Terminal Falling Velocity of a Sand Grain

1)Important Info's: -

➔*The first stop for polluted water entering a water work is normally a large tank, where large particles are left to settle. More generally, gravity settling is an economical method of separating particles.*

➔*If the fluid in the tank is moving at a controlled low velocity, the particles can be sorted in separate containers according to the time it takes for them to reach the bottom.*

➔*This application simulates a spherical sand grain falling in water. The grain accelerates from standstill and rapidly reaches its terminal velocity.*

2)Physics and Equations: -

➔*The fluid flow is described by the Navier–Stokes equations*

$$\rho\frac{\partial \mathbf{u}}{\partial t} - \nabla \cdot \eta(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p = \mathbf{F}$$

$$\nabla \cdot \mathbf{u} = 0$$

- *ρ denotes the density (kg/m^3),*
- *u the velocity (m/s),*
- *η the viscosity (Ns/m^2),*
- *p the pressure (Pa).*
- *The fluid is water with a viscosity* ***of1.51·10^{-3} Ns/m2 and density of 1000 kg/m^3.***

➔*This means that the volume force density F is given by:*

$$F_r = 0, \qquad F_z = -\rho\,(a + g)$$

- ***a (m/s^2)*** *is the acceleration of the grain*
- ***g = 9.81 m/s^2*** *is the acceleration due to gravity.*

➔*The ODE that describes the force balance is:*

$$m\ddot{x} = F_g + F_z$$

- *m (kg) denotes the mass of the particle, x (m) the position of the particle,*
- *F_g (N)the gravitational force,*
- *F_z the z-component of the force that the water exerts on the sand grain*

$$\mathbf{F_g = -\rho_{grain} V_{grain}\, g}$$

- ***V_{grain} (m^3)** is the volume of the sand grain*
- ***ρ_{grain} (kg/m^3)** its density.*

$$F_z = 2\pi \int_S r\mathbf{n} \cdot [-pI + \eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)]dS$$

- *r (m) is the radial coordinate*
- *n is the normal vector on the surface of the grain.*

➔*The initial values for position and velocities are **$u_0=v_0=x_0=\dot{x}_0=0$;***

3)Boundary Conditions: -

➔*At the sphere's surface, the fluid velocity relative the sphere is zero, that is* ***u = 0*** *—a situation described by the no slip wall condition.*

➔*At the inlet of the fluid domain the velocity equals the falling velocity**: u = (0, x·).***

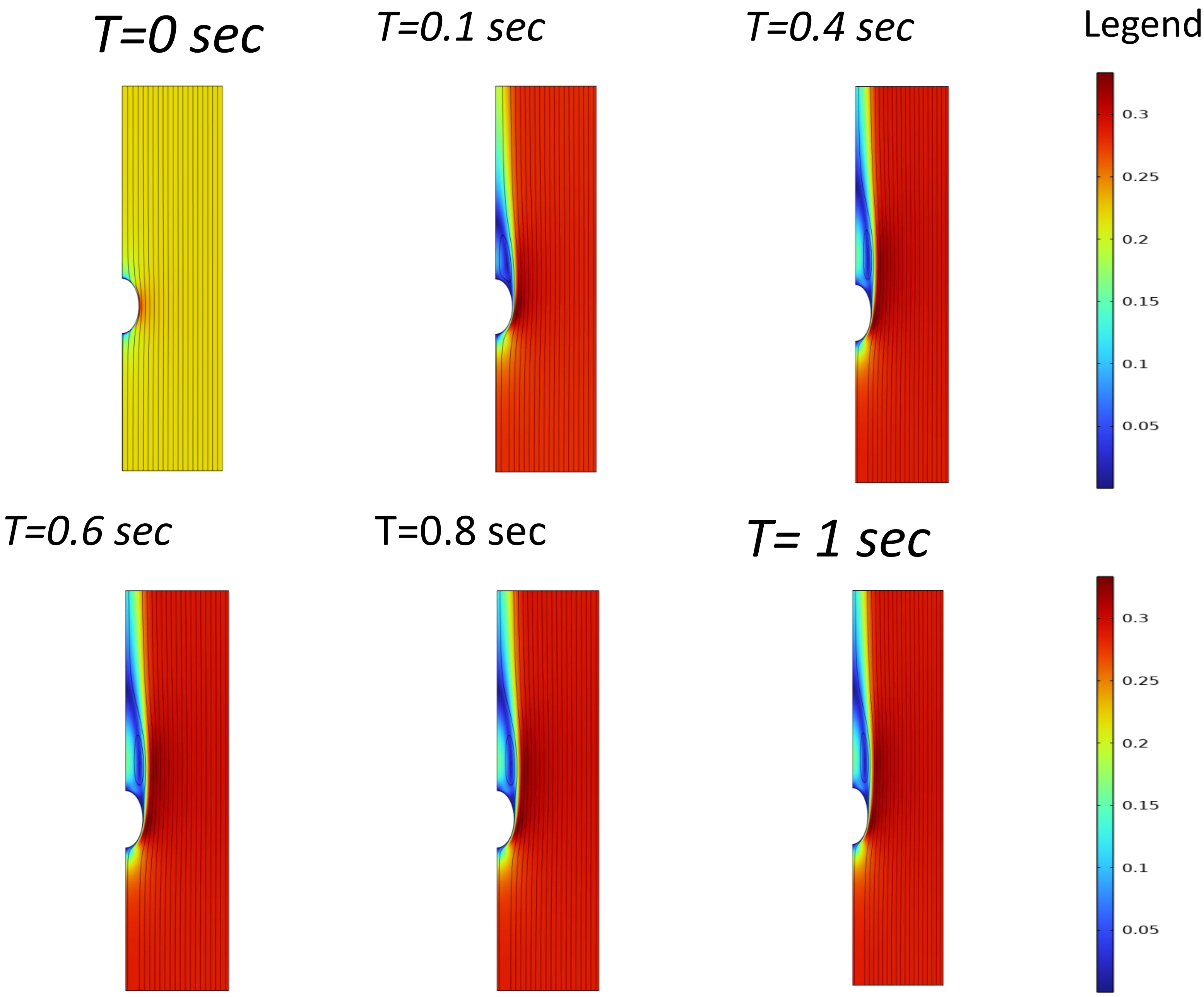
➔ *At the outer boundary of the water domain, the normal velocity and the tangential shear stress both vanish, which means that a symmetry condition applies.*

