# NON LINEAR CONTROL OF A MOBILE MANIPULATOR ROBOT

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## ABSTRACT

This paper describes a nonlinear control designing technique for a mobile manipulator robot model analytically with simulation results to prove the performance of the designed controller. Starting from Introduction to the problem, following up some past research literature studies with some details about Adaptive control. Then, Dynamics of the model selected is explained following which is the mathematics used to design nonlinear control for this model. Finally, there are some results to show how effective the controller is for this type of system.

## INTRODUCTION

This paper consists of research project on mobile manipulator robot model and its control in terms of robot trajectory. The motivation behind this type of model comes from highly trending EOD (Explosive Ordinance Disposal) robot which is being used in a lot of applications such as military, defense, bomb disposal squad. These robots are major transformation of performing tasks which are dangerous or hazardous for a human being. A specific EOD robot model has wheeled vehicle carrying a manipulator on it. Vehicle is designed in such a manner that it can travel on any kind of rough terrains while the manipulator with an arm is capable of handling and maintaining any tasks or objects.

The detailed complexity of model is not considered in this paper and a simpler model is selected to design a controller for the non-linear system. However, this model has some motion constraints which needs to be considered while designing the controller. A model with differentially driven two-wheeled mobile vehicle with a two links manipulator on top of it. So, both of the wheels are driving at two different speeds and the vehicle turns with respect to difference in angular velocities.

Non-linear system has nonlinear dynamics equations of motion. So to control this type of system, a nonlinear controller is needed to be designed which can solve the hard as well as smooth non linearity. Behavior of a nonlinear system to any input is much more complex than a linear system, so control of those systems includes a lot of mathematics and usage of some good control techniques. There are a numerous techniques designed to control nonlinear systems like Lyapunov control, Back-stepping, Adaptive, Robust control. All of these methods require a set of equations that represents the dynamics of particular system and using these equations, control is derived using some technique.

## LITERATURE REVIEW

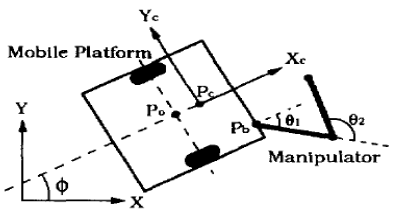
Adaptive control is a technique used to design control for systems like mobile manipulation, that has certain uncertainties in parameters to be estimated and controlled. The control law or adaptation law designed will track the error by reducing uncertainties in model and bring stability in the system. Many nonlinear control techniques were studied using literatures before solving this problem. Adaptive control is chosen to be best suitable for this type of problem.

Further, adaptive control is a method to design an adaptation law. Few parameters are selected based on uncertainties in states of the system. The, parameters are compared to find error which is then fed back via feedback gains to increase the stability of the system. To derive the adaptation law, it is needed to understand the system fully which includes identifying the parameters perfectly to be tracked. The states of system are then controlled and stabilized as per the adaptation law that does the feedback linearization and controlling of provided system.

Adaptive control with output feedback can be classified as one of the very crucial control law that feeds output states back to calculate error and increase the stability of system. As, only obtained results are from output measurement and other information is not available to the system, the control law must have some observer structures are used along with gains for feedback. In this method, some reference output is calculated based on desired output where some of the states are zeros. So, partial information is obtained from the output states while remaining from the observer part of adaptation law.

## MODEL OF ROBOTIC SYSTEM STUDIED

The model for mobile robot as shown in figure 1 is considered




[Figure 1: Robot model pictorial representation]

Some of the parameters of the model which appears in the equations of motion dynamics of system are defined here, $x_g$ is x component of position of center of mass of the robot, $y_g$ is y component, $\phi$ is angular orientation of the body in x-y plane, $\theta_r$ and $\theta_l$ are the angular positions of both the wheels.

