OPTIMAL CONTROL OF ARTIFICIAL KIDNEY SYSTEM

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ABSTRACT

Optimal Control is a strategy of controlling or optimization of any cost function along with satisfying the given constraints within the limits. Performance indices are controlled or minimized to produce optimal results for a wide range of real world application problems using different techniques of optimal control. A problem of renal dialysis technique to operate on the kidney failure is studied here and solution for finding optimal dialysis rate without causing any trouble to patient and successful transfer of bloodstream into artificial kidney device will be the final results. Optimal control policy of Linear Quadratic Regulator with discrete system will be used to solve this control problem. The rate at which dialyzer should work will be the results obtained from this solution. Mathematical model of LQR is simple to interpret with the objective to minimize the performance index of rate at which flow of blood in dialyzer occurs without any danger or risk to patient during the dialysis process.

Setting the dialysate flow rate according to the optimal control policy will reduce any risks and dangers of overshooting pressure dynamically in the patient’s body and gives the minimum time solution for the treatment. So that dialysis can be performed within the minimum time and without any problems. It makes the patients of kidney failure comfortable to have dialysis performed on their failed kidney and improve their life by purifying blood with removal of urea and waste out in an optimal way.

INTRODUCTION

Optimal Control Theory

Optimal Control is a method of mathematical optimization to find optimal solution for various real life problems, using methods and processes which are derived from the works of Calculus of Variations by Edward J. Mcshane. Many derivations are produced using this to get control policies which can solve a wide range of optimal problem formulation. Optimal Control finds a control law for solving problems and the law has the criteria that solves the problem optimally. It includes a cost function which includes state variables and control variables in the equations of mathematical formulation of the problem. The control is formed by set of differential equations which shows paths of control variables to reach the minimized cost function solution.

Problem formulation is the first and most important task while solving an optimal control because if we formulate the mathematical concept of the system of problem, solving it becomes quite easy and simple. The method to formulate an optimal problem is describing the cost function of the system subject to the constraints that are to be satisfied and the boundary conditions or state conditions which are to be considered for the problem. So, when we have all this information written down in a proper formulation mathematically, we can start producing results in terms of a control law which minimizes the cost function. A general representation of mathematical formulation is obtained as, initially the state variables are arranged in a matrix and formed a state vector of the system. Similarly, all the control variables are combined to form a control vector and then N number of ordinary differential equations are formed using those vectors with N variables. Then we show the cost function or performance index for the objective of the problem that is to be solved. So, we have to minimize the cost performance satisfying the ode’s which are constraints for our system.

PROBLEM FORMULATION

Linear Quadratic Regulator problem

It is one of the many types of optimal control problems which solves for a cost function that is quadratic continuous time equation with the dynamic first order linear constraint equation. So, formulation of a LQR problem can be given as follows,
If the state variables vector is given as x(t) and control vector is u(t) for the time varying linear system, then
Minimize the time integral quadratic cost function which is described by a general equation as

$$J = \frac{1}{2}\int_{t0}^{tf} (x^T(t)Qx(t) + u^T(t)Ru(t))\, dt$$

where Q and R both are the weighting matrices on state and control variables respectively and is subject to the constraint equation of dynamic system which is shown as

$$\dot{x} = A(t)x(t) + B(t)u(t)$$

With the initial state condition of $x(t_0) = x_0$,

Study of the actual problem to be solved

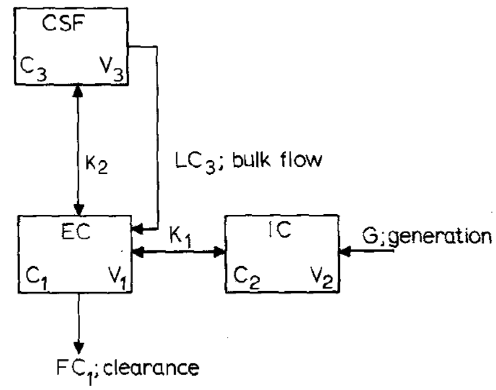
End stage renal disease is a severe disease of complete failure of functioning of kidney which can be caused by the health problems such as Diabetes, High Blood Pressure and some other diseases like Lupus nephritis, which causes damage to the kidney at a slow rate and it reaches the last stage if not cured. This stage called End Stage Renal Disease (ESRD) is highly dangerous and it can result in the patient's death or similar kind of suffering. So, it is required to operate on the disease as soon as possible and at a very faster rate. The only cure or solution for this last stage disease is to operate it with the process of hemodialysis to keep functioning of kidney proper in the body.

