A$ has the Eigen values $\lambda_1, \lambda_2, \lambda_3$

then $A^{-1}$ has the eigen values $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}$.

$\therefore$ The Eigen values of $A^{-1}$ are $1, \frac{1}{2}, \frac{1}{3}$

---

4 (b) Verify cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & 4 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 3 \end{bmatrix}$

The characteristic Equation of $A$ is $|A - \lambda I| = 0$

i.e, $\begin{vmatrix} 2-\lambda & 4 & 7 \\ 0 & 1-\lambda & 8 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda) [(1-\lambda)(3-\lambda) = 0$

$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

By cayley-Hamilton theorem, Every square matrix satisfies its own characteristic equation.

i.e, $A^3 - 6A^2 + 11A - 6I = 0$

consider $A^3 - 6A^2 + 11A - 6I$

$$= \begin{bmatrix} 8 & 28 & 325 \\ 0 & 1 & 104 \\ 0 & 0 & 27 \end{bmatrix} - \begin{bmatrix} 24 & 72 & 402 \\ 0 & 6 & 192 \\ 0 & 0 & 54 \end{bmatrix} + \begin{bmatrix} 22 & 44 & 77 \\ 0 & 11 & 88 \\ 0 & 0 & 33 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ cayley-Hamilton theorem is verified.

5 Diagonalize the matrix $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$ and find $A^4$

using the model matrix 'P'.

The characteristic equation of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 2 & -7 \\ 2 & 1-\lambda & 2 \\ 0 & 1 & -3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\left[(1-\lambda)(-3-\lambda)-2\right] - 2\left[-6-2\lambda\right] - 7(2) = 0$$

$$\Rightarrow -\lambda^3 + 13\lambda - 12 = 0$$

$$\Rightarrow \lambda^3 - 13\lambda + 12 = 0$$

$$\Rightarrow (\lambda-1)(\lambda^2+\lambda-12) = 0$$

$$\Rightarrow (\lambda-1)(\lambda-3)(\lambda+4) = 0$$

$$\Rightarrow \lambda = 1, 3, -4$$

Case (i) :- when $\lambda = 1$, the given system of Eqs becomes

$$x + 2y - 7z = 0$$

$$2x + 2z = 0 \quad \text{— (A)}$$

$$y - 4z = 0 \quad \text{— (B)}$$

Solving eqs (A) & (B) using cross multiplication method,

we get $X_1 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$

Case (ii) :- when $\lambda = 3$, the corresponding Eigen vector

$$X_2 = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$$

Case (iii) :- when $\lambda = -4$, the corresponding Eigen

vector $X_3 = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$

$\therefore$ When $\lambda = 1$, $X_1 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$

When $\lambda = 3$, $X_2 = [5 \quad 6 \quad 1]'$

When $\lambda = -4$, $X_3 = [3 \quad -2 \quad 2]'$

The Modal matrix, $P = [X_1 \; X_2 \; X_3] = \begin{bmatrix} -1 & 5 & 3 \\ 4 & 6 & -2 \\ 1 & 1 & 2 \end{bmatrix}$

$|P| = -(12+2) - 5(8+2) + 3(4-6)$

$= -14 - 50 - 6 = -70. \; (\neq 0)$

$\therefore P^{-1} = -\frac{1}{70}\begin{bmatrix} 14 & -7 & 28 \\ -10 & -5 & 10 \\ -2 & 6 & -26 \end{bmatrix} = \frac{1}{70}\begin{bmatrix} -14 & 7 & -28 \\ 10 & 5 & -10 \\ 2 & -6 & 26 \end{bmatrix}$

$\because |P| \neq 0$

$\therefore \rho(P) = 3$ = No. of L.I. Vectors

= No. of " columns.

$\because$ All the eigen vectors are L.I.

$\therefore$ P diagonalizes A

Hence $P^{-1}AP = D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

W.K.T. $A^n = PD^nP^{-1}$

$\therefore A^4 = PD^4P^{-1}$

$D^4 = \begin{bmatrix} 1^4 & 0 & 0 \\ 0 & 3^4 & 0 \\ 0 & 0 & (-4)^4 \end{bmatrix}$.