## DYNAMICS OF MODEL

The model derived of the model of mobile manipulator as shown in figure 1, has been implemented with non-holonomic constraints of motion. These constraints are because of pure rolling motion of wheels, eliminating any slips while movement. Two of the constraint equations comes from this condition, while one more constraint equation is derived based on obvious condition that the vehicle will move in the direction of axis of symmetry and it cannot have motion in sideways. These constraints make the system non-holonomic. Using Euler-Lagrange approach of analytical dynamics, the following equations of motion of the robot are derived,

$$\boldsymbol{m\ddot{x}_g - m_g d_{og}(\ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi) - \lambda_1\sin\phi - \lambda_2\cos\phi - \lambda_3\cos\phi = 0},$$

$$\boldsymbol{m\ddot{y}_g - m_g d_{og}(\ddot{\phi}\cos\phi + \dot{\phi}^2\sin\phi) + \lambda_1\cos\phi - \lambda_2\sin\phi - \lambda_3\sin\phi = 0},$$

$$\boldsymbol{-m_g d_{og}\ddot{x}_g\sin\phi + m_g d_{og}\ddot{y}_g\cos\phi + I\ddot{\phi} - d_{og}\lambda_1 - b\lambda_2 + b\lambda_3 = 0},$$

$$\boldsymbol{I_w\ddot{\theta}_r + \lambda_2 r = \tau_1},$$

$$\boldsymbol{I_w\ddot{\theta}_l + \lambda_3 r = \tau_2}$$

Considering, the generalized coordinates state vector,

$$\boldsymbol{q = [x_g,\ y_g,\ \phi,\ \theta_r,\ \theta_l]^T}$$

Obtained is general state space equation states as follows,

$M\ddot{q} + V = E\tau - A^T\lambda$

Where M, V, E, A are the matrices with respective coefficients of the five states, with respective dimensions. To better understand the system, states of the system dynamics were plotted using MATLAB command ODE45 which showed that the states were increasing as per the torque value given as input. There were a lot of oscillations in the position and velocity states because of non-holonomic nature of system. So, a controller that linearizes this equation needs to be developed and implemented.

## NON LINEAR FEEDBACK CONTROL

To transform this dynamic system into standard form of state space representation which is used in designing nonlinear control. As we have seen the generalized equation of motion of the model is described by,

$$M(q)\ddot{q} + V(q,\dot{q}) = E(q)T - A^T(q)\lambda \quad (1)$$

This work dealt with three constraints. The rolling constraints and the constraint that the robot must move in the direction of axis of symmetry without any sideways movement are non-holonomic constraints. Representing both of these constraint equations; totally we have three equations, in state space form,

$$A(q)\dot{q} = 0 \quad (2)$$

The above equation denote that the column space of $\dot{q}$ is the subspace of the null space of A(q). There could be a number of possible linear combinations of null space of matrix A. Now let's assume a set of smooth vector fields $s_1(q)\ldots, s_{n-m}(q)$ which are linearly independent and fall in the null space of A. Therefore, we can also write

$$A(q)\dot{s}_i(q) = 0 \quad (3)$$

Where, $i = 1, 2, \ldots, n-m$

Where S(q) is defined as a full rank matrix made from the above vectors such that

$$S(q) = [\ s_1(q)\ldots, s_{n-m}(q)] \quad (4)$$

Since the constraints velocities will always be in the null space of A(q) it is possible to define vector of velocities v(t) such that,

$$\dot{q} = S(q)v(t) \quad (5)$$

Now, by differentiating equation (5), we can get,

$$\ddot{q} = \dot{S}(q)v(t) + S(q)\dot{v}(t) \quad (6)$$

Substituting this equation into original dynamics equation and solving for obtaining simple state space form,

$$M(q)\dot{S}(q)v(t) + M(q)S(q)\dot{v}(t) + V(q,\dot{q}) = E(q)T \quad (7)$$

In this above equation, T represents the torque to both wheels which will be the input for robot, thus the control part will be hidden in this torque input T. Deriving the general equation of T from equation (7), a standard format of the state space equation is required which can be done by changing the state variables which means two more states will be added in it. However, when we manipulate the inverse of a non-square matrix, calculations are further not possible as inverse does not exist. So, multiplying both sides of equation (7) by $S(q)^T$,

