

Mechanics → It is a branch of that deals with the study of forces & their effect on a body.

- 1) Rigid Body.
- 2) Deformable Body

### Classification of Mechanics

#### Mechanics

##### Mechanics of Rigid Body

##### Engineering Mechanics

##### Static

##### Dynamic

##### Mechanics of Deformable body

- Solid Mechanics
- Fluid Mechanics
- Gas Dynamics

##### Kinetics      Kinematics

Force - Force is define as an external Stimuli that changes / tends to changes the position / motion of a body.

#### Characterization :-

- Magnitude
- Point of Application
- Nature (Ex: ~~Ex: Tension or compression~~) (Tensile / compression)
- Line of Action

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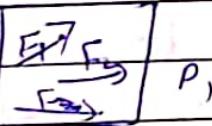
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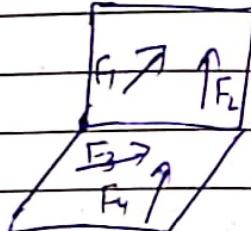
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## Force systems:

1) Coplanar

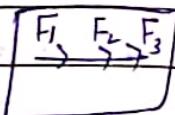


2) Non-coplanar



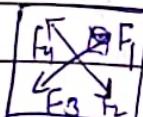
③

Coplanar - collinear

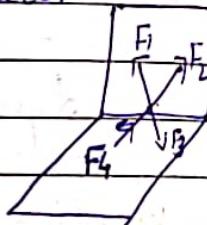


4)

Coplanar concurrent



5) Non-coplanar concurrent



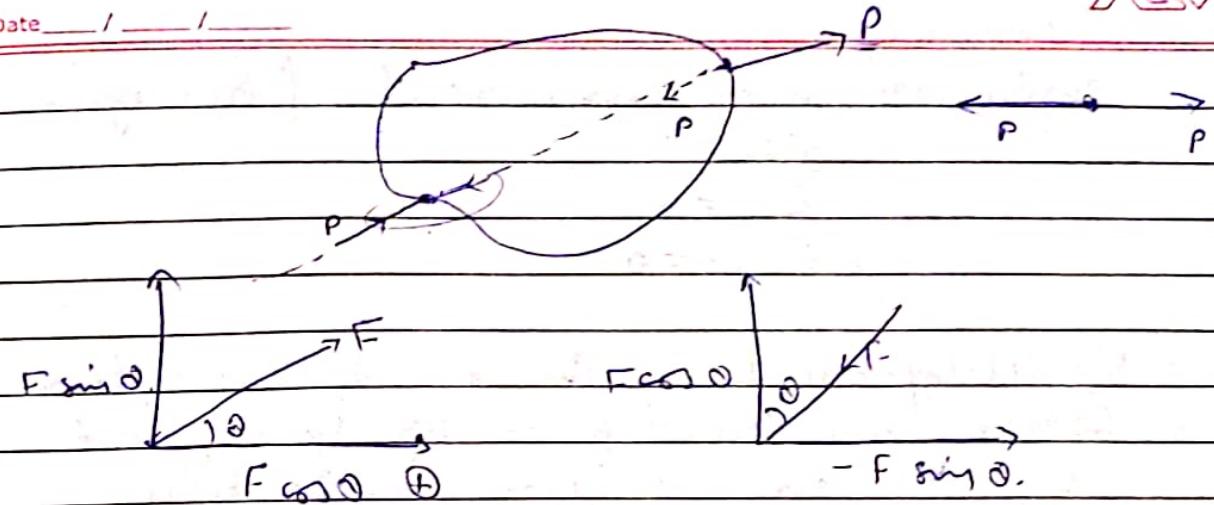
6)

Non-coplanar non concurrent.

7)

Transmissibility Law of forces

The position or motion of a rigid body remain unchanged by replacing a force acting on it by the another force along the same line of action of same magnitude.



Resultant of a forces:

Resultant forces is a forces which could produce the same effect as produced by the system of forces.

Composition of forces: (For two forces)

1) Analytical Method

i) Parallelogram Law of forces.

ii) triangle Law of forces.

iii) Polygon Law of forces

2) Graphical Method

3) Method of Resolution

Equilibrium: When the net effect is zero.

" If ~~is~~ by the application of motion not going to change its position. (without acceleration or retardation).

Condition of equilibrium for coplanar concurrent system.

$$\sum H = 0, \sum F_x = 0$$

$$\sum V = 0, \sum F_y = 0.$$



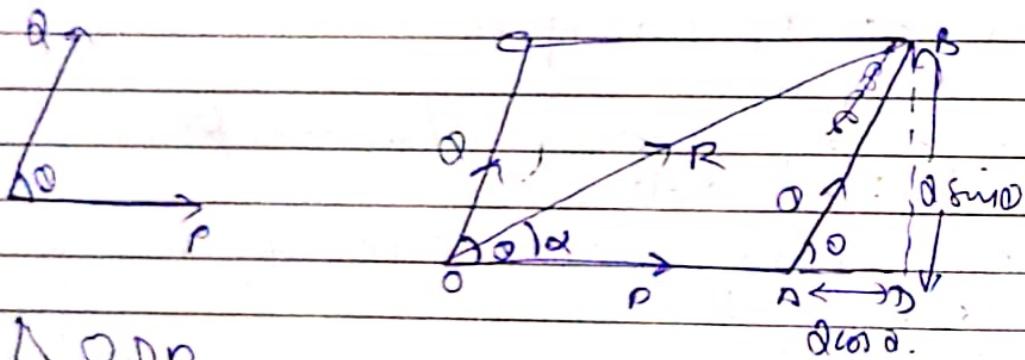
Coplanar, non-concurrent force system

$$\sum H = 0$$

$$\sum V = 0$$

$$\sum M_p = 0$$

Parallelogram law forces: If two coplanar forces are represented by two sides of a parallelogram in the both mag. & dir., then their resultant is represented by the diagonal of that parallelogram in both sides mag. & dir. starting from same point.



