DAA Assignment - 1.

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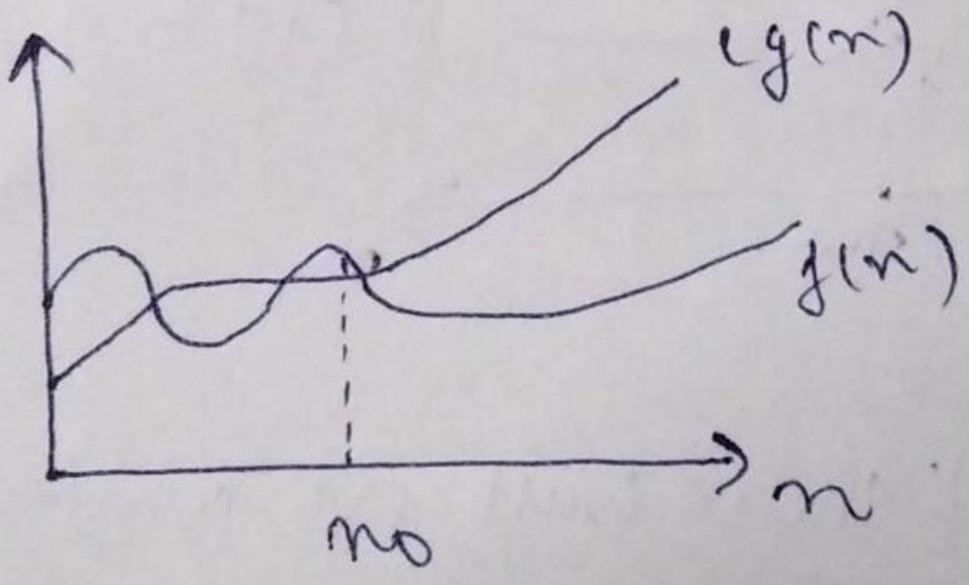
Section - B

A1 → Asymptotic notation are used to represent the complexities of algorithms for asymptotic analysis.

These notation are used for very large input.

1 - Big - Oh (O) -

It gives upper bound for a function $f(n)$ to within a constant factor.

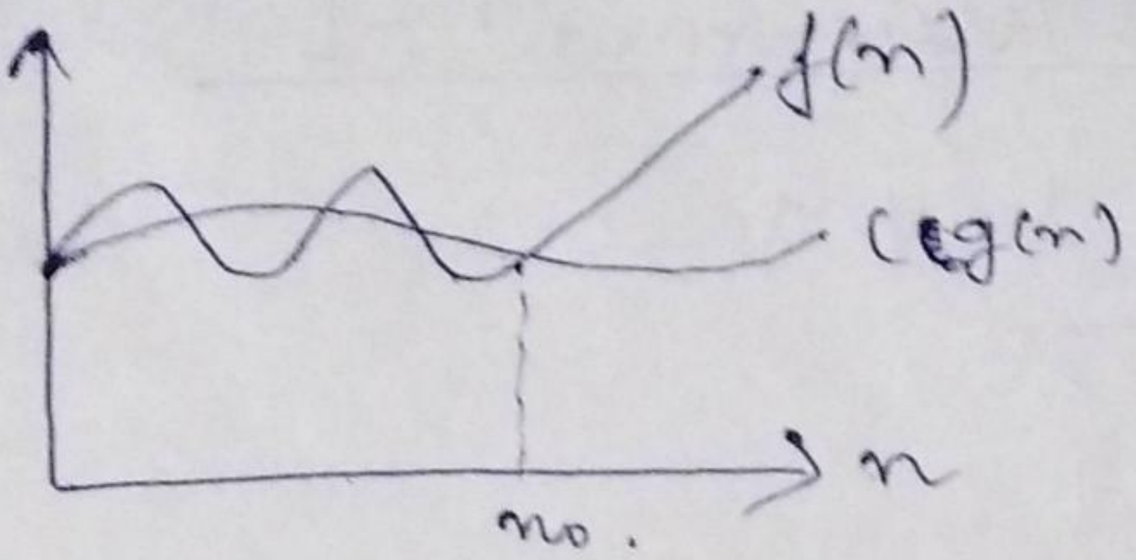


$f(n) \leq c \cdot g(n) \quad \forall \quad n \geq n_0, \; c > 0.$

eg - $O(n^2 + 3n) = O(n^2)$.