$$S^T(M(q)\dot{S}(q)v(t) + M(q)S(q)\dot{v}(t) + V(q,\dot{q})) = S^T E(q)T$$

Transformation of this system into a new simplified state space form, by changing the state variables to x = $[q^T\ \ v^T]^T$,

$$\dot{x} = \begin{bmatrix} Sv \\ (S^TMS)^{-1}(-S^TM\dot{S}v - S^TV) \end{bmatrix} + \begin{bmatrix} 0 \\ (S^TMS)^{-1}S^TE \end{bmatrix} T \quad (8)$$

Now, assuming that the number of actuator inputs is greater than or equal to number of degrees of freedom of the system $(S^TMS)^{-1}S^TE$ has rank n-m, therefore we can apply the following nonlinear feedback.

$T = ((S^TMS)^{-1}S^TE)^{-1}(u - (S^TMS)^{-1}(-S^TM\dot{S}v - S^TV))$ (9)

Substituting this equation for T in equation (8), a very simplified form is obtained after some mathematical calculations are performed.

$$\dot{x} = f(x) + g(x)u$$

Where $f(x) = \begin{bmatrix} S(q)v \\ 0 \end{bmatrix}$, $g(x) = \begin{bmatrix} 0 \\ I \end{bmatrix}$

## ADAPTATION LAW FOR CONTROL

To design the adaptation control law, linearization of the system needs to be done, which seems pretty complex for such system. Because of the complexity of equations, input linearization seems impossible for our model. However, input-output linearization method is useful and applicable using zero dynamics technique from. The first few states of differential model derived goes to zero which means those states could not be used while linearization, all the states are not available for feeding back to input. Thus, output equations will be used to obtain more information of system while integrating with feedback.

$$L_{fg} = [f, g] = \frac{dg}{dx}f - \frac{df}{dx}g = -\begin{bmatrix} S_q \\ 0 \end{bmatrix}$$

$$y = h(q) = [h_1(q) \ldots h_{n-m}(q)]$$

$$\Phi(q) = J_h(q)S(q)$$

Now, introducing a new state space variable system to characterize zero dynamics system, as the decoupled matrix derived using Jacobian and the constrained matrix S is non-singular. Let the new variable called be z, transforming into a valid new state space form, obtained is

$$z = T(x) = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} h(q) \\ L_f h(q) \\ h(q) \end{bmatrix} = \begin{bmatrix} h(q) \\ \Phi(q)v \\ h(q) \end{bmatrix}$$

$$\dot{z_1} = \frac{dh}{dq}\dot{q} = z_2$$

$$\dot{z_2} = \dot{\Phi}(q)v + \Phi(q)u$$

$$\dot{z_3} = J_h S_v = J_h S(J_h S)^{-1} z_2$$

T(x) is the transformation used based on a technique called diffeomorphism, which is a transformation of a differential system into another in such a manner that the mapping of systems is in a way that the function used to map and its inverse both are smooth. Thus, from this method the Jacobian J is obtained and transformation function too, that transformed our system into a new state variables differential system.

Utilizing the following state feedback

$$u = \Phi(q)^{-1}(v - \dot{\Phi}(q)v)$$

$$\dot{z_1} = z_2, \dot{z_2} = v, y = z_1$$

$$\dot{z_3} = 0$$

## DEFINING REFERENCE TRAJECTORY

The output equations of the dynamic system are chosen in such a way that the task performed by the system can be easily described and the controller could be designed. We know that ($x_g$, $y_g$) are the coordinates of the center of mass of the robot platform. We can choose a random point $P_r$ as reference with respect to the coordinates of the center of mass of the platform. We need to control the robot such that it follows the desired trajectory. Now we assume the reference point be $(x_r^g, y_r^g)$ in the robot platform frame. Then the world coordinates$(x_r, y_r)$ of the reference point can be given by

$$x_r = x_g + x_r^g \cos\varphi - y_r^g \sin\varphi$$
$$y_r = y_g + x_r^g \sin\varphi + y_r^g \cos\varphi$$