$\triangle ODB$

$$OB^2 = OD^2 + BD^2$$

$$OB^2 = (OA + AD)^2 + BD^2$$

$$R^2 = P^2 + Q^2 + 2PQ \cos \alpha$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha} \quad \textcircled{1}$$

$$\tan \alpha = \frac{BD}{OD} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

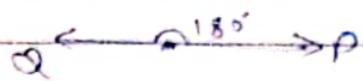
$$\alpha = \tan^{-1} \left( \frac{Q \sin \theta}{P + Q \cos \theta} \right) \quad \textcircled{2}$$



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Case I

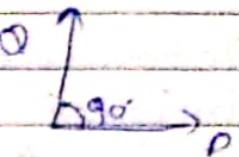
$$\theta = 180^\circ$$



①  $R = \sqrt{P^2 + Q^2}$   $P > Q$   
 $= \sqrt{Q^2 + P^2}$   $Q > P$

②  $\alpha = 0^\circ$

③  $\theta = 90^\circ$



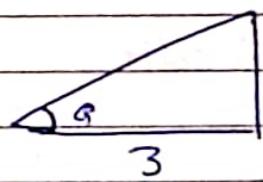
Case II

$$\theta = 0^\circ$$

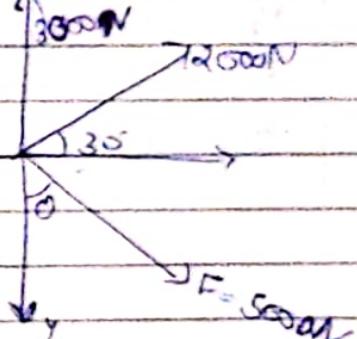
$R = \sqrt{P^2 + Q^2}$   
 $\alpha = 0^\circ$

$R = \sqrt{P^2 + Q^2}$   
 $\alpha = \tan^{-1}\left(\frac{Q}{P}\right)$

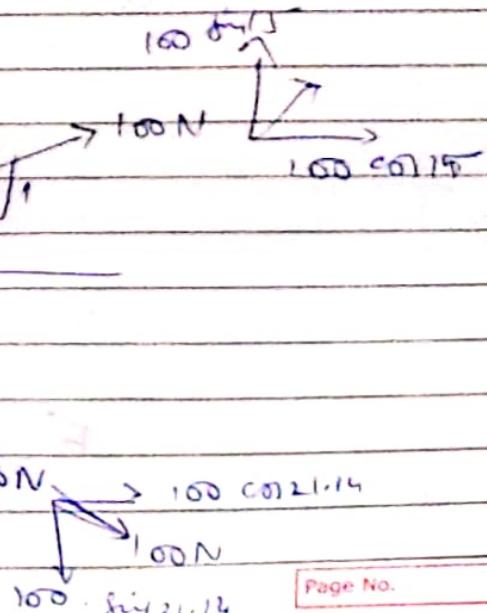
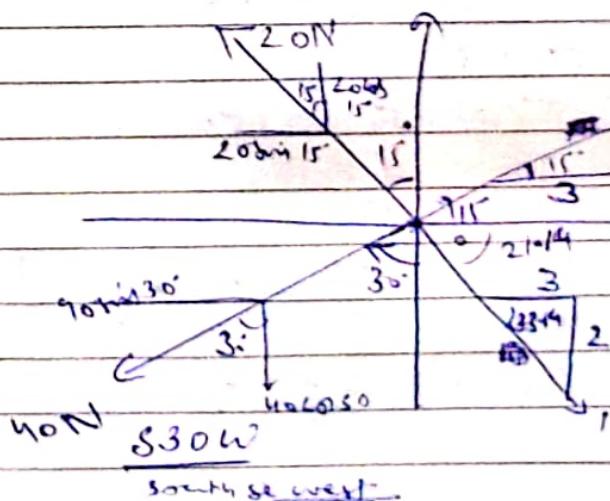
- Q: A body is subjected to three forces as shown in fig. Identify the dir<sup>n</sup> of force F if resultant dir<sup>n</sup> of  $\perp$  x-axis.



$$\tan \alpha = \frac{4}{3}$$
 $\alpha = \tan^{-1}\left(\frac{4}{3}\right)$ 
 $= 53.13^\circ$



Q:



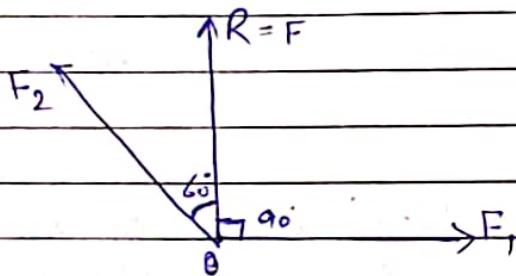
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$$\sum H = 100 \cos 15^\circ + 100 \cos 33.6^\circ - 20 \sin 18.43^\circ - 40 \tan 55^\circ$$

$$\sum V = 100 \sin 15^\circ - 100 \sin 33.6^\circ + 20 \cos 18.43^\circ - 40 \cot 55^\circ$$

- Q. Determine the horizontal force of a force inclined at angle  $60^\circ$  with a vertical whose resultant equals to the vertical force  $F$ .

Sol:



$$P_0 = F_1 \quad R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 150^\circ}$$

$$\alpha = F_2$$

$$\theta = 150^\circ \quad F = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 150^\circ}$$

$$\tan \alpha = \frac{F_1 \sin 150^\circ}{F_1 + F_2 \cos 150^\circ}$$

$$\frac{1}{\alpha} = \frac{F_2 \sin 150^\circ}{F_1 + F_2 \cos 150^\circ}$$

$$F_1 + F_2 \cos 150^\circ = 0$$

$$F_1 = -F_2 \cos 150^\circ$$

$$F_1 = \frac{\sqrt{3}}{2} F_2$$

$$F^2 = \frac{3}{4} F_2^2 + F_1^2 - \sqrt{3} \times \frac{\sqrt{3}}{2} F_2^2$$

$$F^2 = \left( \frac{3}{4} + 1 - \frac{3}{2} \right) F_2^2$$

$$F_2^2 \left( \frac{3+4-6}{4} \right) = F^2$$

$$F^2 \times \frac{1}{4} = F^2$$

$$\boxed{F_2 = \alpha F}$$

Q - The resultant of two forces  $P$  &  $\alpha P$  acting at a point is  $R$ . The resultant  $R$  gets doubled when  $\alpha$  is either doubled or its direction is reversed show that  $P$ ,  $\alpha$  &  $R$  confirm to the ratio of  $\frac{P}{\sqrt{2}} : \frac{\alpha}{\sqrt{3}} : \frac{R}{\sqrt{2}}$