2 - Big Omega notation (Ω)

It gives a lower bound for a $f(n)$ to within a constant factor.

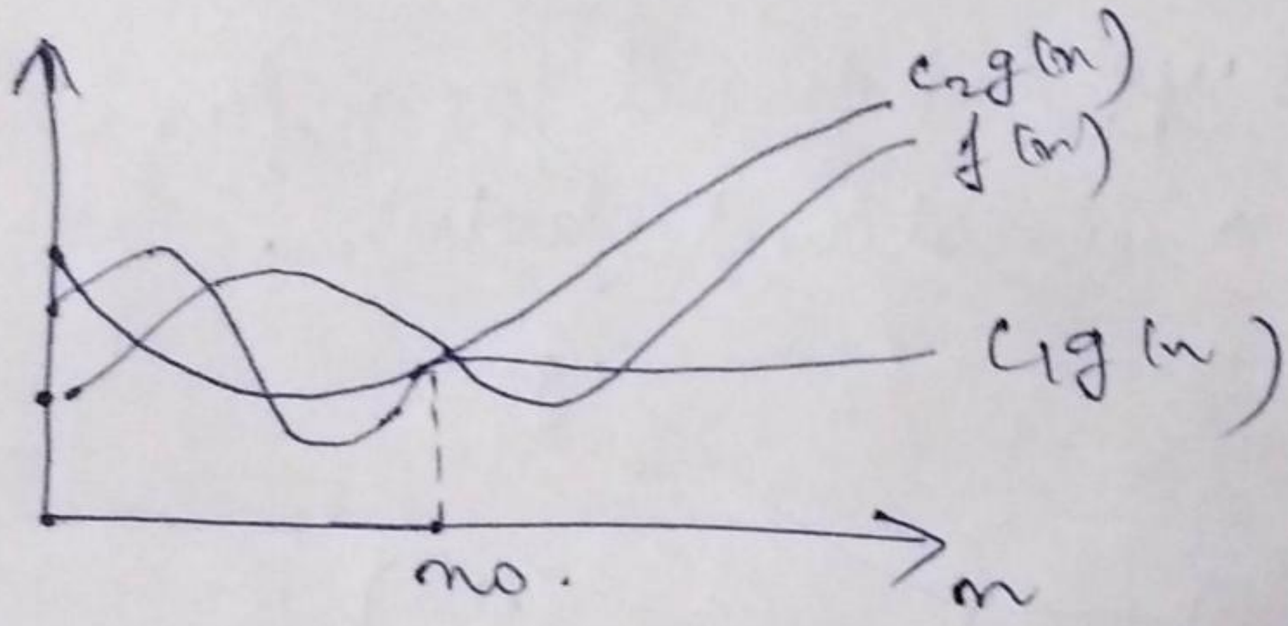


$\Omega(g(n)) = \{ f(n)$: There exist (+ve) constant c and no. such that $0 \leq cg(n) \leq f(n)$ $\forall\ n \geq no\}$.

eg → $\Omega(n \log n)$

3 → Big theta notation (Θ)

It gives bound of function within a constant factor.



$\Theta(g(n)) = \{ f(n)$: there exist +ve constant c1, c2 and no. such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ $\forall\ n \geq no\}$.

eg → $\Theta(n^2)$.

A2 → for (i=1 to n)
{
i = i*2);
}

1, 2, 4, 8 ..... n

$T(n) = O(\log_2 n)$

A3 → $T(n) = \begin{cases} 3T(n-1) & n>0 \\ 1 & n=0. \end{cases}$

$T(n) = 3T(n-1)$ — (1).

$T(n-1) = 3T(n-2)$

$T(n) = 9T(n-2)$ — (2).

$T(n) = 3^3 T(n-3)$ — (3).