We have a trajectory tracking problem by taking the coordinates of the reference point to the output equation.

$$y = h(q) = [x_r \quad y_r]^T$$

The decoupling matrix corresponding the output is given by

$$\varphi(q) = J_h(q)S(q) = \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix}$$

where

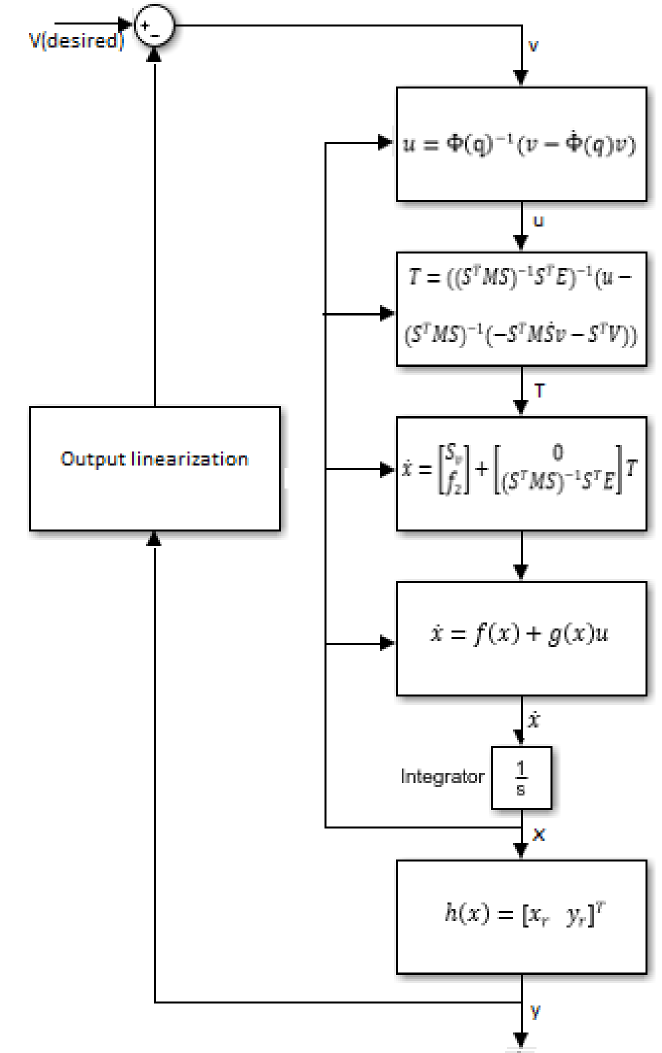
$$\varphi_{11} = c((b - y_r^g)\cos\varphi - (d + x_r^g)\sin\varphi)$$
$$\varphi_{12} = c((b + y_r^g)\cos\varphi + (d + x_r^g)\sin\varphi)$$
$$\varphi_{21} = c((b - y_r^g)\sin\varphi + (d + x_r^g)\cos\varphi)$$
$$\varphi_{22} = c((b + y_r^g)\sin\varphi - (d + x_r^g)\cos\varphi)$$

$$T = (S^TMS)u + S^TM\dot{S}v + S^TV$$
$$u = \varphi^{-1}(q)(v - \dot{\varphi}(q)v)$$
$$\ddot{y}_1 = v_1$$
$$\ddot{y}_2 = v_2$$

This approach shown above in analytical form, is used to simulate the system model and produce results for the experiment.

