Latvia University Contest 2

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Hello Muscat Programming Bootcamp 2019

A. Thomas the Tank Engine

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The graph is a *cactus*: every vertex is located on at most one simple cycle.

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Then, run any graph search algorithm (BFS or DFS) from the locomotive vertex (mark all vertices of the train as used, so you don't visit them during BFS).

If any cycle of length $\geqslant k$ is reachable, then every vertex of the graph is reachable.

Otherwise, output all vertices visited by BFS. Complexity of this solution is $O(n + m)$.

B. Broken line

Given N points on the perimeter of a convex polygon. Find a shortest polygonal chain through all these vertices.

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How to calculate $f_{i,j}$: either make a segment from point i to point $i + 1$, then take $f_{i+1,j}$, or make a segment from i to j, then take $f_{j,i+1}$.

All these calculations take $O(n^2)$ time.

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Fix any point (say, the n-th one). It can be the end vertex of a chain, or an interior one.

If it's an endvertex, check two neighbors. If it is an interior vertex, check all possible other points to be connected to the n-th one, and take the minimum.

C. Columns

Given an $n \times m$ grid, find the number of divisions into *oriented columns*.

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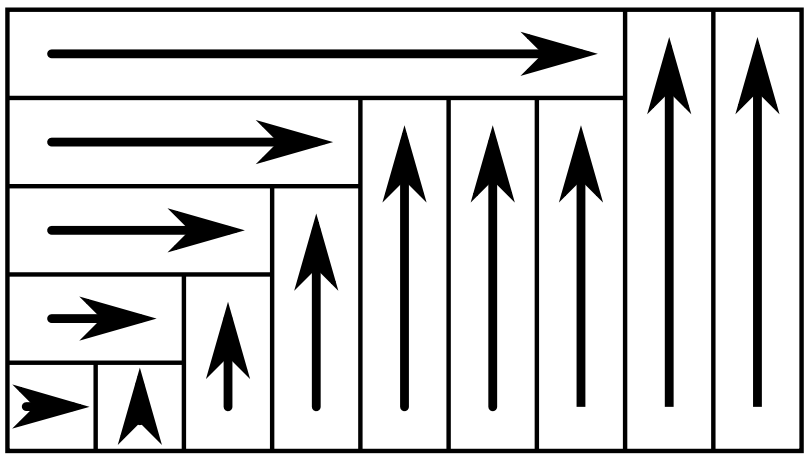
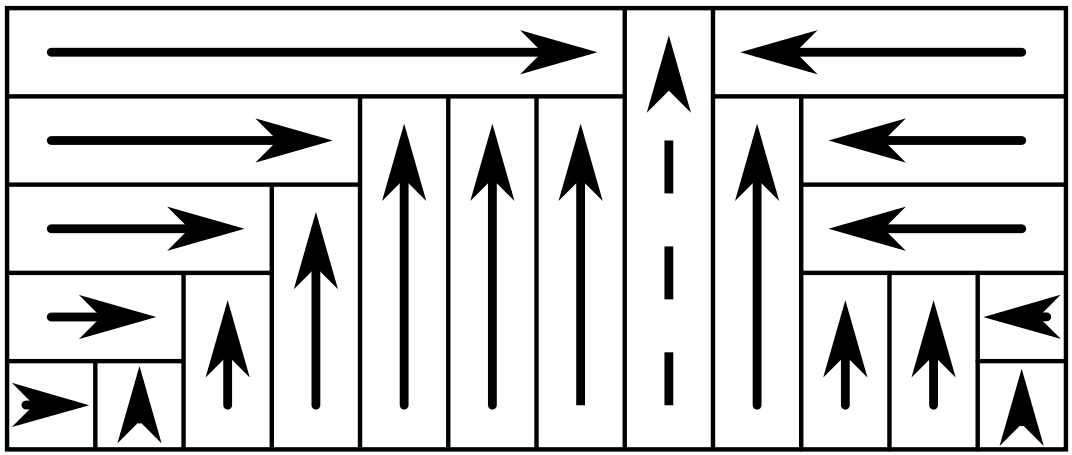
First auxiliary pattern ("2-diff"): only two different directions are allowed (e.g. "right" and "up").



The number of such patterns is C_{w+h}^{h}.

C. Columns

Second auxiliary pattern ("3-diff"): three different directions allowed (e.g. "right", "left" and "up"). At least one "up" tile must touch the top of the rectangle (dashed in the picture).