➔*Furthermore, a neutral condition,*

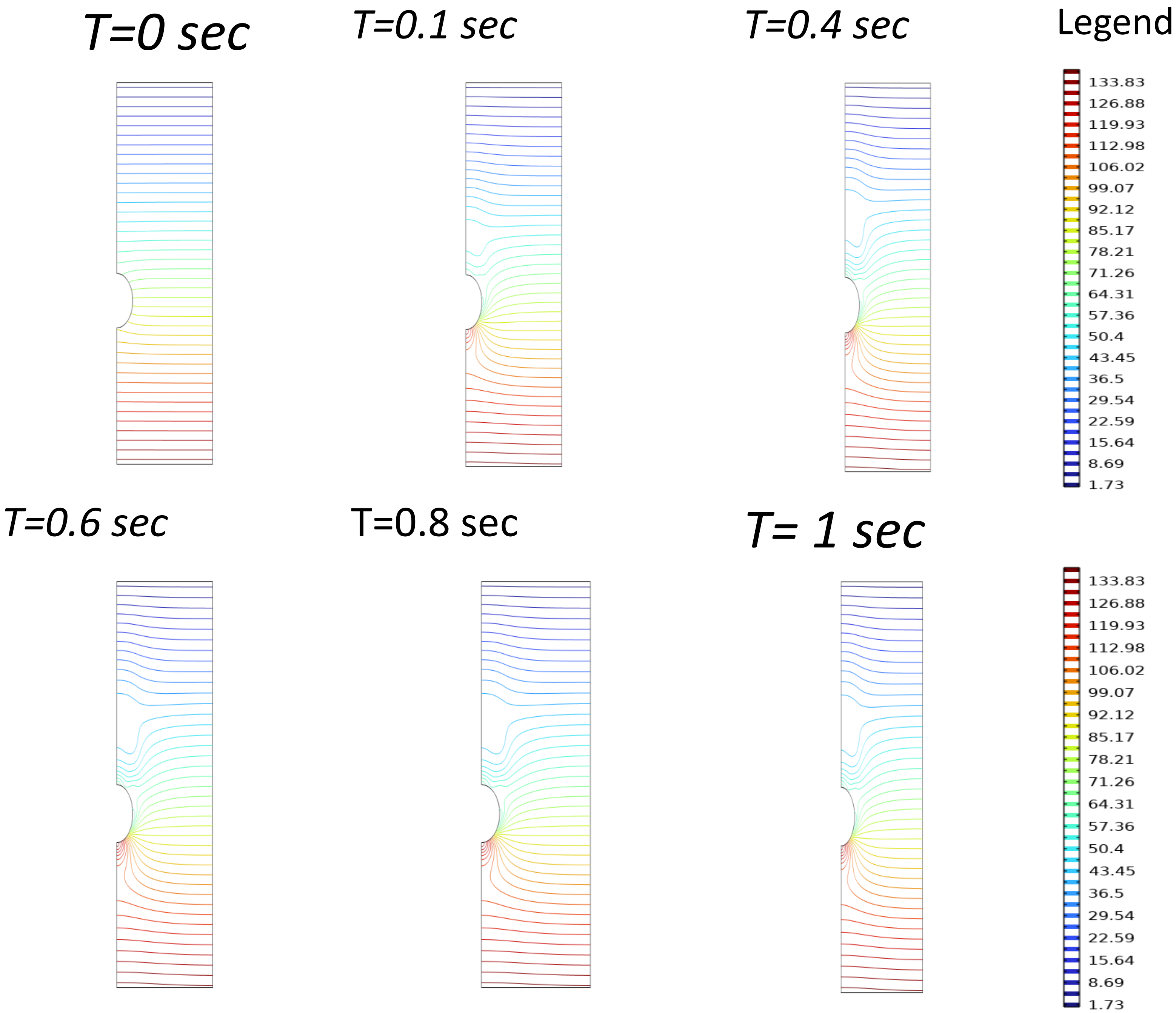
n · [−pI + η (∇u +(∇u) T)] = 0*, describes the outlet, and an axial symmetry condition models the symmetry axis at r = 0.*