$T(k) = 3^k T(n-k)$ — (4).

for $T(n-k) = T(0)$

$n-k = 0$

$n = k$.

$T(n) = 3^n T(0)$

$T(n) = 3^n$

$T(n) = \Theta(3^n)$ =.

A4 → $T(n) = \begin{cases} 2T(n-1)-1, & n>0 \\ 1, & n=0 \end{cases}$

$T(n) = 2T(n-1) - 1$ — (1).

$T(n-1) = 2T(n-2) - 1$

$T(n) = 4T(n-2) - 1 - 2 - ②$.

$T(n) = 8T(n-3) - (1+2+4) - ③$.

$T(k) = 2^k T(n-k) - (\underbrace{1+2+4+\cdots 2^{k-1}}_{k \text{ terms}})$

$T(n-k) = T(0)$

$n = k$

$T(k) = 2^n T(0) - (1+2+4+\cdots)$ k terms.

Its a GP

$a = 1$

$r = 2$

$T(n) = 2^n - \left(1\frac{(2^n - 1)}{2-1}\right)$

$T(n) = 2^n - 2^n + 1$

$T(n) = 1$

$T(n) = \Theta(1)$.

AS →
```
int i = 1, s = 1;
while (s <= n) {
    i++;
    s = s + i;
    printf ("#");
}
```

1, 3, 6, 10, 15, ... n

← k terms →

Its kth term is $\frac{k(k+1)}{2} = n$.

$k = \sqrt{n}$

$T(n) = O(\sqrt{n})$ =

A6 → void function (int n) {

int i, count = 0;

for (int i = 1; i * i < n; i++)

count++;

}

$T(n) = O(\sqrt{n})$.

A7 → $T(n) = O(n * \log_2 n * \log_2 n)$

$T(n) = O(n * (\log_2 n)^2)$

$T(n) = O(n(\log n)^2)$

A8 → function (int n) { T(n)

if (n == 1) return;

for (i = 1 to n) { n^2

for (j = 1 to n) }

printf ("*");

}

}

function (n-3); T(n-3)

}

$T(n) = T(n-3) + n^2$ — (1).

$T(n-1) = T(n-4) + (n-1)^2$

$T(n) = T(n-4) + n^2 + (n-1)^2$

$T(n) = T(n-5) + n^2 + (n-1)^2 + (n-2)^2$

$T(n) = T(n-k) + (n^2 + (n-1)^2 + (n-2)^2 + \cdots)$

$(k-2)$ terms.

for $T(n-k) = 1$

$k = n-1$

$T(n) = T(1) + (n^2 + (n-1)^2 + (n-2)^2 + \cdots)$

$(n-3)$ terms.

$T(n) = T(1) + (4^2 + 5^2 + \cdots n^2)$

$T(n) = T(1) + \left(\frac{(n-3)(n-2)(2n-5)}{6}\right)$

$T(n) = 1 + \left(\frac{2n^3 + \cdots}{6}\right)$

$T(n) = n^3$

$T(n) = O(n^3)$

A9 → void function (int n) {

for (i=1 to n) {

for (j=1; j<=n; j=j+i)

printf ("*.");

}

i=1	n times.		
i=2	1, 3, 5, ... n	n.	
i=3	1, 4, 7, ... n	n/2	
i=n	0	n/3	

$T(n) = (n + \frac{n}{2} + \frac{n}{3} \cdots)$ n times

$\Rightarrow \boxed{T(n) = O(n \log n)}$

A10 → for the functions n^k and a^n, what is the relation.

$k \geq 1$ & $a > 1$

relation is n^k is $O(c^n)$.

A11 →

```
void fun (int n)
{
  int j = 1, i = 0;
  while (i < n)
  {
    i = i + j;
    j++;
  }
}
```

0, 3, 6, 10, 15, ... n

k terms

so for this series is

k^{th} term is $\frac{k(k+1)}{2}$

$n = \frac{k^2 + k}{2}$

$k \simeq \sqrt{n}$

$T = \Theta(\sqrt{n})$ ⇌.