[Figure: Artificial kidney system implemented]

Hemodialysis is the technique in which an artificial kidney is used to keep the function of kidney working normally to avoid the risk of damaged kidney not functioning. As we know that, kidney is the organ that removes all the biological waste from the bloodstream and maintains amount of water in body to keep the energy levels properly maintained. Now, hemodialysis uses a device called artificial kidney, which performs tasks same as what a kidney does. It takes out all the blood from the body to transfer it into a dialyzer inside of which the blood is purified by removing all the waste materials from it and separating it from the clean blood which is transferred back in body to bring back the normal condition along with maintaining proper flow of clean and purified blood in body.

As we are using the dialysis for cleaning the urea from the body, we have to work on the process at very faster rate to keep the clean flow of bloodstream maintained in minimum time. But it is usual that when we transfer blood into artificial kidney at very fast rate, it dynamically overshoots which could be highly risky during the treatment. Hence, to obtain perfect rate of dialysis in the artificial kidney technique, we need to find optimal solution for the dialysis rate of the device. Optimal control theory is used to find the dialysis rate for the treatment to get optimum rate at which the patient can be operated without causing any trouble. To obtain the desired optimal rate of dialysis, first it is required to produce mathematical formulation for the problem.

There is a 3 pool model developed by Gormley and Bell in 1970 after testing the urea transfer dynamics in animals which can be adopted directly to be used to obtain the mathematical cost function for urea system dynamics. The model consists of three pools, namely Extracellular fluid pool, intracellular fluid pool and cerebrospinal fluid which are the basic transfer devices in the system. Now, these pools are tested already, so it is feasible to implement and use the results of these pools in our system by using the results in formulating structure of our problem formulation.



[Figure: Three pool model for urea transfer]

Elements of the pool model are EC= Extracellular fluid pool with concentration C_1 that exchanges mass with IC= Intracellular fluid pool with a mass transfer coefficient K_1 and a CSF= Cerebrospinal Fluid pool with coefficient K_2. G is the biological generation term which is where we obtain the fluids pressure in system dynamics. During our dialysis process, clearance term is replaced by the artificial kidney to transfer all the blood fluids in dialyzer to purify it. When chemical and material balance is performed on each of the both sides of elements in this three pool model system, we get these equations using concentration deviation in different pools.

$$V_1 \frac{dC_1}{dt} = K_1(C_2 - C_1) + K_2(C_2 - C_1) + Q_B(C_{Bout} - C_1) + LC_3$$

Also, the effectiveness factor is given as,

$$\varepsilon = \frac{QB(CBin - CBout)}{Qmin(CBin - CDin)}$$

So, using this relation, new equation is formed which is as below,

$$V_1 \frac{dC_1}{dt} = K_1(C_2 - C_1) + K_2(C_2 - C_1) + Q_B\varepsilon(C_{Din} - C_1) + LC_3 \;, \quad ,C_{Din}=0$$

Above equation is used to obtain the equations for perturbation variables that are concentration deviations from the steady state which are named as x_1, x_2 and x_3 here in our equations and u is the control.

For the formulation of problem to solve it using controls, differential equations are derived from the state and control variables and are linearized to obtain formulation of optimal control problem in mathematical representation. So above equation is used to obtain the state equations for dialyzer and in terms of state variables, the differential equations are derived which represents basics of the actual matrix formation.

Actually, for cerebrospinal fluid, there arises a need to consider the pressure of the fluid instead of concentration deviation in cerebrospinal fluid pool. So our variable for cerebrospinal fluid will be y_3 which is the pressure deviation of fluid.

METHODOLOGY

Mathematical model of system of artificial kidney process is developed by taking concentration of the pools as states of system. Solving the Linear Quadratic Regulator problem for this system of equations results into obtaining feedback gain matrix K. This feedback signal is used to design a state feedback controller with control effort 'u' where

$$u = -Kx$$

For external weights of the cost function of LQR problem solution, the values of weights are changed and simulation of the system is performed in Matlab which shows how the overshoot and oscillations in response of control effort is changed. By visual inspection, weight vectors with better responses of signals are selected to implement on system. The values of certain quantities are taken from the data available from an existing artificial kidney patient system by W. Fred Ramirez, David W. Lewis and Michael C. Mickley.