$\therefore A^4 = +\frac{1}{70}\begin{bmatrix} -1 & 5 & 3 \\ 4 & 6 & -2 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 256 \end{bmatrix}\begin{bmatrix} -14 & +7 & -28 \\ 10 & 5 & -10 \\ 2 & -6 & 26 \end{bmatrix}$

$= \frac{1}{70}\begin{bmatrix} -1 & 5 & 3 \\ 4 & 6 & -2 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} -14 & 7 & -28 \\ 810 & 405 & -810 \\ 512 & -1536 & 6656 \end{bmatrix}$

$= \frac{1}{70}\begin{bmatrix} 5600 & -2590 & 15946 \\ 3780 & 5530 & -18284 \\ 1820 & -2660 & 12474 \end{bmatrix}$

$= \begin{bmatrix} 80 & -37 & 227 \\ 54 & 79 & -258 \\ 26 & -38 & 179 \end{bmatrix}$

6 (a) write the Taylor's Series expansion for $f(x) = \log(1-x)$ about $x=0$

Given that $f(x) = \log(1-x) \Rightarrow f(0) = 0$

$$\left.\begin{aligned} f'(x) &= \frac{-1}{1-x} \Rightarrow f'(0) = -1 \\ f''(x) &= \frac{-1}{(1-x)^2} \Rightarrow f''(0) = -1 \\ f'''(x) &= \frac{-2}{(1-x)^3} \Rightarrow f'''(0) = -2 \\ f^{IV}(x) &= \frac{-6}{(1-x)^4} \Rightarrow f^{IV}(0) = -6 \end{aligned}\right\} (I)$$

Taylor's Series expansion for $f(x)$ about $x=0$ is

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{IV}(0) + \cdots \quad \text{— (A)}$$

Substituting all the values of (I) in (A), we get the required Maclaurin series as

$$\therefore \log(1-x) = -x + \frac{x^2}{2!}(-1) + \frac{x^3}{6}(-2) + \frac{x^4}{4!}(-6) + \cdots$$

$$= -x - \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{4} - \cdots$$

6 (b) Verify Rolle's theorem for $f(x) = |x|$ in $[-1, 1]$

we have $f(x) = |x| = \begin{cases} -x, & -1 \leq x < 0 \\ x, & 0 \leq x < 1 \end{cases}$

modulus function is continuous

(i) $f(x)$ is continuous for every value of $x$

$\therefore f(x)$ is continuous in the closed interval $[-1, 1]$

(ii) $f(x)$ is not derivable at $x = 0$

we have $f(0) = |0| = 0$

$$L.H.D = Lf'(0) = \underset{x \to 0^-}{Lt} \frac{f(x) - f(0)}{x - 0}$$

$$= \underset{x \to 0^-}{Lt} \frac{|x| - 0}{x}$$

$$= \underset{x \to 0^-}{Lt} \frac{-x}{x}$$

$$= \underset{x \to 0^-}{Lt} (-1) = -1$$

$$R.H.D = Rf'(0) = \underset{x \to 0^+}{Lt} \frac{f(x) - f(0)}{x - 0}$$

$$= \underset{x \to 0^+}{Lt} \frac{|x| - 0}{x}$$

$$= \underset{x \to 0^+}{Lt} \frac{x}{x}$$

$$= \underset{x \to 0^+}{Lt} (1) = 1$$

Since $Lf'(0) \neq Rf'(0)$

$\therefore f(x)$ is not derivable at $x = 0$.

$\therefore f(x)$ is not derivable in the open interval $(-1, 1)$ at

$x = 0$.

Hence Rolle's Theorem is not applicable to

$f(x) = |x|$ in $[-1, 1]$.

7 If $a<b$ prove that $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$

consider $f(x) = \tan^{-1}x$ in $[a, b]$ for $0<a<b<1$.