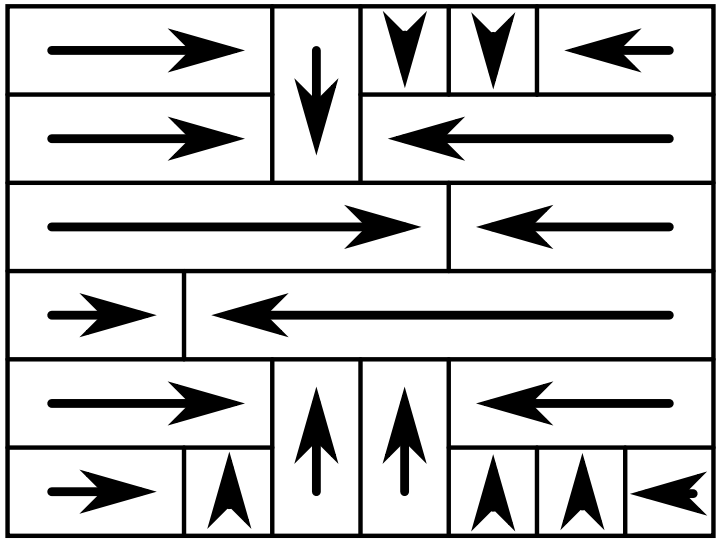


To calculate the number of such patterns, fix the position of the leftmost "up" column touching the top. Now the rectangles to the right and to the left contain exactly "2-diff" patterns.

Computation is done in $O(w)$.

C. Columns

Patterns with horizontal separator: one or several lines containing only horizontal columns.

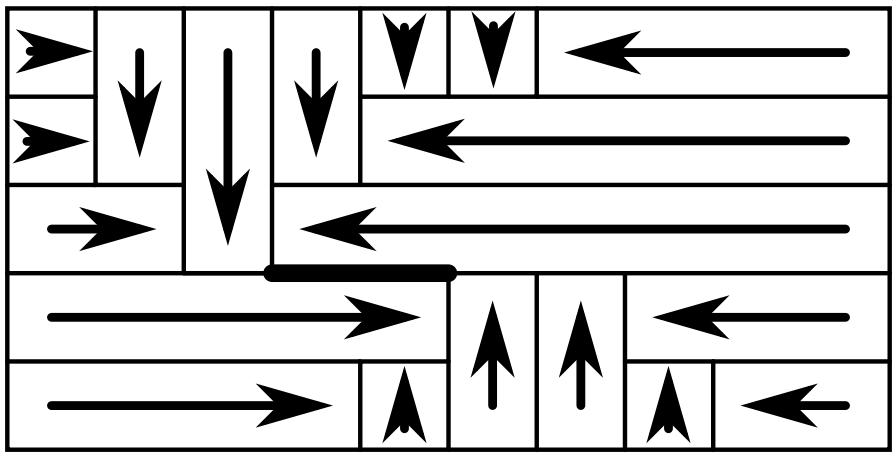


To calculate the number of ways, fix the first and the last separator line. Let there be s of them. Patterns to the top and to the bottom are "3-diff", and the number of ways to fill separator lines is $(w+1)^s$.

Computation is done in $O(h^2)$

C. Columns

General case: neither vertical nor horizontal separators exist.



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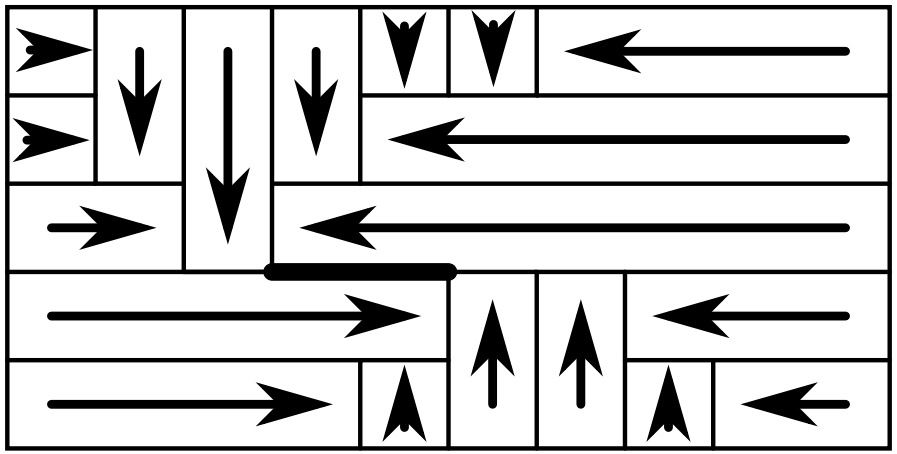
General case: neither vertical nor horizontal separators exist.