A12 → Recurrence relation of fibonacci series is

$$T(n) = \{ T(n-1) + T(n-2) + 1 \}.$$

$$T(n) = 2T(n-2) + 1)$$

$$T(n) = 4T(n-4) + 3$$

$$T(n) = 8T(n-6) + 7$$

$$T(n) = 16T(n-8) + 15$$

$$T(n) = 2^k T(n-2k) + (2^k - 1)$$

for $T(n-2k) = T(0)$

$$n = 2k$$

$$k = \frac{n}{2}.$$

$$T(n) = 2^{n/2} T(0) + (2^{n/2} - 1)$$

$$T(n) = 2^n - 1$$

$$T(n) = O(2^n).$$

Hence space complexity of fibonacci series is $O(n)$ as it depends on height of recursive tree and it is equal to n in fibonacci series.

A13 → $n(\log n)$

```
void fun. { for (int j=0; j<n ; i++ {
        for (int i=0 ; i<n ; i=i*2)
        {
```

```
        print("*");
      }
    }
}
void main()
{
    fun();
}
```

→ n^3

```
#include <stdio.h>
    void main()
    {
      int n;
      cin >> n;
      for (int i=0; i<n; i++){
        for (int j=0; j<n; j++){
        for (int k=0; k<n; k++){
          n++;
        }
      }
    }
}
```

→ $\log(\log(n))$.

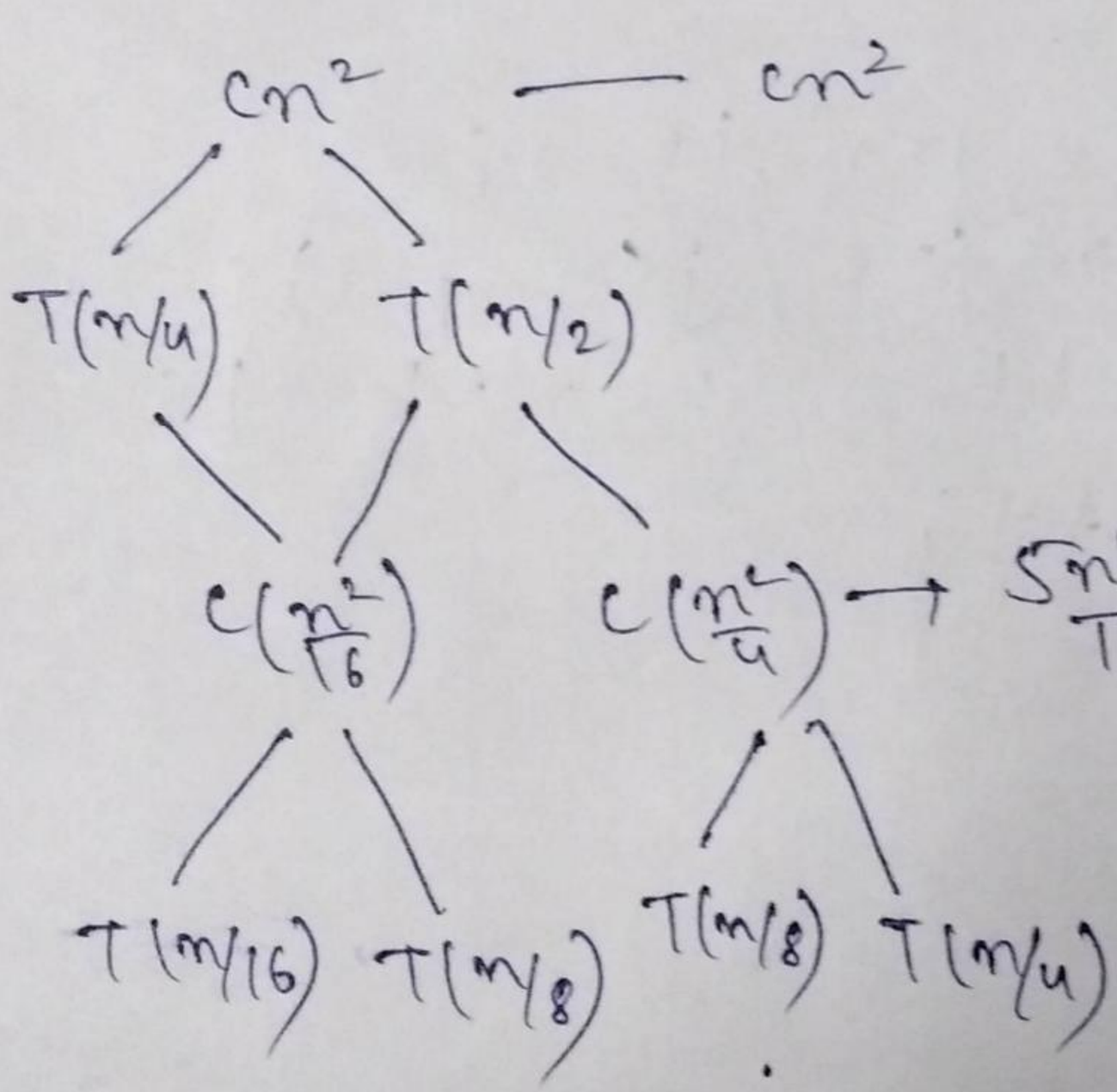
```
#include <bits/stdc++.h>
    void fun(int n)
    {
      if (n==2)
        return 1;
      else
```

```
    fun (sqrt (n));
  }
void main ()
  {
    fun (100);
  }
```