Since $f(x)$ is continuous in closed interval $[a, b]$ and derivable in open interval $(a, b)$, we can apply Lagrange's mean value theorem

Hence there exists a point $c$ in open interval $(a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

Here $f'(x) = \frac{1}{1+x^2}$ and hence $f'(c) = \frac{1}{1+c^2}$

$\therefore$ There exists a point $c$, $a<c<b$ such that

$$\frac{1}{1+c^2} = \frac{\tan^{-1}b - \tan^{-1}a}{b-a} \quad \text{——— (1)}$$

we have $1+a^2 < 1+c^2 < 1+b^2$

$$\therefore \quad \frac{1}{1+a^2} > \frac{1}{1+c^2} > \frac{1}{1+b^2} \quad \text{——— (2)}$$

$\because a<c<b$
$\therefore a^2<c^2<b^2$
$\Rightarrow 1+a^2<1+c^2<1+b^2$

Using eqs (1) & (2), we have

$$\frac{1}{1+a^2} > \frac{\tan^{-1}b - \tan^{-1}a}{b-a} > \frac{1}{1+b^2}$$

(or) $\frac{b-a}{1+b^2} > \tan^{-1}b - \tan^{-1}a > \frac{b-a}{1+a^2}$

$$\Rightarrow \frac{b-a}{1+a^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+b^2}$$

Hence the result

# UNIT – IV

8 (a) Expand $f(x,y) = xy^2 + \cos(xy)$ in powers of $(x-1)$ and $(y-\frac{\pi}{2})$ using Taylor's Series

Given that $f(x,y) = xy^2 + \cos xy$, $a=1$, $b=\frac{\pi}{2}$

Taylor's Expansion of $f(x,y)$ in powers of $(x-a)$ & $(y-b)$

is $f(x,y) = f(a,b) + (x-a) f_x(a,b) + (y-b) f_y(a,b)]$

$$+\frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b) f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right] + \cdots \quad \text{(A)}$$

we have $f(x,y) = xy^2 + \cos(xy) \Rightarrow f(1,\frac{\pi}{2}) = \frac{\pi^2}{4} + \cos\frac{\pi}{2} = \frac{\pi^2}{4} + 1$

$f_x(x,y) = y^2 - y\sin(xy) \Rightarrow f_x(1,\frac{\pi}{2}) = \frac{\pi^2}{4} - \frac{\pi}{2}\sin\frac{\pi}{2} = \frac{\pi^2}{4}$

$f_y(x,y) = 2xy - x\sin(xy) \Rightarrow f_y(1,\frac{\pi}{2}) = \pi - \sin\frac{\pi}{2} = \pi$

$f_{xx}(x,y) = -y^2\cos(xy) \Rightarrow f_{xx}(1,\frac{\pi}{2}) = -\frac{\pi^2}{4}\cos\frac{\pi}{2} = \frac{-\pi^2}{4}$

$f_{xy}(x,y) = 2y - \sin xy - xy\cos(xy) \Rightarrow f_{xy}(1,\frac{\pi}{2}) = \pi - \frac{\pi}{2}\cos\frac{\pi}{2} = \frac{\pi}{2}$

$f_{yy}(x,y) = 2x - x^2\cos xy \Rightarrow f_{yy}(1,\frac{\pi}{2}) = 2 - \cos\frac{\pi}{2} = 2-1 = 1$

substituting these values in eq (A), we get

$$f(x,y) = \frac{\pi^2}{4} + 1 + (x-1)\frac{\pi^2}{4} + (y-\frac{\pi}{2})\pi + \frac{1}{2!}\left[(x-1)^2\left(-\frac{\pi^2}{4}\right) + 2(x-1)(y-\frac{\pi}{2})\cdot\frac{\pi}{2} + (y-\frac{\pi}{2})^2\right] + \cdots$$

8 (b) Given that $u = \frac{y}{z} + \frac{z}{x}$

we have $\frac{\partial u}{\partial x} = \frac{-z}{x^2}$, $\frac{\partial u}{\partial y} = \frac{1}{z}$, $\frac{\partial u}{\partial z} = \frac{-y}{z^2} + \frac{1}{x}$

consider $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = \frac{-z}{x} + \frac{y}{z} - \frac{y}{z} + \frac{z}{x} = 0$

(OR)

