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Problem Set 3: Written Part

Q1) Listing vertices of a negative cycle:

We know that the Bellman-Ford algorithm detects ~~wther~~ whether there is a negative cycle but does explicitly name it.

Once the Bellman-Ford is performed (i.e. all edges have been relaxed), if can still lower the distance to a node, they a negative cycle is preset. Ind order to find the lists of vertices in the negative cycle, we look at the predecessors of edge that lower the weight. The list of vertices in the cycle will be all the predecessors that we trace back to until we reach a node already present in the list (i.e. detect a cycle).

We know this algorithm find a negative cycle because ~~part~~ if total distance between two edges can be lowered after relaxation, there has tobe a negative weighted edge.

Q2) The part of the algorithm that breaks down is the assuption that when you ~~relax~~ find a relaxed path, this path is minimal thus adding a number to this will keep the path minimal. ~~th~~ This assumption only holds for positive numbers and not negative numbers.

3) Eulerian ~~cir~~circuits

Prove that every even graph decomposes into cycles:

Suppose G is even.

Proof by induction:

Base case: $|E(G)| = 0$

- In this case, all vertices are isolated, thus they are decomposed into cycles.

IH: Suppose that $\forall$ even graph for $|E(G)| < m$, there exists a cycle decomposition.

IS: Consider a graph G with $|E(G)| = m$.

If we only ~~condiso~~ consider a subgraph of G, H such that H ~~const~~ ~~containts~~ contains all the vertices of degree non-zero. Therefore all the vertices of H have degree greater than 2.

Given that all the vertices have degree ~~at~~ atleast 2, there must exist ~~in~~ a cycle ~~in H~~. C, in H.

If we take a subgraph F of H, such that we remove the edges present in the cycle C.

As F has $|E(F)| < m$, we know that it contains a cycle decomposition C' by the inductive hypothesis.

Therefore G has a cycle decomposition ~~fo~~ of $C' \cup C$.

Thus we have proved every even graph decomposes into cycles.

Q4) Graph coloring

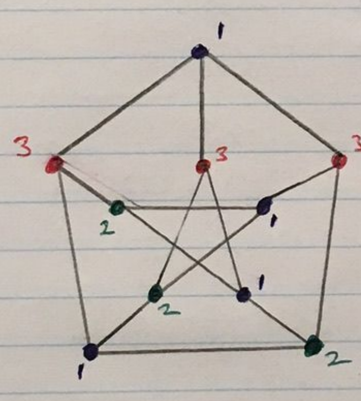
a) A complete bipartite graph is 2-colourable

Given that there 5 independent sets, the chromatic number is 3.

(Not sure about this question)

b) The graph is not 2-colorable as there are many vertices which are connected and at the same depth (in a DFS tree).

The smallest posibble k is 3 :



Let :

Colour 1 = Blue

Colour 2 = Green

Colour 3 = Red