Three cases are possible.

- Two horizontal opposite-facing columns touch (depicted in the previous slide)
- Two vertical opposite-facing columns touch
- None of the above happens

We calculate the first two cases allowing the touch length to be zero and then subtract the result for the third case because we double-counted it.

C. Columns



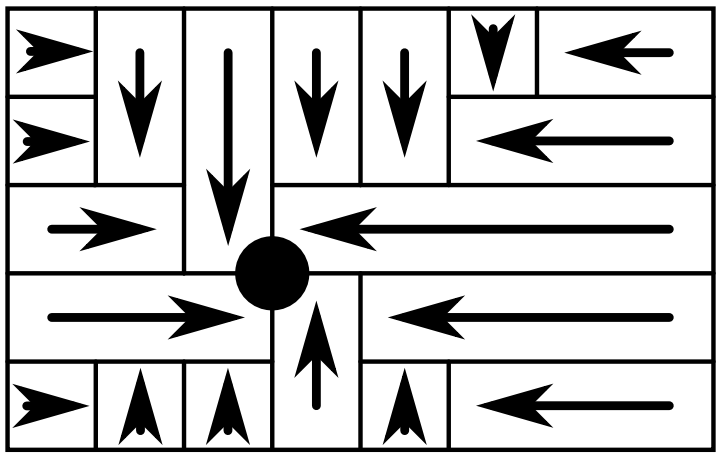
We always consider the pattern look like in the picture: two horizontal columns touch and the one facing left is at the top.

We fix the height and iterate through the position of the first "full" vertical column in the bottom part. Now the left and the right parts of the bottom parts resemble "2-diff" pattern with fixed dimensions. We use prefix sums to count all possibilities for the top part, which are also look as combinations of two "2-diff"-s.

Complexity: $O(wh)$.

C. Columns

General case where the length of the touch is zero.



We iterate through the position of the center and use "2-diff" pattern to calculate the number of ways in each of the four corners.

Complexity: $O(wh)$.

C. Columns

Outline:

- Add cases where either horizontal or vertical separator line(s) exist
- Add cases where no separator exist and two opposite-facing lines touch by a (possibly empty) segment
- Subtract cases where lines of four different directions meet at the same point

E. Zebra

Given a polygon, and a zebra-like coloring of a plane, find the white area inside the polygon.

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We will select trapezoids that have bases parallel to zebra lines. Areas of intersections of whites line with this trapezoid form an arithmetic progression, sum of which can be easily found.

We consider n trapezoids, each in $O(1)$ time.

F. Fractal NV

Traverse a colored fractal polyline efficiently.

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To increase level, increase level of all superedges with maximum level in your current path.

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You can prove that after the i-th stage the first element is equal to XOR of all elements j where j is a submask of i.

$$result_i = \bigoplus_{j \subset i} input_j$$

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It is possible to calculate this array in $O(n \log n)$ time using an algorithm from Fast Subset Convolution.

Might require some optimizations, since time limit is really tight.

Towers of Hanoi

Find the minimum number of ways to solve the Hanoi Towers puzzle from the given starting configuration.

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If the current disk is on the target rod, skip it. Otherwise, you have to move everything to the third rod, then move the current disk to target rod, then move all others into the target rod.

This takes $2^x + y$ operations, where x is the number of current disk, and y is the cost of moving all disks to a new target rod.

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In other words, just add 2^x each time the current disk is not on target rod, then switch targets. $O(n)$ time.

I. Interesting sequence

Find p-th occurrence of n in the given sequence.

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By the way, a_i is the bit count of standard Gray code:
$a_i = \text{bitcount}(i \oplus \lfloor \frac{i}{2} \rfloor)$

J. Transformation on numbers

Find the number of operations of adding 10^y and changing a digit to transform one given integer into another.

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When you process all digits, the answer for a state with no carry is the final result.

K. Kingdom division

Split a tree into three connected components A, B and C such that $|A| = |B|$. Maximize $|A|$.

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If u is not a root, then everything above it is either A, or $A \cup B$. In the first case you have to check if u has a subtree of size $n - size_u$, in the second case you have to check whether the $A \cup B$ part can be split into two equal subtrees.

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If u is a root, you need to find two subtrees of equal sizes. This all can be done, if you store a multiset of all subtree sizes for every vertex and merge them small-to-large.