9 (a) Find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ if $u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$

$$\frac{\partial u}{\partial x}=\frac{1}{\sqrt{1-\frac{x^2}{y^2}}}\left(=\frac{y}{\sqrt{y^2-x^2}}\right)\frac{1}{y}+\frac{1}{1+\frac{y^2}{x^2}}\left(=\frac{x^2}{x^2+y^2}\right)\left(-\frac{y}{x^2}\right).$$

$$\frac{\partial u}{\partial y}=\frac{1}{\sqrt{1-(x/y)^2}}\cdot\left(-\frac{x}{y^2}\right)+\frac{1/x}{1+(y/x)^2}=\frac{-(x/y)}{\sqrt{y^2-x^2}}+\frac{x}{x^2+y^2}.$$

$$\therefore x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\cancel{\frac{x}{\sqrt{y^2-x^2}}}-\cancel{\frac{xy}{x(x^2+y^2)}}-\cancel{\frac{x}{\sqrt{y^2-x^2}}}+\cancel{\frac{xy}{x^2+y^2}}$$

$$=0.$$

9 (b) show that $u=\frac{x}{y-z}$, $V=\frac{y}{z-x}$, $\omega=\frac{z}{x-y}$ are functionally dependent.

$$\frac{\partial(u,v,w)}{\partial(x,y,z)}=\begin{vmatrix}\frac{\partial u}{\partial x}&\frac{\partial u}{\partial y}&\frac{\partial u}{\partial z}\\\frac{\partial v}{\partial x}&\frac{\partial v}{\partial y}&\frac{\partial v}{\partial z}\\\frac{\partial w}{\partial x}&\frac{\partial w}{\partial y}&\frac{\partial w}{\partial z}\end{vmatrix}=\begin{vmatrix}\frac{1}{y-z}&\frac{-x}{(y-z)^2}&\frac{x}{(y-z)^2}\\\frac{y}{(z-x)^2}&\frac{1}{z-x}&\frac{-y}{(z-x)^2}\\\frac{-z}{(x-y)^2}&\frac{z}{(x-y)^2}&\frac{1}{x-y}\end{vmatrix}$$

$$=\frac{1}{y-z}\left[\frac{1}{(x-y)(z-x)}+\frac{yz}{(z-x)^2(x-y)^2}\right]$$

$$+\frac{x}{(y-z)^2}\left[\frac{y}{(z-x)^2(x-y)}-\frac{yz}{(x-y)^2(z-x)^2}\right]$$

$$+\frac{x}{(y-z)^2}\left[\frac{yz}{(z-x)^2(x-y)^2}+\frac{z}{(x-y)^2(z-x)}\right]$$

$$=\frac{1}{(y-z)(x-y)(z-x)}+\frac{yz}{(y-z)(z-x)^2(x-y)^2}$$

$$+\frac{xy}{(y-z)^2(z-x)^2(x-y)}-\frac{xy\cancel{z}}{(y\cancel{-z})^2(x-y)^2(z-x)^2}$$

$$+\frac{xyz}{(y-z)^2(z\cancel{-x})^2(x-y)^2}+\frac{xz}{(y-z)^2(x-y)^2(z-x)}\neq 0$$

$\therefore u, v, w$ are functionally Independent

## UNIT – V

10. Evaluate by change of order of Integration

$$\int_0^{2a}\int_{y^2/4a}^{3a-y} dx\,dy.$$

**Sol.** let $I = \int_0^{2a}\int_{y^2/4a}^{3a-y} dx\,dy$

The given limits are $x = \frac{y^2}{4a} \rightarrow ①$ , $x = 3a - y \rightarrow ②$

and $y = 0$ to $2a$. (i.e., the horizontal strip varies from ① to ②)

① represents a parabola $y^2 = 4ax$.