A14 → $T(n) = T(n/4) + T(n/2) + cn^2$

$T(1) = c$

$T(0) = 0$



$T(n)$ = cost of each level

$T(n) = cn^2 + \frac{5cn^2}{16} + \frac{25cn^2}{256} + \dots$

it is a G.P.

with $a=n^2$

$r=5/16$

So sum of sp

$$T(n) = cn^2 / \left(1-\frac{5}{16}\right) = \frac{16cn^2}{11} = \frac{16cn^2}{11}$$

$$T(n) = O(n^2)$$

A15→

```
for (int i to n)
{
    for (int j=1; j<n; j+=i)
    {
      //O(1)
    }
}
```

$n, \frac{n}{2}, \frac{n}{3}, \frac{n}{4}, \frac{n}{5}, \ldots 1$

k-times

$K = \log_2 n$

$n(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{n})$

$(n(\log n))$

$T(n) = O(n \log n)$

A16→

```
for (int i=2; i<=n; i= pow (i, k))
{
    //O(1)
}
```

$2, 2^k, 2^{(k)^2}, 2^{(k)^3}, \ldots n$

It is a GP $a = 2$

$r = 2^k$

k^{th} term $= ar^{k-1}$

$n = 2(2^k)^{k-1}$

let $k^{(k-1)} = x$

$k = k \log_k k = \log x$

$k = \log x$ — (1).

$n = 2^x$

$\log_2 n = x \log_2 2$

$x = \log_2 n$

$\log x = \log(\log n)$

from (1).

$k = \log(\log(n))$.