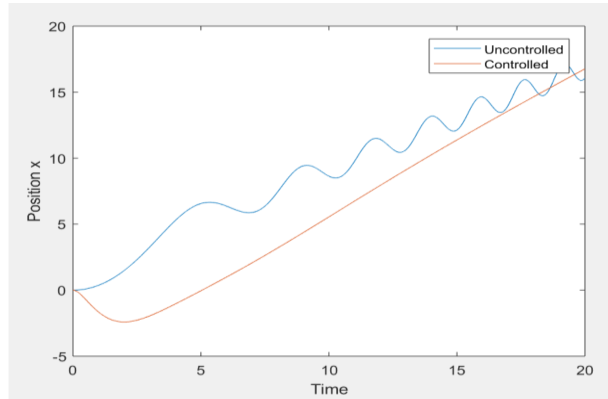
**BLOCK DIAGRAM FOR THIS APPROACH**



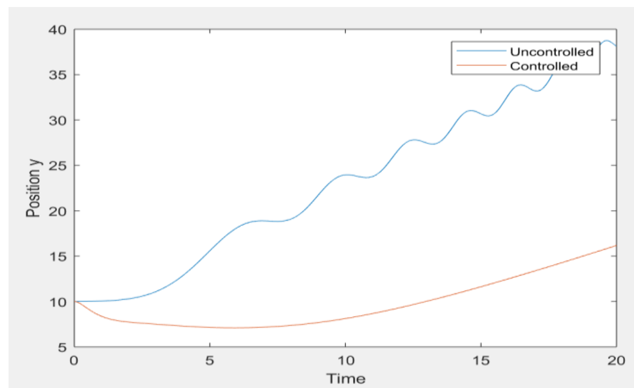

[Figure: Block Diagram for adaptive controller designed]

**SIMULATION RESULTS**

The control designed mathematically was used to simulate the system model using MATLAB and the graphs for states were plotted which shows how this controller removes all the oscillations and settles down the system within short period of time. All the irregularities due to non-holonomic nature of the model are eliminated when controller is implemented in system and input signal is provided. The simulation is performed for the time span of 20 seconds and both the system model without and with the designed controller are plotted.

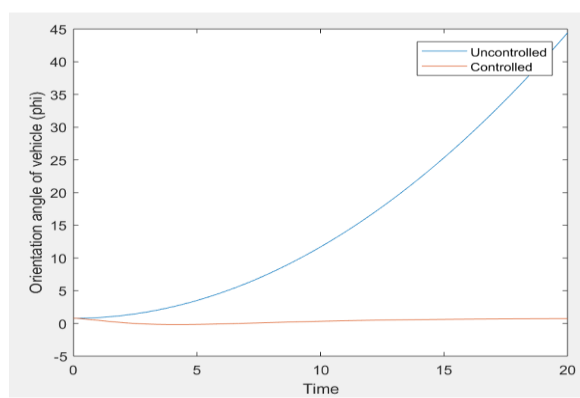



[Figure: position of mobile manipulator, x-component]



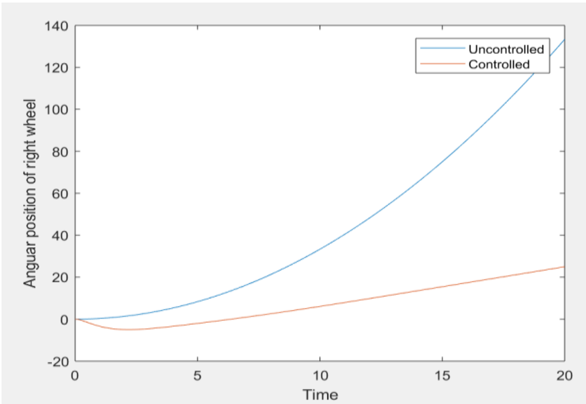

[Figure: position of mobile manipulator, y-component]

As, it is clearly seen from the graph plot that motion of the vehicle became smooth with no oscillations or fluctuations in system and based in initial conditions, as the angle of vehicle is 45˙ i.e. directed towards y axis, so it moves more in x direction and less in y direction to have sufficient movements and moves in positive direction. Also, the initial condition for position is given as (0,10) so using the reference trajectory, it follows the path by initially coming to reference point and then follows the trajectory path given.