② " " st. line $x + y = 3a$

Intersection points of ① & ②

Substituting ① in ②, we get

$\frac{y^2}{4a} + y = 3a \Rightarrow y^2 + 4ay = 12a^2$

$\Rightarrow y^2 + 4ay - 12a^2 = 0$

$\Rightarrow (y + 6a)(y - 2a) = 0$

$\Rightarrow y = -6a, 2a$

when $y = -6a$,

$x = 3a - y = 3a + 6a$

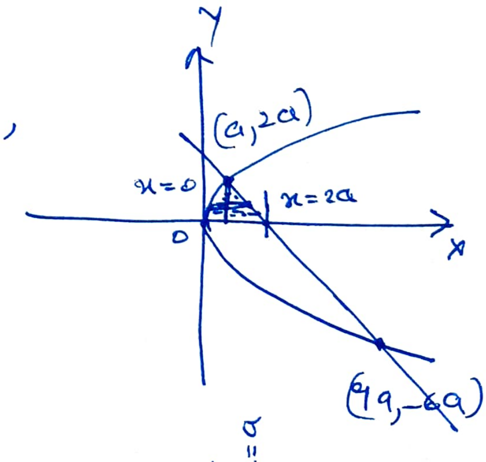
$= 9a$

when $y = 2a$,

$x = 3a - 2a = a$

$\therefore$ (~~-6a, 9a~~) ; $(a, 2a)$ & $(9a, -6a)$ are intersection points

The region of Integration, R is the shaded portion of the figure.




To change the order of Integration, Introduce vertical strip.

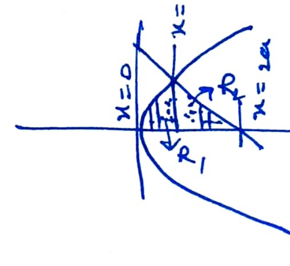
This strip divides R in to two parts $R_1$ & $R_2$.

In $R_1$ : $0 \le y \le 2\sqrt{ax}$

$0 \le x \le a$

In $R_2$ : $0 \le y \le 3a - x$

$a \le x \le 2a$.




$$\therefore I = \iint_{R_1} dy\,dx + \iint_{R_2} dy\,dx.$$

$$\Rightarrow I = \int_0^a \int_0^{2\sqrt{ax}} dy\,dx + \int_a^{2a} \int_0^{3a-x} dy\,dx .$$

$$= \int_0^a (y)_0^{2\sqrt{ax}}\, dx + \int_a^{2a} (y)_0^{3a-x}\, dx .$$

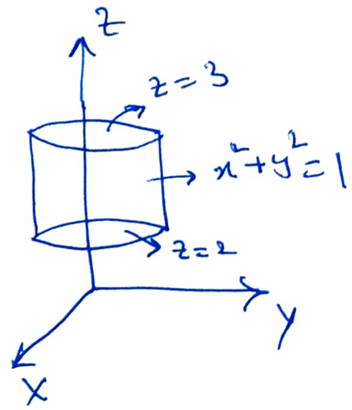
$$= \int_0^a 2\sqrt{ax}\, dx + \int_a^{2a} (3a-x)\, dx .$$

$$= 2\sqrt{a} \left(\frac{x^{3/2}}{3/2}\right)_0^a + \left(3ax - \frac{x^2}{2}\right)_a^{2a}$$

$$= \frac{4\sqrt{a}}{3}\left(a^{3/2} - 0\right) + \left(6a^2 - 2a^2 - 3a^2 + \frac{a^2}{2}\right)$$

$$= \frac{4}{3}a^2 + \frac{3}{2}a^2 = \frac{17}{6}a^2 .$$

(11) Evaluate $\iiint_R z(x^2+y^2)\,dx\,dy\,dz$ where $R$ is the region bounded by the cylinder $x^2+y^2=1$ & the planes $z=2$ and $z=3$ by changing it to cylindrical co-ordinates.




Sol.: Given $I = \iiint_R z(x^2+y^2)\,dx\,dy\,dz$

$R$: A cylinder $x^2+y^2=1$, $2 \le z \le 3$

Introducing, cylindrical polar co-ordinates, we get

$x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$, $dx\,dy\,dz = \rho\,d\rho\,d\phi\,dz$

$$I = \int_2^3 \int_0^{2\pi} \int_0^1 z\rho^2 \rho\,d\rho\,d\phi\,dz \quad (\because x^2+y^2 = \rho^2) .$$

$$= \int_2^3 z\,dz \int_0^{2\pi} d\phi \int_0^1 \rho^3\,d\rho$$ ($\because$ In integrand, the variables are independent)

$$= \left(\frac{z^2}{2}\right)_2^3 (\phi)_0^{2\pi} \left(\frac{\rho^4}{4}\right)_0^1 = \frac{1}{2}(9-4)\,2\pi\left(\frac{1}{4}\right) = \frac{5}{4}\pi$$