Volume of EC pool, V_1	15 liters
Volume of IC pool, V_2	25 liters
Volume of CSF pool, V_3	0.135 liters
Mass transfer coefficient between EC and IC, K_1	56.7 liters/h
Mass transfer coefficient between EC and CSF,K_2	0.085 liters/h
Bulk flow rate from CSF pool	0.00197 liters/h
Concentration Pressure deviation to CSF pool, P_0	84 mm H_2O liters/g
Generation rate for urea, G	0.312 g/liters
Blood flow rate, Q_B	12 liters/h
Concentration of EC pool during steady state	0.0433 g/liters
Effectiveness factor, ε_{ss}	0.6

[Table: System parameters from existing data available]

After deciding the weights for the system, there is a need to identify initial states. Objective of this problem solving is to bring the system to a terminal state of x=0 from the initial state which is to be defined. So, considering the values of initial states based on requirements of flow rate in the deviation of cerebrospinal fluid pressure pool, simulation is performed. In general case, initial states are taken as [1,1,1] from which system will reach to [0,0,0] at steady state with acceptable level of control and minimizing the quadratic cost function.

The cost function which is to be optimized is

$$I = \frac{1}{2}\int_{t0}^{tf}(x^T(t)Qx(t) + u^T(t)Ru(t))\,dt$$

In the above equation, Q weight penalizes the states while R weight matrix penalizes control during minimizing this performance index. Solving the Riccati equation to obtain feedback state along with Eigen values gives the state feedback using which state is fed back into input of the system by control efforts. This procedure minimizes the cost function and obtain optimal solution for this problem. Two separate solutions are obtained, one using the output weights and another without that to get different controllers for analysis. However, the feedback gain considering output weights gives more accurate and good results compared to solution obtained without using the weights.

Closed Loop Controller Design

In state space approach, controller is designed by substituting Kalman feedback gain and adding back into the input signal, system is controlled to perform at its optimal. State matrix A is replaced by (A-BK) where K is feedback gain to control the system. This way system reaches to its terminal states from the initial states defined using the state feedback control law derived.

1). Using output weights: Here, cost function is minimized considering the constraints as dynamic state equation and output equation both. In this case, dynamic state equation and output constraints are simply the equations,

$$\dot{x} = A(t)x(t) + B(t)u(t)$$
$$y = Mx,$$

Where, matrix A is defined as linearized model of the system dynamics of urea transfer and is

$$A = \begin{matrix} -\frac{K_1}{V_1} - \frac{K_2}{V_1} - \frac{Q_B}{V_1}\epsilon_{ss} & \frac{K_1}{V_1} & \frac{L}{V_1} + \frac{K_2}{V_1} \\ \frac{K_1}{V_2} & -\frac{K_1}{V_2} & 0 \\ \frac{K_2}{V_3} & 0 & -\frac{L}{V_3} + \frac{K_2}{V_3} \end{matrix},$$

B is the matrix with first element as $\frac{Q_B}{V_1}C_{1ss}$ and rest all elements of the matrix as zero, where ϵ_{ss} and C_{1ss} are effectiveness factor or efficiency and concentration of extracellular pool at steady state of the system respectively, and

M is perturbation measurement matrix and it is defined as

$$M = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -P_0 & 0 & P_0 \end{matrix}$$

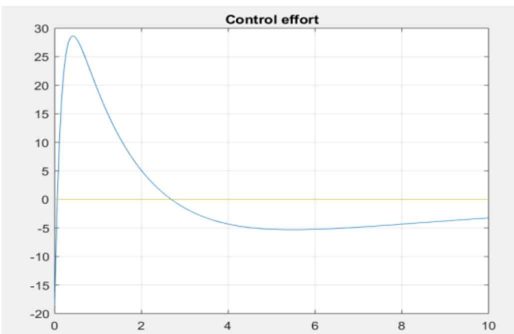
(P_0=84 mm of H_2O liters/g) is the concentration pressure deviation in cerebrospinal fluid pool and hence output equation can also be written as $y_3 = P_0(x_3 - x_1)$ in terms of scalar value.

2). Without using output weights: Only the system dynamics equation is used as constraints, and output equation does not come in the picture while solving for optimal control. So, in state space model, perturbation measurement matrix M is not required.