cub  $R^2 = P^2 + \alpha^2 P^2 + 2P\alpha \cos 0^\circ \quad \text{--- (1)}$

$$\alpha = \alpha_0, \quad R = \alpha R_0$$

$$2R^2 = P^2 + 4\alpha^2 P^2 + 4P\alpha \cos 0^\circ \quad \text{--- (2)}$$

$$\alpha = -\alpha_0, \quad R = \alpha R_0$$

$$4R^2 = P^2 + \alpha^2 P^2 - 2P\alpha \cos 0^\circ \quad \text{--- (3)}$$

$$(1) + (3) \quad 5R^2 = \alpha P^2 + 2\alpha^2 P^2 \quad \text{--- (4)}$$

$$(2) + (3) \times 2 \quad 12R^2 = 3P^2 + 6\alpha^2 P^2 \quad \text{--- (5)}$$

$$(5) \times 2 - (4) \times 3$$

$$9R^2 = 0 + 6\alpha^2 P^2$$

$$\therefore 9R^2 = 6\alpha^2 P^2$$

$$\boxed{\sqrt{3} R = \sqrt{2} \alpha P}$$

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### Lami's theorem

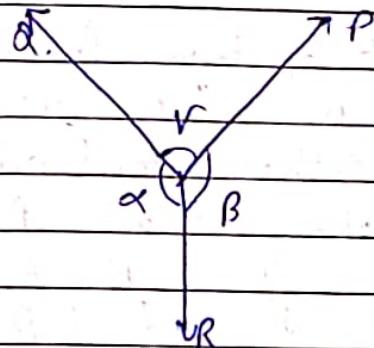
If three coplanar concurrent forces are in equilibrium then each force is directly proportional to the ~~sin~~<sup>sign</sup> of angle between the other two forces.

Condition

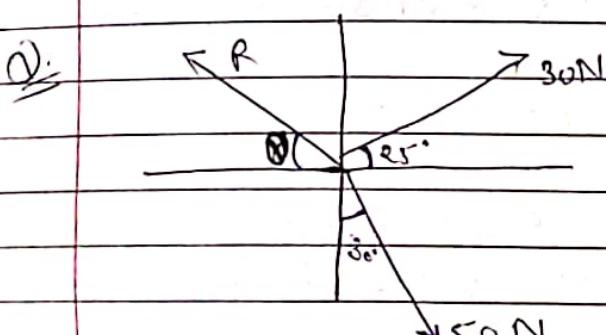
$$P \propto \sin \alpha$$

$$\alpha \propto \sin \beta$$

$$R \propto \sin \gamma$$



$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma} = C$$



By method of Resolution

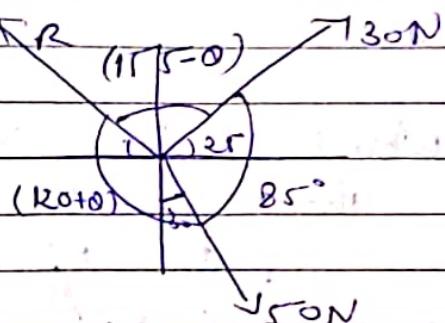
$$\sum H = 0, \quad 30 \cos 25^\circ + 50 \sin 30^\circ - R \cos \theta = 0$$

$$30 \cos 25^\circ + 50 \sin 30^\circ = R \cos \theta. \quad \text{---(1)}$$

$$\sum V = 0, \quad 30 \sin 25^\circ + R \sin \theta - 50 \cos 30^\circ = 0.$$

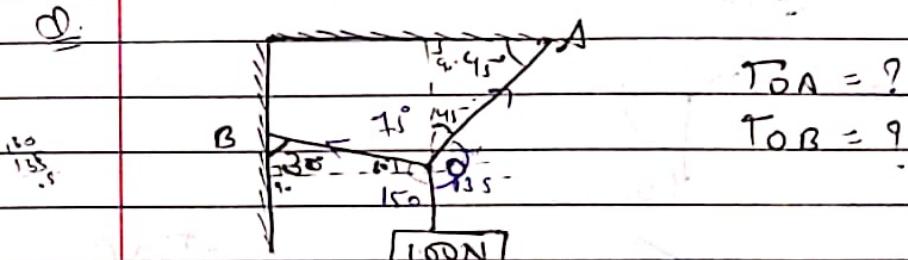
$$30 \sin 25^\circ + 50 \cos 30^\circ - 30 \sin 25^\circ = R \sin \theta. \quad \text{---(2)}$$

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1) Methods for

$$\frac{30}{\sin(120+0)} = \frac{R}{\sin 85} = \frac{50}{\sin(15.5-0)}$$

Q:



$$T_{OA} = ?$$

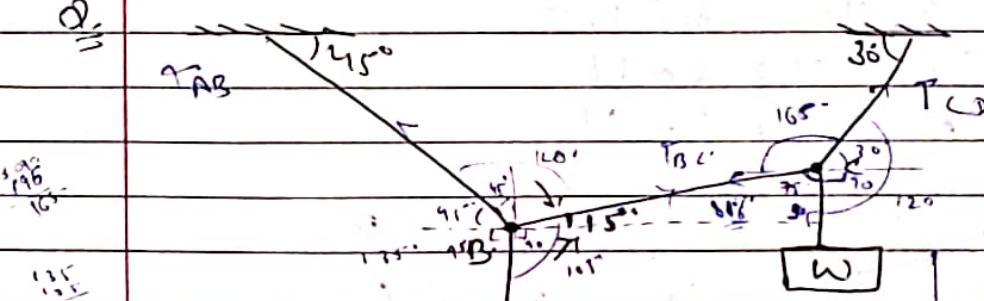
$$T_{OB} = ?$$

$$\frac{100}{\sin 75} = T_{OA} = T_{OB} = \frac{100}{\sin 150}$$

$$T_{OA} = 52N$$

$$T_{OB} = 73.6N$$

Q:

Point C

$$W = 81.6 = T_{CD}$$

$$\sin 165^\circ \quad \sin 120^\circ \quad \sin 75^\circ$$

$$\therefore W = 24.5 N$$

$$T_{CD} = 91.001 N$$

Point B

$$\frac{100}{\sin 150} = T_{AB} = T_{BC}$$

$$\sin(105) \quad \sin 135^\circ$$

$$T_{AB} = 111.53 N$$

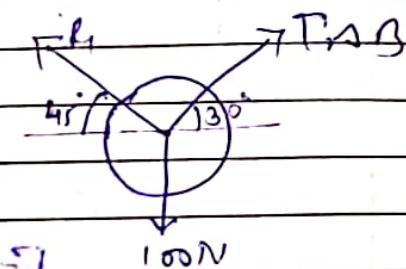
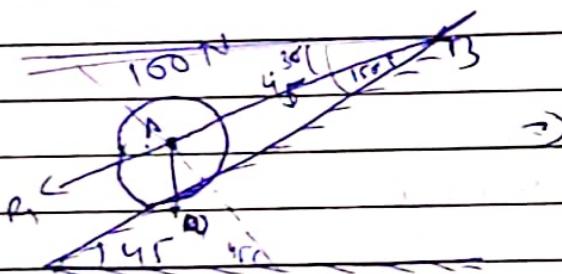
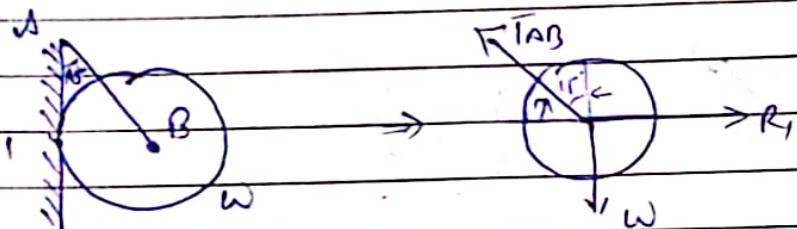
$$T_{BC} = 81.6 N$$

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## \* Free Body Diagram

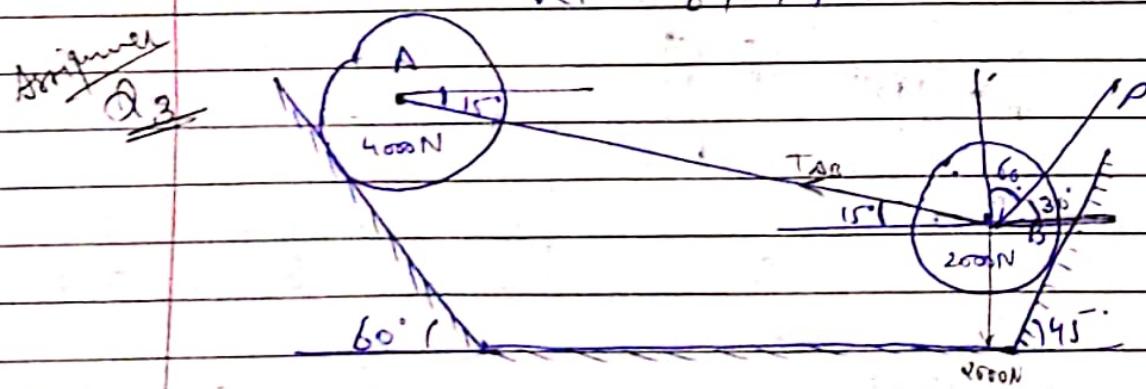
Diagram of a Body which is free from all the contacting surfaces shows all the forces acting on it.



$$T_{AB} = \frac{R_1}{\sin 137^\circ \cdot \sin 120^\circ \cdot \sin 105^\circ} = 100$$

$$T_{AB} = 73.20 \text{ N.}$$

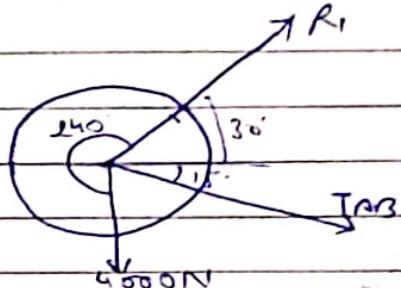
$$R_1 = 29.77 \text{ N.}$$





$$\cancel{2000} = \cancel{P} \quad \cancel{\sin 30^\circ} = \cancel{T_{AB}} \quad \cancel{\sin 120^\circ}$$

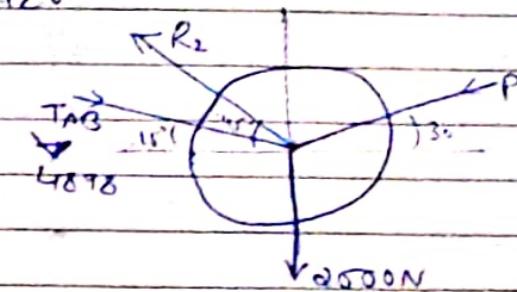
$$P = \frac{0.9 \times 2000}{0.7} = 1.2 \times 2000$$



$$\frac{4000}{\sin 45^\circ} = \frac{T_{AB}}{\sin 240^\circ} = \frac{R_1}{\sin 75^\circ}$$

$$T_{AB} = -4898 \text{ N}$$

$$R_1 = 5464 \text{ N}$$



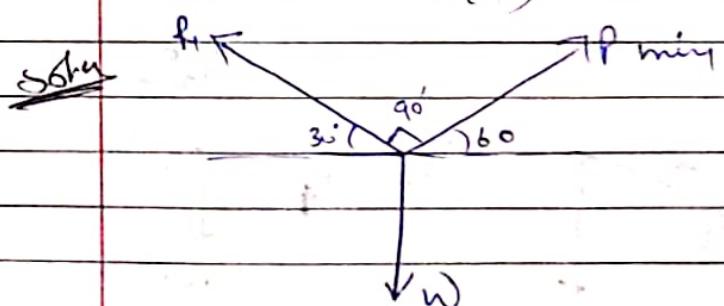
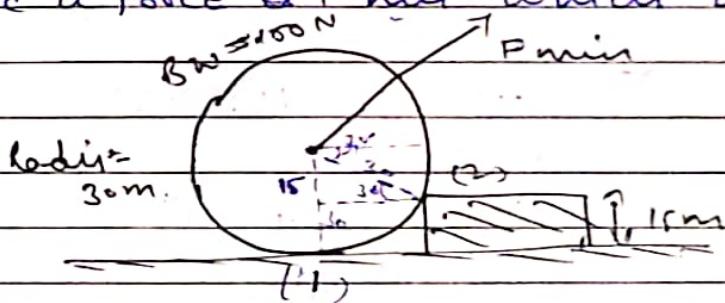
$$P \cos 30^\circ + R_2 \cos 45^\circ = T_{AB} \cos 15^\circ \\ = 4898 \cos 15^\circ$$