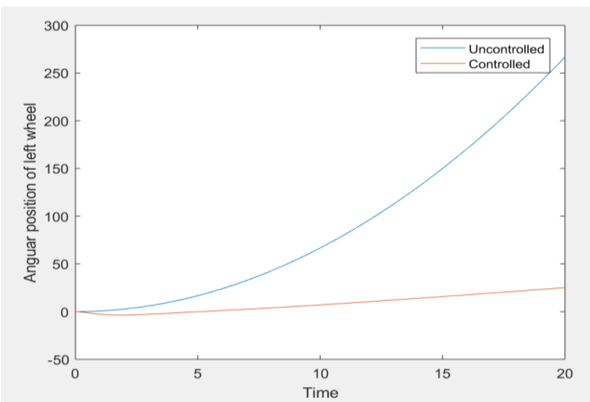



[Figure: Orientation/Heading angle of robot vehicle]

Initial condition for the robot is given as 45˙ so the vehicle starts rotation to come to a stable state of zero due to controller effect. Hence, the controller works perfectly for vehicle angular position in the Cartesian space.

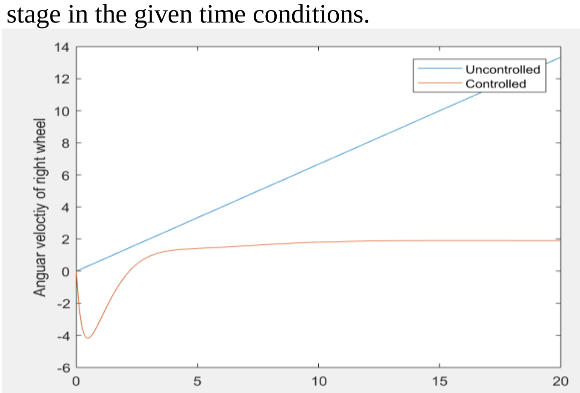



[Figure: Angular position of the right wheel]

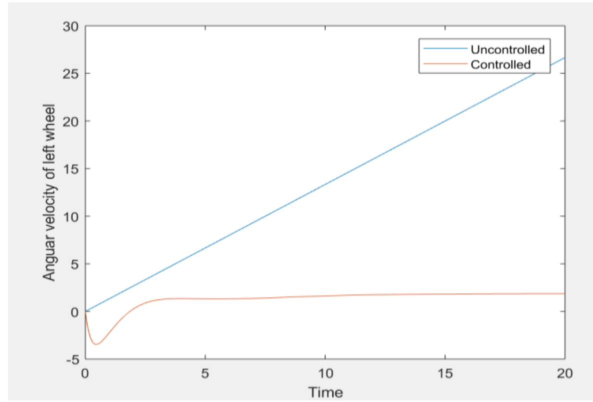



[Figure: Angular position of the left wheel]

The wheels of the vehicle seem to be stabilizing and moving at a slower controllable rate to produce effective movement for robot. Thus, it will follow the desired trajectory and reach to a final stage in the given time conditions.




[Figure: Angular velocity of right wheel]




[Figure: Angular velocity of left wheel]

The last two graphs clearly show how the angular velocities of both the wheels are controlled and goes to a stable state within 3-4 seconds duration and stays stable later on. These states also act as input signals as they are directly controlled by controller, making the signal of rates of rotation as input to system. There is a slight overshoot while stabilizing which is due to difference in the initial value and reference values of desired trajectory.

## CONCLUSION

Thus, the nonlinear controller; designed using the input-output linearization feedback technique with zero dynamics system formed by transformation of state variables, works accurately for the system. All the states become stable with the provided input signal by controller. Any non-holonomic mobile manipulator model would reach up to final position defined by following the given desired path. Hence, this approach of designing a control is perfect for nonlinear control of a mobile manipulator system dynamics model.

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