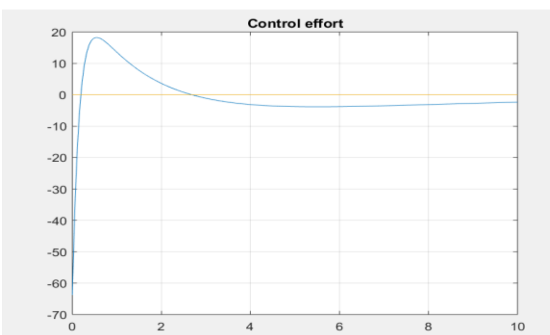
Using both of the above methods, controller is designed and responses are simulated to determine values of concentrations of all the pools at different time instants. Maximum values, overshoots, and settling time of the responses can be measured using simulation results. These numerical results which will be obtained could be used in implementing actual dialyzer while transferring the biological wastes from body into artificial kidney device to avoid dynamic overshoot in pressure deviation. Also, urea transfer rate within minimum time is calculated based on these results and hence, the problem of optimal control is solved.

RESULTS

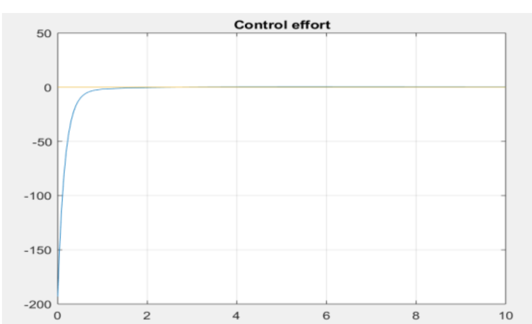
When the weights were changed by increasing from 0 to higher numbers, it is clearly observed and noted that the time response of control changes based on weights. If Q is increased, higher penalty is imposed on states which results in higher maximum peak in response of control signal. This eventually increases settling time so that it takes longer time to reach steady state, this higher jump in signal response is not a good thing for control. Hence it is required to have small value for Q. Then the value decreased to 0.1 from 1 to see control for lower value. Final value of matrix Q is selected as 0.1 by this simulation results.



[Figure: Control signal for Q=25 times]

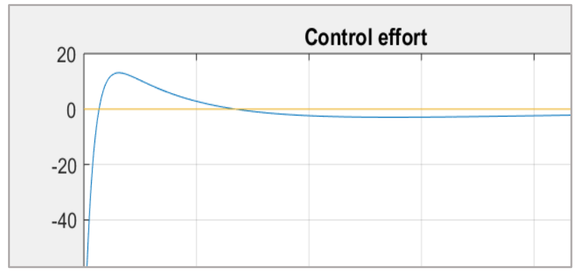


[Figure: Control signal for Q=2 times]

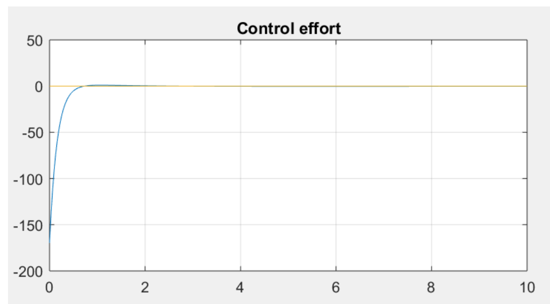


[Figure: Control signal for Q=0.1]

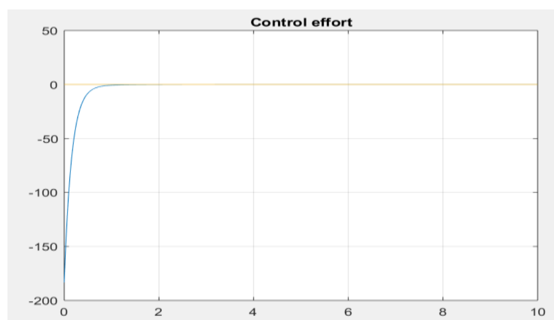
Similarly studying simulations for weight matrix R which penalizes control of the system by changing values from 1 and increasing it to higher values until no change is found in the control responses. Overshoot occurs for lower weight while as value of R is increased, overshoot goes away and settling time of the system is quite reduced and system reaches its steady state sooner comparatively. These are the simulation results by changing matrix value of R weight.