$$R_2 \sin 45^\circ - P \sin 30^\circ = 2000 + 4898 \sin 15^\circ$$

$$P = 1071.28 \text{ N}$$

$$R_2 = 5378.73 \text{ N}$$

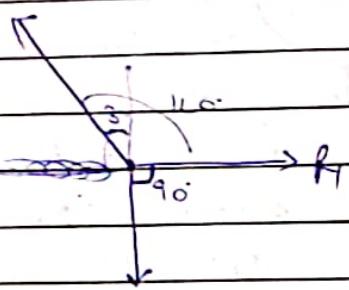
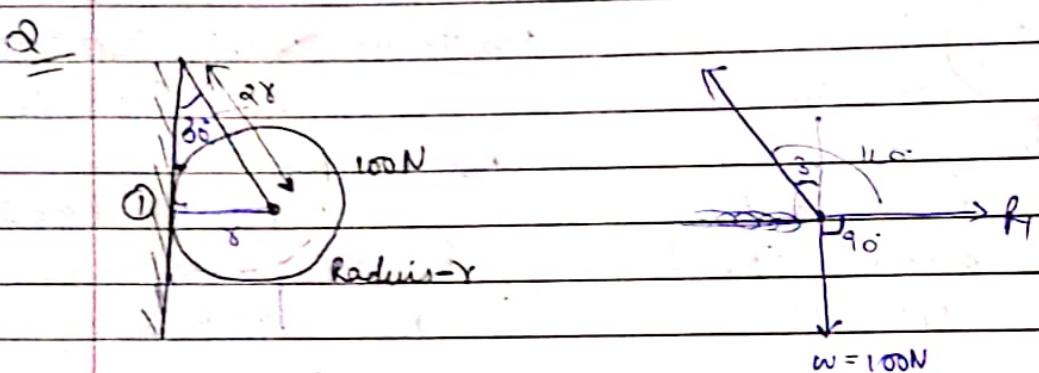
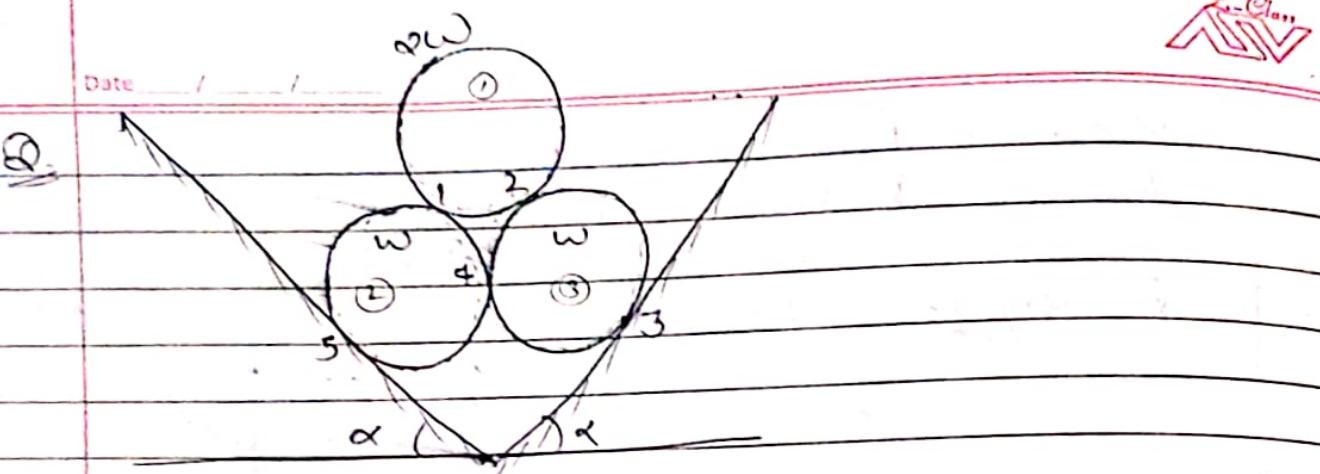
Q: Find a force & Point which contact is (2) not (1).



For P\_min sine will

be 90° at (2) front

$$\frac{100}{\sin 90^\circ} = \frac{R_1}{\sin 150^\circ} = \frac{P_{min}}{\sin 120^\circ}$$

