Swapnil Daxini (V00861672) Due: Oct 15, 2018

CSC 349A: Assignment 3

Question #1

$$f(x) = \frac{1 + \cos x}{(x - \pi)^2}$$

a) $b = 10,\ k = 4$, Find $fl(f(3.154))$

$fl(\cos x) = -0.9999$

$fl(1 + \cos x) = 0.0001$

$fl(x - \pi) = 0.0120$

$fl((x-\pi)^2) = 0.000144 = 0.144 \times 10^{-3}$

$$fl\left(\frac{1 + \cos x}{(x - \pi)^2}\right) = 0.6944$$

(b) $$f(x) = \sum \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$\cos(x)$ to 4^{th} order about $a = \pi$:

$$\cos(x) = \cos(\pi) + \underbrace{(-\sin(\pi))(x-\pi)}_{0} + \frac{(-\cos(\pi))}{2!}(x-\pi)^2 + \underbrace{\frac{\sin(\pi)}{3!}(x-\pi)^3}_{0} + \frac{\cos(\pi)}{4!}(x-\pi)^4$$

$$\cos(x) = -1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}$$

(c) $$f(x) = \frac{1}{(x-\pi)^2}\left(\cancel{1} + \left(\cancel{-1} + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}\right)\right)$$

$$f(x) = \frac{1}{2} - \frac{(x-\pi)^2}{24}$$

(d) In order for the process of computing $f(3.154)$ to be well-conditioned, we need show that $f(3.154 + \varepsilon)$ is approximately equal to exact value given that $\left|\frac{\varepsilon}{3.154}\right|$ is small.

$$f(x) = \frac{1}{2} - \frac{(x-\pi)^2}{24}$$

$$f(3.154+\varepsilon) = \frac{1}{2} - \frac{(3.154-\pi+\varepsilon)^2}{24}$$

$$= \frac{1}{2} - \frac{(0.0124076(\ldots)+\varepsilon)^2}{24}$$

$$= \frac{1}{2} - \frac{0.15394\times10^{-3}}{24} - \frac{0.024814(\ldots)\varepsilon}{24} - \frac{\varepsilon^2}{24}$$

$$f(3.154+\varepsilon) = 0.499993585 - 0.001034\varepsilon - \frac{\varepsilon^2}{24}$$

For small $\left|\frac{\varepsilon}{3.154}\right|$, this number is approximately equal to 0.49999, which is the exact value of $f(3.154)$.

e) If the computation of $fl(f(3.154))$ is stable, then there exists a small value of ε, such that $f(3.154+\varepsilon) \approx 0.6944$ for which $\left|\frac{\varepsilon}{3.154}\right|$ is small. Otherwise, it is unstable.

$$f(3.154+\varepsilon) = 0.49999 - 0.001034\varepsilon - \frac{\varepsilon^2}{24}. \quad (1)$$

Clearly, for all small values of ε, this is approximately equal to 0.49999, thus this computation is unstable. Furthermore, given that $fl(f(3.154)) = 0.6944$, which is greater than 0.499999, ε would have to be complex to satisfy equation (1). Thus, no such real value of ε exists.

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Question #2

a) Copy of m-file attached to pdf.

b) $Q = 20 \, m^3/s$, $g = 9.81 \, m/s^2$

$$0 = 1 - \frac{Q^2}{g A_c^3} B \quad (1)$$

$$B = 3 + y \quad (2)$$

$$A_c = 3y + \frac{y^2}{2} \quad (3)$$

We can maniplulate (1) :

$$1 = \frac{Q^2}{g A_c^3} B$$

$$g A_c^3 = Q^2 B$$

We can substitute A_c and B:

$$g\left(3y + \frac{y^2}{2}\right)^3 = Q^2(3+y)$$

$$\underline{\underline{g\left(3y + \frac{y^2}{2}\right)^3 - Q^2(3+y) = 0}}$$

Thus we can find the roots of this equation to get the critical depth.