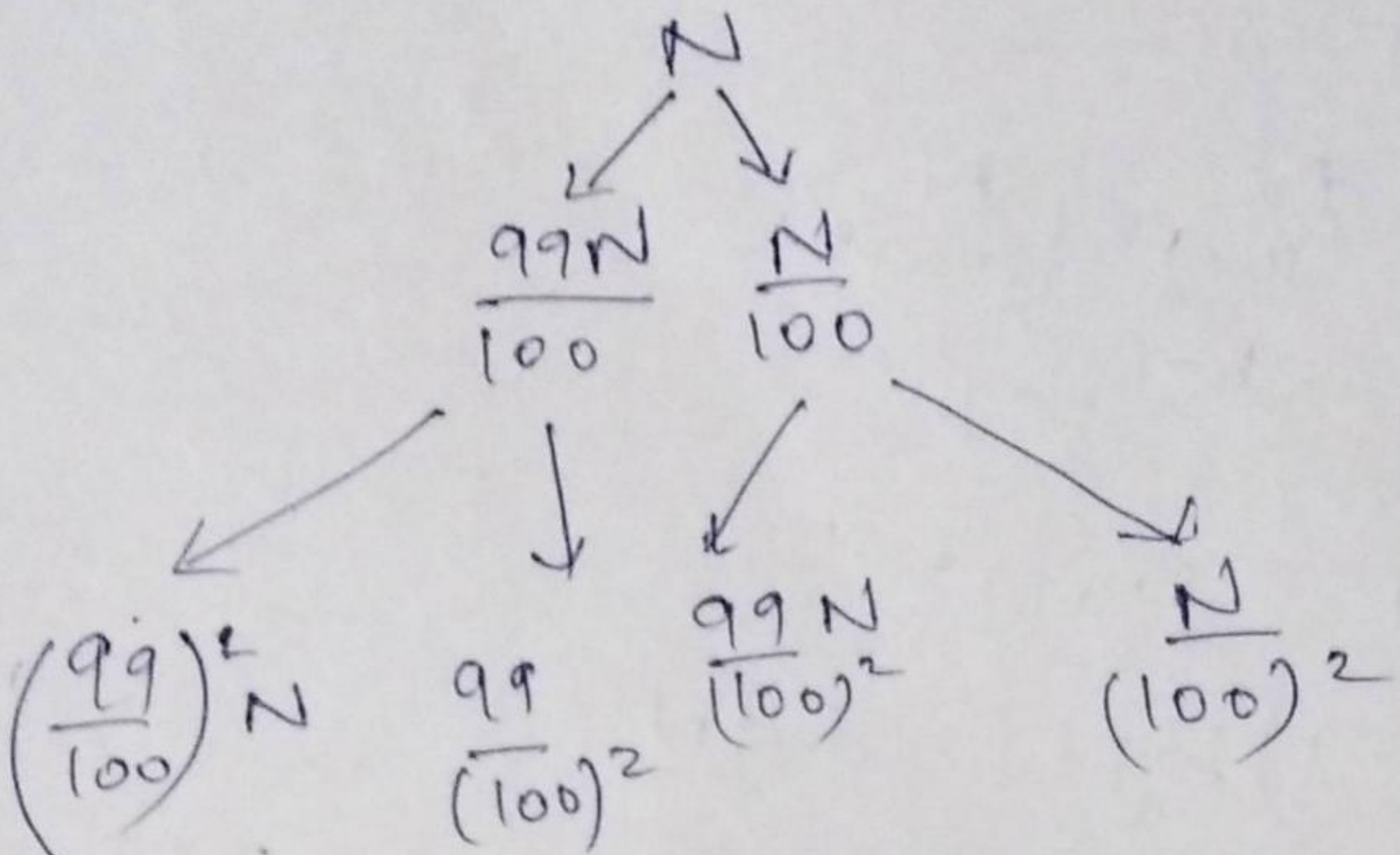
$T(n) = O(\log(\log(n)))$

A17 → Since pivot is divided in 99% & 1%

do

$$T(n) = T\left(\frac{99}{100} N\right) + T\left(\frac{N}{100}\right) + N$$

Now as here we can use 2 extremes of a tree.
where starting point is N.



$$N\left(\frac{(99)(99)}{100\times100} + \frac{99(1)}{100\times100}\right) + \frac{100}{100\times100}N = \frac{99}{100}N + \frac{N}{100}$$

$$= N$$

$$= N$$

So cost of each level is N only

Total cost = height * cost of each level

So for 1st stream — $N, \frac{99N}{100}, \left(\frac{99}{100}\right)^2 N, \ldots$

$$\left(\frac{99}{100}\right)^{h-1} \cdot N = 1$$

$$\left(\frac{99}{100}\right)^{h-1} = \frac{1}{N}$$

$$N = \left(\frac{100}{99}\right)^{h-1}$$

$$\log N = h \log(1)$$

$$h = \log N \text{ or}$$

$$h = \frac{\log N}{\log(100/99)} + 1 =$$

height of 2nd stream →

$$N, \frac{N}{100}, \frac{N}{(100)^2}, \frac{N}{(100)^3}, \ldots\ldots 1$$

$$N\left(\frac{1}{100}\right)^{h-1} = 1$$

$$N = (100)^{h-1}$$

$$(h-1)\log 100 = \log N$$

$$h = \frac{\log N}{\log 100} + 1 \text{ & } h = \log N \text{ (approx)}.$$

$$T(n) = O(N \log N)$$

So time complexity is $O(N \log N)$

height of both extre is $\frac{\log N}{\log 100} + 1$ of $\left(\frac{1}{100}\right)$

and $\frac{\log N}{\log\left(\frac{100}{99}\right)} + 1$ of $\left(\frac{99}{100}\right)$

So we can conclude that if division is done more ~~then~~ height of tree will be more & and when division ratio is less ~~then~~ height is less.