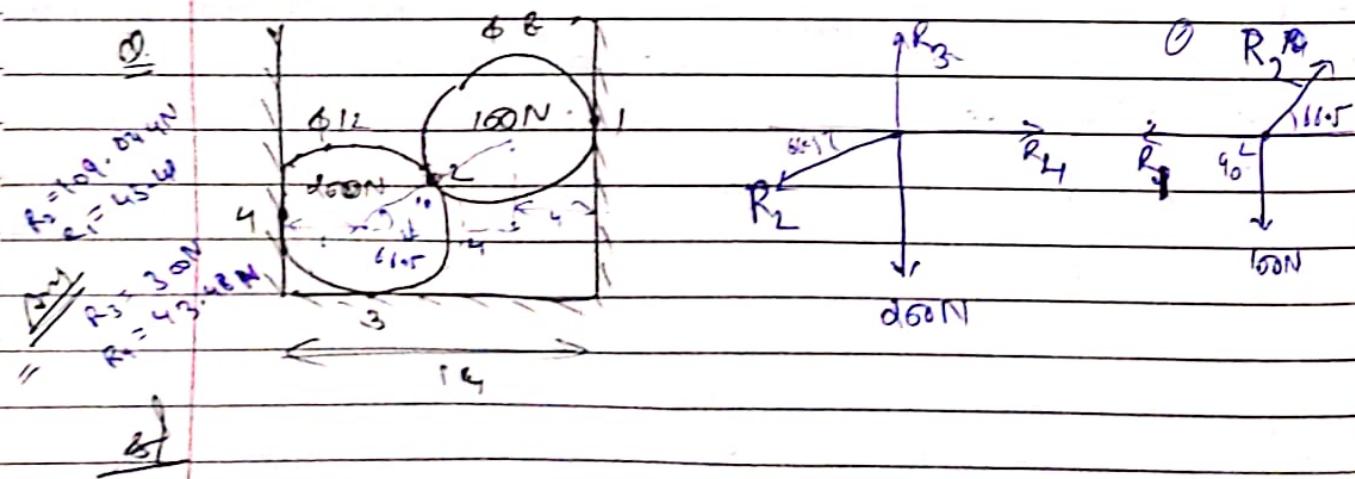


$$\frac{100}{\sin 120} = R_1 = \frac{T}{\sin 150}$$

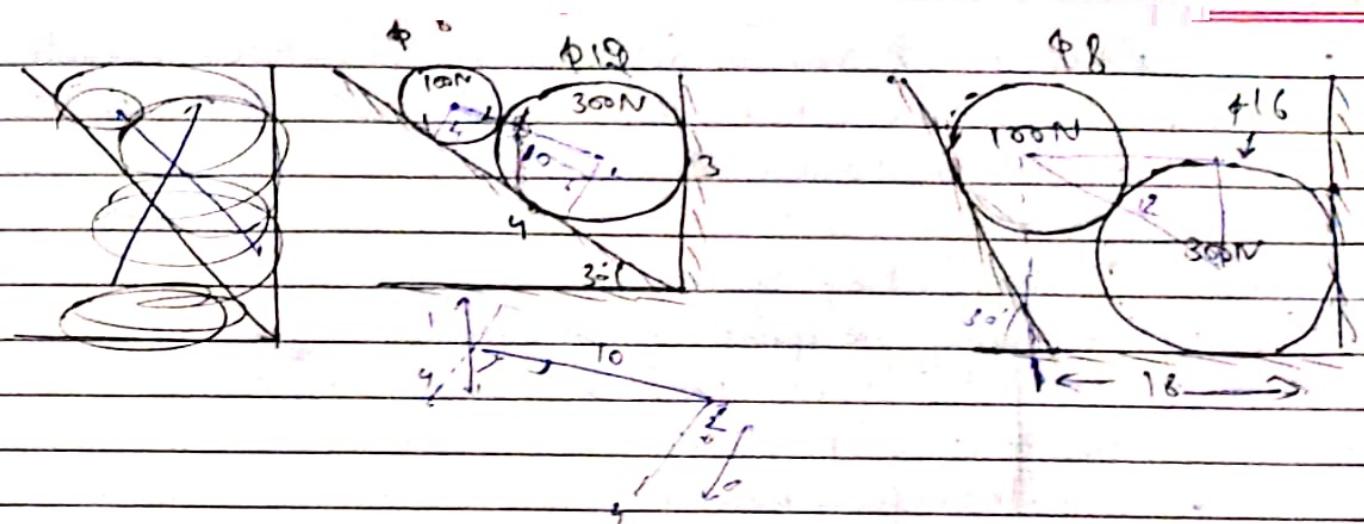
$$T = \frac{100}{\sin 120} = 115.46 \text{ N}$$

$$R_1 = 115.46 \times \sin 150^\circ$$

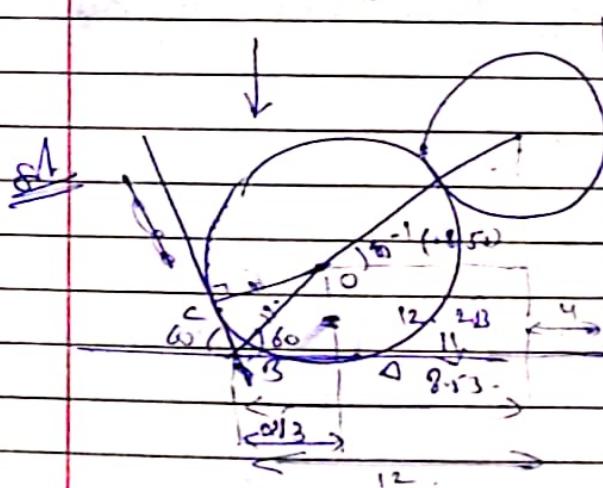
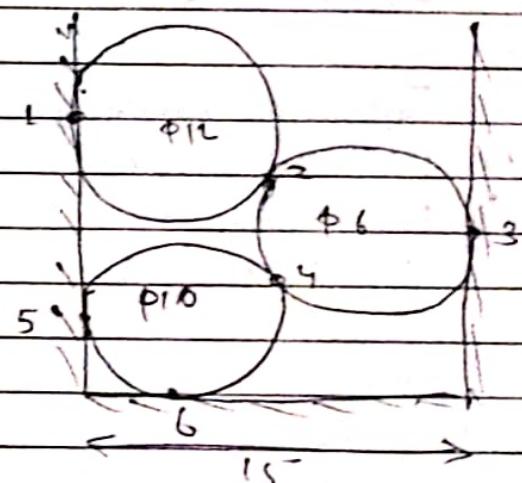
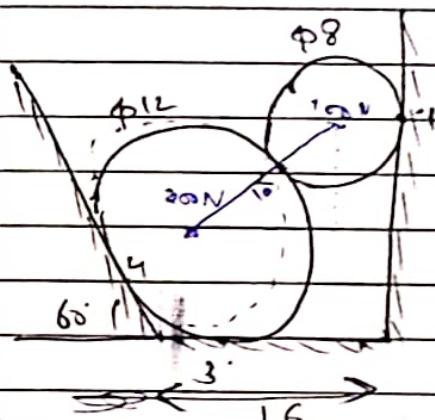
$$= 57.46 \text{ N}$$



Q.



Q.



$\Delta OAB / \Delta OCB$

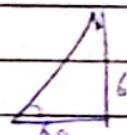
SAS

$$\tan 60 = \frac{6}{?}$$

$$? = 6$$

$$\tan 60$$

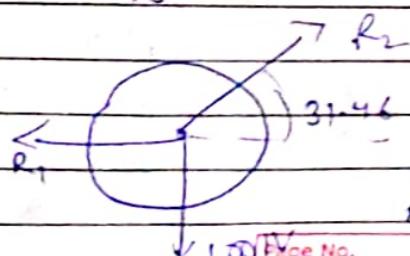
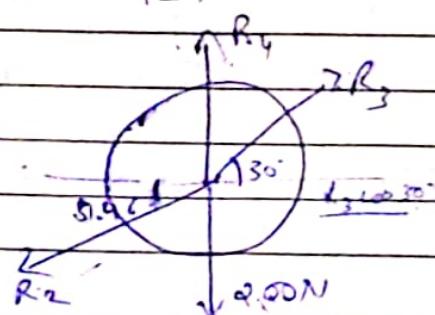
$$= \sqrt{3}$$



$$R_1 = 204.80N$$

$$R_2 = 160.48N$$

$$R_2 = 160.48N$$



$$R_2 = 191.98$$

$$R_1 = 163.47$$

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Beam: Beam is defined as structural member that is subjected to a load parallel to its axis or in lateral direction.

