Swapnil Darini (V00861672) Due: 30/11/18

CSC 349A: Assignment 6

Q1 a) $P(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2)$

$$L_0(x) = \frac{(x-2h)(x-3h)}{(-2h)(-3h)} = \frac{x^2-5hx+6h^2}{6h^2}$$

$$L_1(x) = \frac{x(x-3h)}{2h(2h-3h)} = \frac{x^2-3hx}{-2h^2}$$

$$L_2(x) = \frac{x(x-2h)}{3h(3h-2h)} = \frac{x^2-2hx}{3h^2}$$

$$P(x) = \left(\frac{x^2-5hx+6h^2}{6h^2}\right)f(0) - \left(\frac{x^2-3hx}{2h^2}\right)f(2h) + \left(\frac{x^2-2hx}{3h^2}\right)f(3h)$$

(b) $I = \int_0^{3h} f(x)\,dx \approx \int_0^{3h} P(x)\,dx$

$$\therefore I \approx \int_0^{3h} \left(\frac{x^2-5hx+6h^2}{6h^2}\right)f(0) - \left(\frac{x^2-3hx}{2h^2}\right)f(2h) + \left(\frac{x^2-2hx}{3h^2}\right)f(3h)$$

$$I \approx \frac{1}{h^2}\left[\frac{f(0)}{6}\left[\frac{x^3}{3} - \frac{5hx^2}{2} + 6h^2x\right]_0^{3h} - \frac{f(2h)}{2}\left[\frac{x^3}{3} - \frac{3hx^2}{2}\right]_0^{3h} + \frac{f(3h)}{3}\left[\frac{x^3}{3} - hx^2\right]_0^{3h}\right]$$

$$I \approx \frac{1}{h^2}\left[\frac{f(0)}{6}\left(9h^3 - \frac{45h^3}{2} + 18h^3\right) - \frac{f(2h)}{2}\left(9h^3 - \frac{27h^3}{2}\right) + \frac{f(3h)}{3}\left(9h^3 - 9h^3\right)\right]$$

$$\approx \frac{1}{h^2}\left[\frac{3f(0)h^3}{4} + \frac{9h^3f(2h)}{4}\right]$$

$$I \approx \frac{h}{4}\left(3f(0) + 9f(2h)\right)$$

c) $\int_0^{0.36} f(x)\,dx$ given that

x	$f(x)$
0	0.5
0.24	0.50727
0.36	0.51656

From b), we know that

$I \approx \frac{h}{4}\left(3f(0) + 9f(2h)\right)$

In this case, $h = 0.12$, $f(0) = 0.5$, $f(2h) = 0.50727$

Therefore:

$$I \approx \frac{0.12}{4}\left(3(0.5) + 9(0.50727)\right)$$

$$\underline{I \approx 0.1819629}$$

Compared to the actual value, $\int_0^{0.36} f(x)dx = 0.1819695..$, the relative error is $\underline{0.0036\%}$

Q2 $\quad I \approx \frac{5}{9}f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}f(0) + \frac{5}{9}f\left(\sqrt{\frac{3}{5}}\right)$

Thus, we need to test $f(x) = 1, x^1, x^2 \dots x^d$ until the quadrature formula no longer matches the exact result:

$f(x)$	$\int_{-1}^{1} f(x)dx$	$\frac{5}{9}f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}f(0) + \frac{5}{9}f\left(\sqrt{\frac{3}{5}}\right)$	
1	$1 - -1 = 2$	$\frac{5}{9} + \frac{8}{9} + \frac{5}{9} = \frac{18}{9} = 2$	✓
x	$\left[\frac{x^2}{2}\right]_{-1}^{1} = 0$	$\frac{5}{9}\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}(0) + \frac{5}{9}\left(\sqrt{\frac{3}{5}}\right) = 0$	✓
x^2	$\left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{2}{3}$	$\frac{5}{9}\left(\frac{3}{5}\right) + \frac{8}{9}(0) + \frac{5}{9}\left(\frac{3}{5}\right) = \frac{2}{3}$	✓
x^3	$\left[\frac{x^4}{4}\right]_{-1}^{1} = 0$	$\frac{5}{9}\left(-\left(\frac{3}{5}\right)^{3/2}\right) + \frac{8}{9}(0) + \frac{5}{9}\left(\frac{3}{5}\right)^{3/2} = 0$	✓
x^4	$\left[\frac{x^5}{5}\right]_{-1}^{1} = \frac{2}{5}$	$\frac{5}{9}\left(\left(\frac{3}{5}\right)^2\right) + \frac{8}{9}(0) + \frac{5}{9}\left(\left(\frac{3}{5}\right)^2\right) = \frac{2}{5}$	✓
x^5	$\left[\frac{x^6}{6}\right]_{-1}^{1} = 0$	$\frac{5}{9}\left(-\left(\frac{3}{5}\right)^{5/2}\right) + \frac{8}{9}(0) + \frac{5}{9}\left(\left(\frac{3}{5}\right)^{5/2}\right) = 0$	✓
x^6	$\left[\frac{x^7}{7}\right]_{-1}^{1} = \frac{2}{7}$	$\frac{5}{9}\left(\left(\frac{3}{5}\right)^3\right) + \frac{8}{9}(0) + \frac{5}{9}\left(\left(\frac{3}{5}\right)^3\right) = \frac{6}{25}$	X

<u>Thus the degree of precision is 5</u>

(b) $\int_{-1}^{1} e^{-x}\sqrt{x+2}\,dx$

$f(x) = e^{-x}\sqrt{x+2}$

Thus $\int_{-1}^{1} f(x)\,dx \approx \frac{5}{9}\left(e^{\sqrt{\frac{3}{5}}}\sqrt{-\sqrt{\frac{3}{5}}+2}\right) + \frac{8}{9}\left(e^{0}\sqrt{0+2}\right) + \frac{5}{9}\left(e^{-\sqrt{\frac{3}{5}}}\sqrt{\sqrt{\frac{3}{5}}+2}\right)$

$\approx \underline{\underline{3.01793494}}$ (calculated on matlab)

Q3 See attached Matlab published code.

Swapnil Daxini (V00861672) CSC349A Assignment 6

Contents

Question 3

Part A

```
type trap.m
```

```
function trap(a, b, maxiter, tol, f)
m = 1;
x = linspace(a, b, m+1);
y = f(x);
approx = trapz(x, y);
disp('      m     integral approximation');
fprintf('  %5.0f %16.10f \n ', m, approx);

for i = 1 : maxiter
    m = 2^i;
    oldapprox = approx;
    x = linspace ( a, b, m+1) ;
    y = f(x);
    approx = trapz(x, y);

    fprintf(' %5.0f %16.10f \n ', m, approx);
    if abs(approx - oldapprox) < tol
        return
    end
end
fprintf('Did not converge in %g iterations', maxiter)
```

Part B

```
type f1.m

% Solve first integral
trap(0.1, 3, 20, 10^-6, @f1)

type f2.m

% Solve second integral
trap(0, 1, 20, 10^-10, @f2)
```

```
function y = f1(x)

    y = sin(1./x);

end
```

```
       m     integral approximation
       1    -0.3143983004
       2     0.7147254605
       4     1.3447434609
       8     1.5589483255
      16     1.4776583126
      32     1.4679626280
      64     1.5197926883
     128     1.5355585774
     256     1.5386514853
     512     1.5393496800
    1024     1.5395196356
    2048     1.5395618423
    4096     1.5395723764
    8192     1.5395750089
   16384     1.5395756669
```

```
function y = f2(x)

    y = exp(3.*x)./sqrt(x.^3+1);

end
```

```
       m     integral approximation
       1     7.6013096811
       2     5.9133433291
       4     5.4710046573
       8     5.3585418274
      16     5.3303053079
      32     5.3232385483
      64     5.3214713803
     128     5.3210295584
     256     5.3209191010
     512     5.3208914866
    1024     5.3208845830
    2048     5.3208828571
    4096     5.3208824256
    8192     5.3208823177
   16384     5.3208822908
   32768     5.3208822840
   65536     5.3208822823
   131072     5.3208822819
   262144     5.3208822818
   524288     5.3208822818
```

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