A18→

a) $n, n!, \log n, \log\log n, \text{root}(n), n\log n$

$2^n, 2^{2n}, 4^n, n^2, 100$.

Ans →

$O(100) < O(\log\log n) < O(\log n) < O(\sqrt{n}) < O(n) <$

$O(n\log n) < O(n^2) < O(2^{2n}) < O(4^n)$

(b) $2(2^n), 4n, 2n, 1, \log(n), \log(\log(n)), \sqrt{\log(n)}$,
$\log 2n, 2\log n, n, \log(n!), n!, n^2, n \log(n)$

$O(1) < O(\log(\log(n))) < O(\log(n)) < O(\log 2n) <$
$O(2\log n) < O(n) < O(n\log(n)) < O(\log(n!)) <$
$O(2n) < O(4n) < O(n^2) < O(n!) < O(2(2^n))$.

(c) $8^{2n}, \log_2 n, n\log_6(n), n\log_2(n), \log(n!), n$.
$\log_8(n), 96, 8n^2, 7n^3, 5n$.

Ans → $O(96) < O(\log_8(n) < O(\log_2 n) < O(\log(n!) <$
$O(n\log_6(n)) < O(n\log_2(n)) < O(5n) < O(8n^3)$
$O(7n^3) < O(n!) < O(8^{n2n})$

A19 → void linearSearch(int arr[], int n, int key)
{
for (i=0 to i=n) ...
if arr[i] == key;
cout << "found";
else
continue
}

A20 → Iterative Insertion Sort

void insertionSort(arr, n) {
int i, temp, j
for i-1 to n.

```
{
  temp = arr[i]
  j = i-1
  while j>=0 && arr[j]>temp
  {
     arr[j+1]= arr[j];
     j--
  }
  arr[j+1]=temp
}
```

Recursive Insertion sort →

```
insertion sort (arr, n)
{
   if n<=1
     return;
   insertion (arr, n-1);
   last = arr[n-1];
   j=n-2;
  while (j>=0 and arr[j]>last)
  {
     arr[j+1]=arr[j];
     j--;
  }
  arr[j+1]=last;
}
```

Insertion sort is called online sorting because it don't know the whole input, it might make decision that later turn out to be not optimal. other algorithm are off-line algorithms that are discussed.

A21 →

	Time complexity			Space
	Best	Avg	worst	
Bubble sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(1)$
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(1)$
Insertion sort	(n)	$O(n^2)$	$O(n^2)$	$O(1)$
Merge sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$	$O(n)$ {due to recursion}
Quick sort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$	$O(n)$
Heap sort	$O(n \log n)$	$O(n \log n)$	$O((n \log n)$	$O((1)$

A22 →

	implace	Stable	online sorting
Bubble sort	Yes	Yes	NO
Selection sort	Yes	NO	NO
Insertion sort	Yes	Yes	Yes
Merge sort	NO	Yes	NO
Quick sort	Yes	NO	No
Heap sort	Yes	NO	No

A23 → Binary search (arr, int n, key)

```
{
  beg = 0
end = n-1
while (beg <= end)
{
   mid = (beg + end)/2
  if[arr [mid] == key]
     found
   else if arr[mid] < key
        beg = mid + 1
   else
        end = mid - 1
}
}
```

Time complexity of linear search - O(n)

Space complexity of linear search - O(1)

Time complexity of Binary search - O(log n)

Space complexity of Binary search = O(n)

A24 → $T(n) = T\left(\frac{n}{2}\right) + 1 =$.