Classification of Beams:

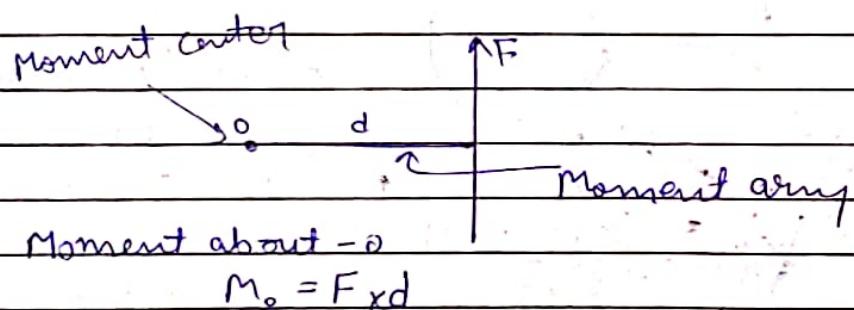
- 1) Simply supported Beam.
- 2) Cantilever beam.
- 3) Continuous Beam.
- 4) Over hanging Beam.
- 5) Fixed Beam.

Types of supports:

- 1) Roller support
- 2) Hinge support
- 3) Fixed support

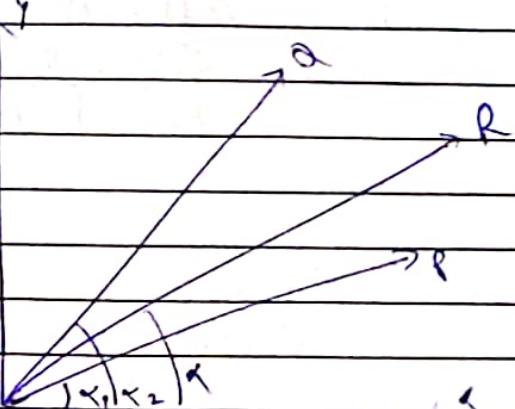
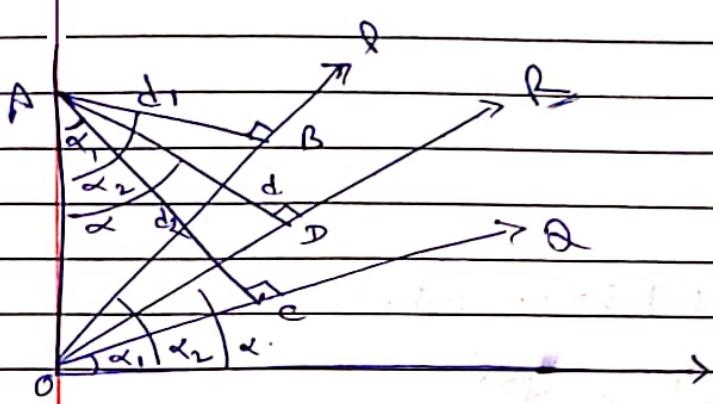
Moment

Measuring of rotational effect produced by force  
also force & distance



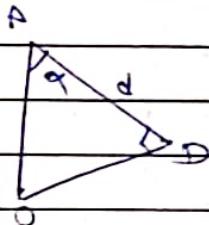
4) Principal of Moment or Varigny's Theory.

Moment of a resultant force is equal to the algebraic sum of moments of individual forces forming a system about the same moment center.



$$R \cos \alpha \Rightarrow P \cos \alpha, + Q \cos \alpha$$

$$\begin{aligned}\sum M_A &= P \times d_1 + Q \times d_2 \\&= P \times OA \cdot \cos \alpha_2 + Q \times OA \cdot \cos \alpha_1 \\&= OA (P \cos \alpha_2 + Q \cos \alpha_1) \\&= OA \times R \cos \alpha \\&= R \cdot OA \cos \alpha \\&= R \times d\end{aligned}$$



$$R \times d = P \times d_1 + Q \times d_2$$

$R$  = Resultant

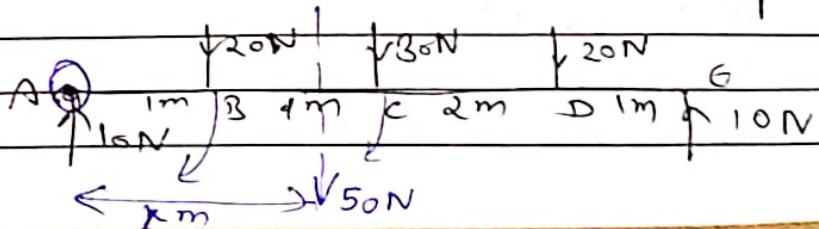
$P, Q$  = forming the co-planar, concurrent force system

$d_1, d_2$ ,  $d$  = moment arms.

Moment- Nxm unit

+ve.
-ve.
  
cw
Acw.