c) See attached Matlab published code

Swapnil Daxini (V00861672) Assignment 2 Question 2

Contents

Part a) Bisect M-file

```
type Bisect.m
```

```
function root = Bisect ( xl , xu , eps , imax, f, enablePlot )

    i = 1;

    f_l = f(xl);
    f_u = f(xu);

    if enablePlot == 1
        hold on;
    end

    fprintf ('iteration approximation \n')

    while( i <= imax)
        xr = (xl+xu)/2;

        f_r = f(xr);

        plotCondition = [1 2 4 6];

        y = ismember(i, plotCondition);

        if (enablePlot == 1 && y)
            x = [xl: 0.01: xu];
            z = [xl xr xu];
            fz = f(z);

            plot(x, f(x), z, fz, '*g');
        end

        fprintf (' %6.0f %18.8f \n', i, xr );

        if (f_r == 0 || ((xu-xl)/abs(xu+xl))< eps)
            root = xr;
            return
        end
        i = i+1;

        if (f_l*f_r < 0)
            xu = xr;
        else
            xl = xr;
            f_l = f_r;
```

```
        end
    end

    if enablePlot == 1
        hold off
    end

    fprintf ('failed to converge in %g iterations\n', imax)
end
```

Part c)

your_function M-file

```
type your_function.m
```

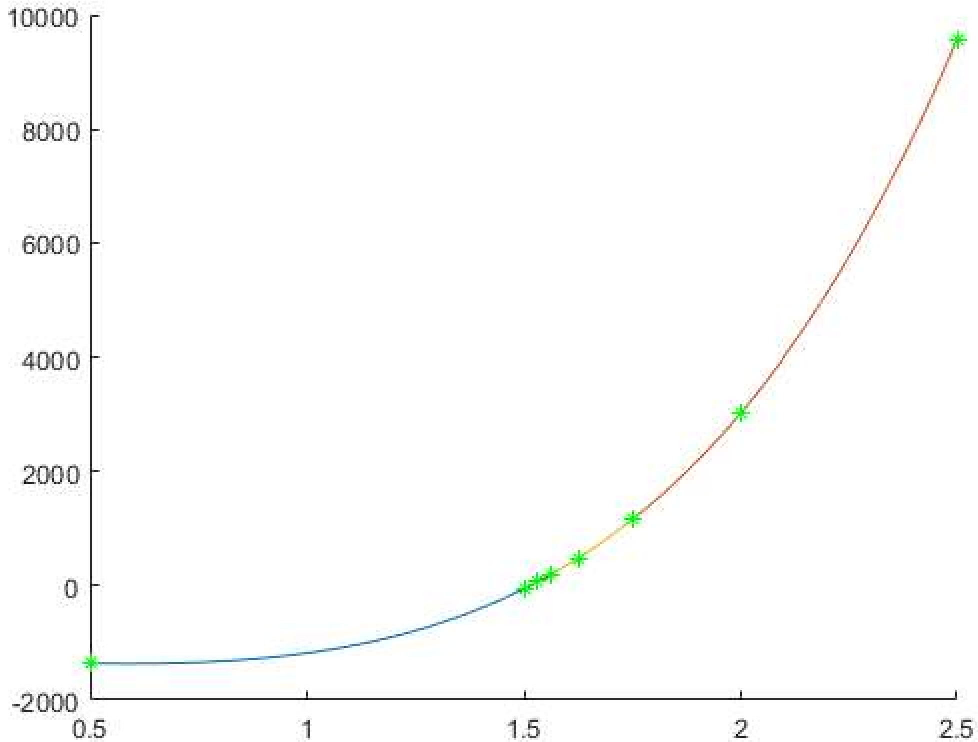
```
function y = your_function(x)
    g = 9.81;
    Q = 20;

    y = g*(3*x+x.^2/2).^3-Q^2*(3+x);
end
```

Bisect function call

```
Bisect(0.5, 2.5, 0.01, 10, @your_function, 1);
```

```
iteration approximation
      1         1.50000000
      2         2.00000000
      3         1.75000000
      4         1.62500000
      5         1.56250000
      6         1.53125000
      7         1.51562500
      8         1.50781250
```



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