[Figure: Plot of control u w.r.t. time t for R=I]



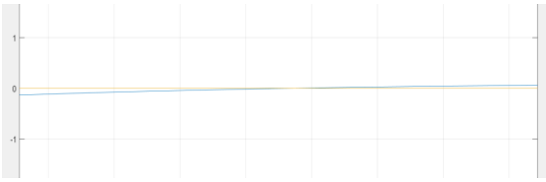
[Figure: Plot of control u w.r.t. time t for R=10*I]



[Figure: Plot of control u w.r.t. time t for R=25*I]

After visual inspection of these results of control signals obtained for different values of weight matrices, the best results were obtained for specific values which are Q=0.1 and R=25 times the identity matrix. If these weight matrices used to solve our LQR problem, it will produce better solutions for controller system needed to be designed. Equation of optimal control is changed based on the penalties decided to have a better control response of dynamics of the artificial kidney system.

There is a slight variations form steady state when high value of R is chosen and this deviates the control from going zero by a very small displacement after a certain time.


[Figure: Slight deviation of control from going to zero]

For this reason, not very high value of R is recommended even though steady state is reached faster.

Solving analytically the LQR problem after deciding outer penalty weights, gave the feedback gain matrix, solution of the Riccati equation and closed loop Eigen values for state feedback controller. So, these values for both open loop control system as well as closed loop controller with a signal fed back into input from the state using gain matrix obtained from open loop solution. Results for open loop state space system are as, feedback gain matrix,

$$K = 179.046 \quad -165.4186 \quad -0.1816$$

solution of Riccati equation,

$$S = 10^5 * \begin{matrix} 8.2177 & -7.5923 & -0.0083 \\ -7.5923 & 7.0144 & 0.0077 \\ -0.0083 & 0.0077 & 0 \end{matrix}$$

and closed loop Eigen values,

$$E = \begin{matrix} -0.6464 \\ -0.1572 \\ -0.1698 \end{matrix}$$

These results are used to feed the state feedback signal back to input of the system to design the closed loop controller. So, state equation is modified by using input signal $u = -Kx$, added in system which makes

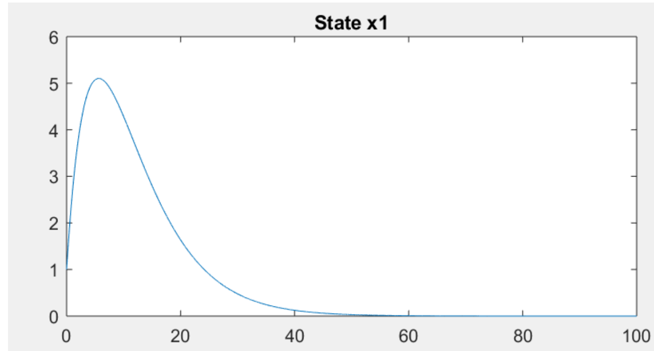
$$\dot{x} = Ax + Bu = Ax + B(-Kx) = (A - BK)x$$

Eigen values of (A-BK) should be such that the stability of system is maintained and its controllability. After modifying the controller, checking for new closed loop Eigen values at this system gives the results such as,

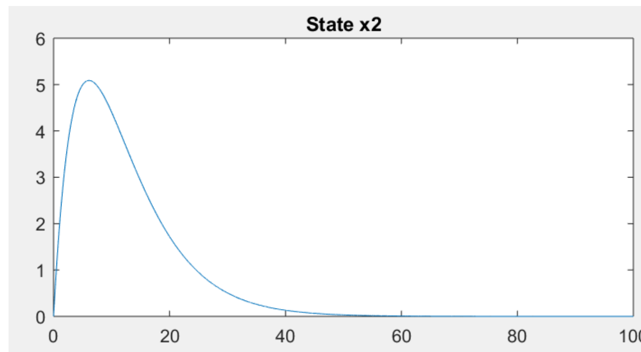
$$E = \begin{matrix} -0.1597 + 0.0244j \\ -0.1597 - 0.0244j \\ -0.6252 \end{matrix}$$

So, two complex conjugate poles and one real Eigen value pole, each of these are negative. Means the system is stable and controllable, acceptable level of control can be applied to this system to bring it to steady state after a fixed duration of time for successful performance of optimal control.