Q. Calculate resultant & location of its resultant force.



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$$\sum H = 0$$

$$\sum V = 50$$

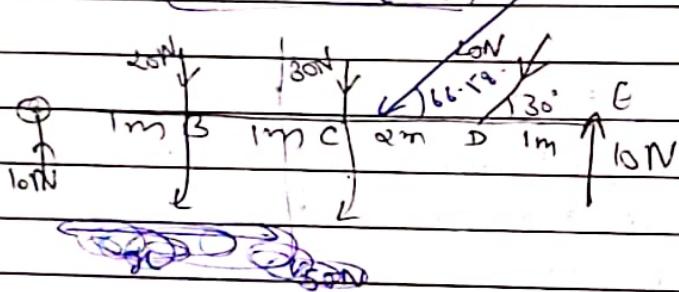
$$R = \sqrt{\sum H^2 + \sum V^2}$$
$$= 50 N \downarrow$$

$$\sum M_A$$

$$50 \times x = 10 \times 0 + 20 \times 1 + 30 \times 2 + 20 \times 4 - 10 \times 5$$

$$x = 110/50$$

$$x = 2.2 \text{ m}$$

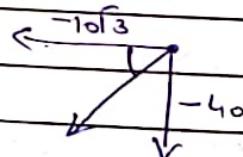


$$\sum H = -20 \cos 30 = -10\sqrt{3}$$

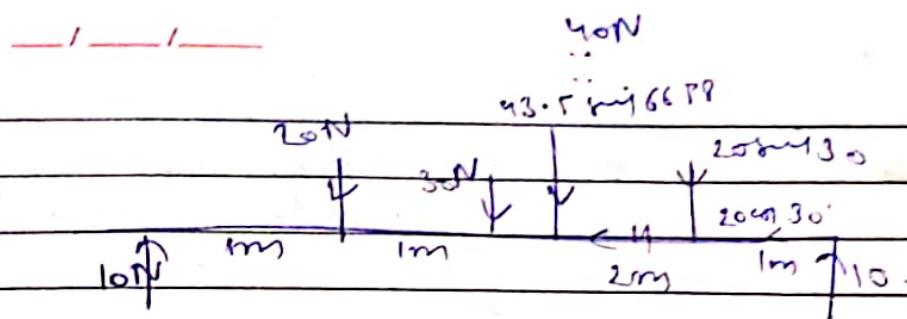
$$\sum V = -30 - 20 \sin 30 = -40$$

$$R = \sqrt{\sum H^2 + \sum V^2}$$
$$= \sqrt{1900}$$
$$= 43.5 N$$

$$\tan \theta = \frac{\sum V}{\sum H}$$
$$= \frac{-40}{-10\sqrt{3}}$$
$$= \frac{4}{\sqrt{3}}$$



$$\theta = 66.58^\circ$$

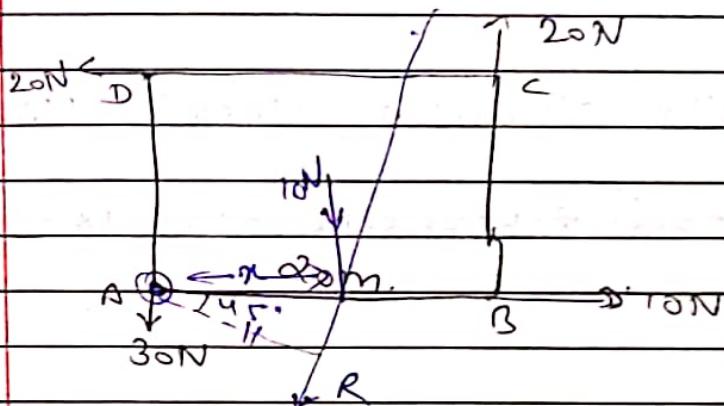


$\sum MA$

$$40 \times n = 10 \times 0 + 20 \times 1 + 30 \times 2 + 20 \times 3 + 20 \times 4 - 10 \times 4$$

$$n =$$

Q



calculate resultant forces  
& its location

$$\sum H = 10 - 20 = -10N$$

$$\sum V = 20 - 30 = -10N$$

$$R = \sqrt{10^2 + 10^2} = 10\sqrt{2} = 14.14$$

$$\tan \alpha = -10 = 1$$

$$-10$$

$$\alpha = 45^\circ$$

$$\sum Ma = 30 \times 0 + 10 \times 0 - 20 \times 20 - 20 \times 20$$

$$10 \times x = -800 \text{ Nxm}$$

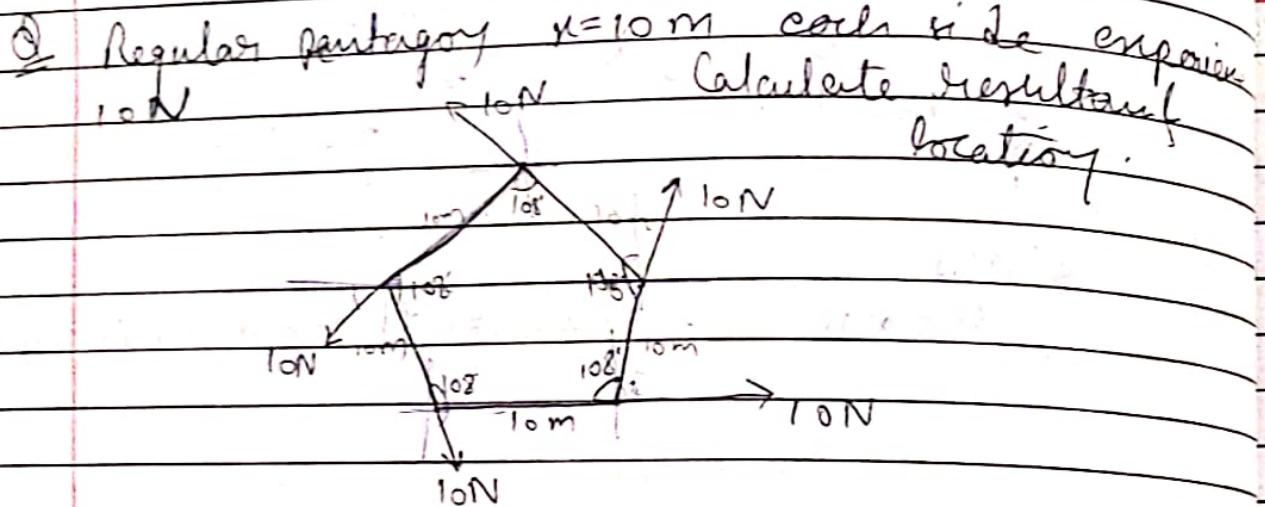
$$(x = -80 \text{ m})$$

$$\underline{\underline{R}} \times x \cos 45^\circ = -200$$

$$10 \times \frac{1}{2} \times x \times \frac{1}{\sqrt{2}} = -800$$

$$(x = -80 \text{ m})$$

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$$\sum H = 10 + 10\cos 72^\circ - 10\cos 72^\circ - 10\cos 72^\circ + 10\cos 72^\circ \\ = 10$$

~~Ex = 10sin72 + 10sin72 + 10sin72 + 10sin72~~

$$= 3.09$$