3)Results: -

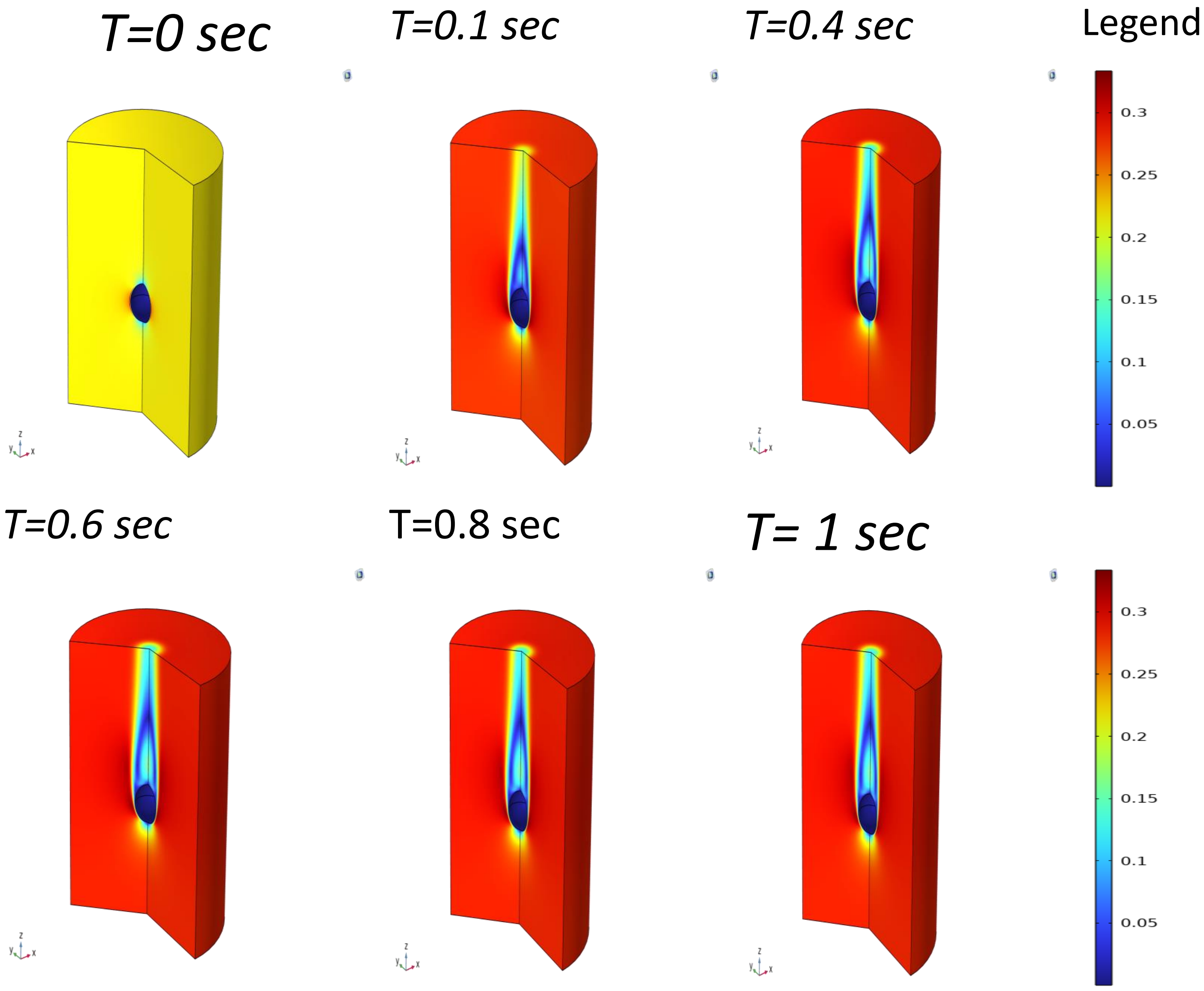
➔Velocity (Surface & Streamline) Plot:



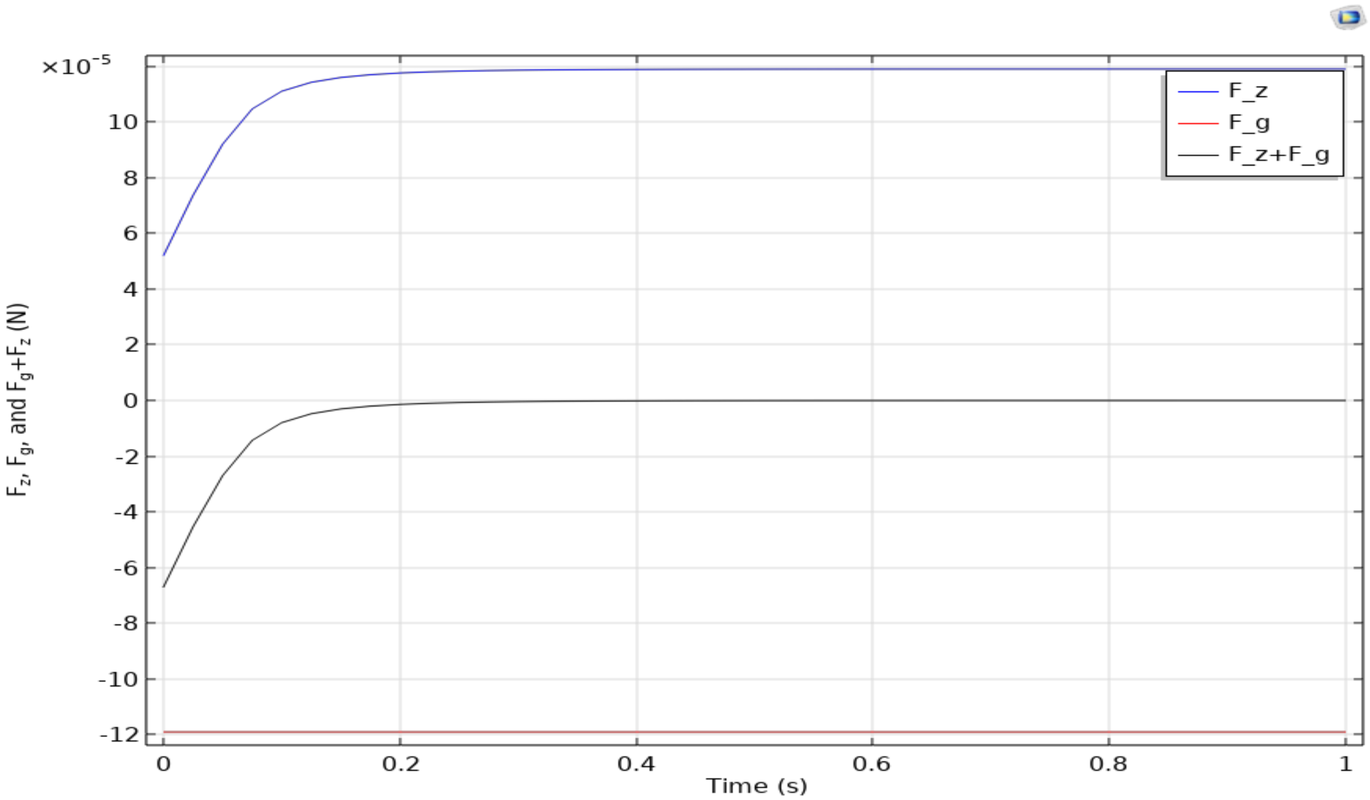
➔Pressure plots: -



➔Velocity 3D Plots: -



➔ F_Z, F_g and F_Z+F_g vs Time graph: -



➔ Grain Speed vs Time graph: -