CLOSED LOOP SYSTEM SIMULATION RESULTS

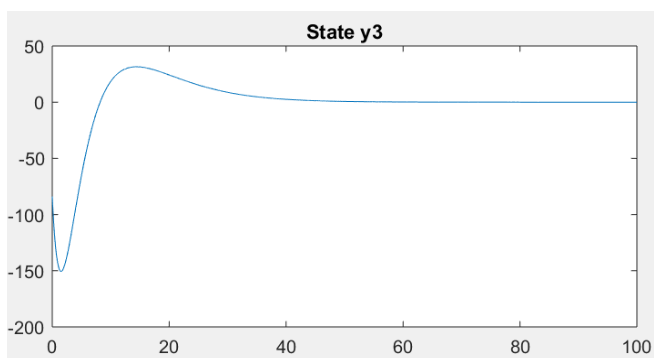


[Figure: response of concentration deviation of extracellular state]



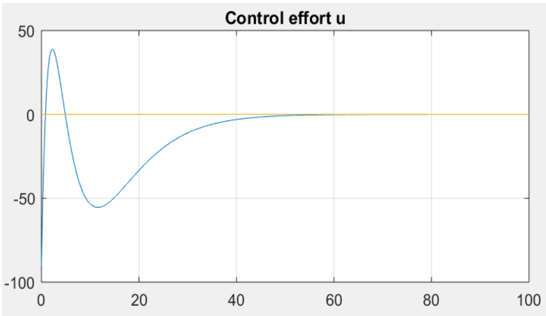
[Figure: response of concentration deviation of intracellular state]

It is clearly visible that concentration deviation in both the intracellular and extracellular pools is similar, if initial values are given as 1 for x1(extracellular state) and 0 for x2(intracellular state). Both reaches at zero at 50 seconds simulation and becomes steady at that time.



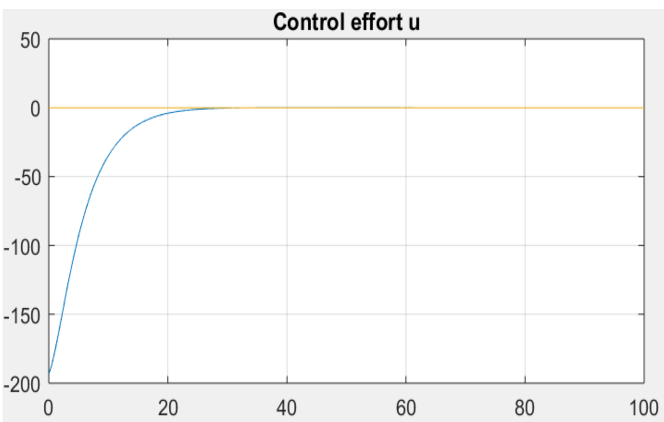
[Figure: cerebrospinal fluid pressure deviation]

Initial value of this state y_3 is calculated based on the mathematical formulation of states as $y_3 = P_0(x_3 - x_1)$ and as initial values of x_3 and x_1 are provided as 0 and 1 respectively. Also P_0 is the constant multiplied to concentration difference to CSF pressure and its value is 84mm H_2O liters/g. Hence initial value for y_3 is -84 and it also goes to zero at steady state because of control effect. There is a large overshoot or oscillation in the system observed if penalty over control is not as desired which is not good for operation.



[Figure: Lower penalty on control results]

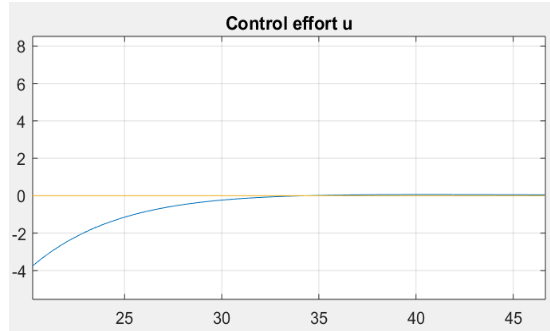
Thus, as observed to reduce or totally avoid any such oscillations in the system, higher LQR weight on control must be imposed to get smooth flow of biological fluids in the process without any risks. Finally, introducing controller designed earlier considering stability of the system, and obtaining optimal performance of system by using the weights and feedback gain values calculated before.



[Figure: Control u for controller system]

The control response of our system goes from starting value which is calculated based on state feedback gain value as input is zero at that time. It settles down without any overshoot and goes to zero value at steady state. This controller will control the concentration deviation overshoots and make the system stable at a certain level such that urea transfer process is achieved without any difficulties in human body while treatment. Hence system is controlled using optimal control without any pressure overshoot in dynamics during concentration transferring of pools in the model.

Thus, it is concluded that system is controlled using this optimal control technique using linear quadratic regulator approach and artificial kidney patient system treatment is improved and made risk free for any patient.



[Figure: Smooth settling without any pressure overshoot]

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