

ECE1512 - Digital Image Processing and Applications - Project B

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1. State Space Models (SSMs) - Mamba: Linear-Time Sequence Modeling with Selective State Spaces

In this section, Mamba, as introduced in the paper "Mamba: Linear-Time Sequence Modeling with Selective State Spaces" [3], is discussed. Mamba is a novel sequence modeling architecture that leverages Selective State Space Models (SSMs) to address the computational inefficiencies of Transformers while maintaining their ability to perform complex reasoning. The architecture introduces input-dependent dynamics in SSM parameters, enabling selective propagation or forgetting of information along sequences.

This work represents a significant step forward in designing models that combine efficiency and performance, achieving **linear scaling** in sequence length and demonstrating up to 5× higher inference throughput compared to Transformers. Mamba eliminates the need for attention or MLP blocks, offering a simplified design suitable for multimodal tasks. It achieves state-of-the-art results across various domains, including language, audio, and genomics, and matches the performance of Transformers twice its size in language modeling tasks.

Mamba is designed to solve the computational inefficiency and scalability issues of Transformers, particularly for handling long sequence data across various modalities (e.g., language, audio, and genomics). The primary challenges that Mamba aims to address are:

Inefficiency of Self-Attention in Transformers.

- **Quadratic Complexity:** The self-attention mechanism in Transformers scales quadratically with sequence length, making it computationally expensive and memory-intensive for long sequences [11].
- **Limited Context:** Transformers can only process a finite context window efficiently, which limits their ability to model long-range dependencies.

Limitations of Existing Efficient Architectures.

- Several subquadratic-time models (e.g., linear attention [8], gated convolution [10], recurrent models [1], and structured state space models [4]) have been proposed to improve efficiency. However they have their own drawbacks.
 - They often fail to match the performance of Transformers, especially in discrete and information-dense data such as natural language.
 - They lack content-based reasoning capabilities, which are crucial for tasks like language modeling and in-context learning.

The author’s key contribution to State Space models are threefold:

- 1) **Selection Mechanism:** A key limitation of previous models was their inability to perform input-dependent selection, crucial for focusing on relevant information and ignoring irrelevant data. To address this, the authors parameterize SSMs based on input, enabling the model to selectively retain or discard information. This mechanism is validated on synthetic tasks such as selective copying and induction heads.
- 2) **Hardware-aware Algorithm:** The input-dependent nature of Selective SSMs posed computational challenges. The authors introduce a **hardware-efficient recurrent algorithm** that leverages a scan operation instead of convolutions, avoiding the need to materialize large intermediate states in GPU memory. This implementation achieves true linear scaling in sequence length and is up to $3\times$ faster than previous SSM methods on modern hardware.
- 3) **Simplified Architecture:** By combining the principles of previous SSM architectures with the MLP block of Transformers, the authors design a unified, homogenous block for the Mamba architecture. This streamlined design effectively integrates selective state spaces while maintaining simplicity and scalability.

1.1. Mamba Architecture

The Mamba architecture builds on Selective State Space Models (SSMs) with the following architectural innovations: enhanced selection strategies, hardware-aware computational algorithms, and a simplified yet powerful block design. These advancements enable Mamba to efficiently handle long sequences with linear scaling, achieving performance on par with or surpassing Transformers.

1.1.1. Selection strategy. To improve the selection strategy, Mamba transitions SSMs from time-invariant to time-varying dynamics. This is achieved by making parameters such as B , C , Δ input-dependent, allowing the model to selectively propagate or ignore information along the sequence based on its content. This is presented in Algorithm 2 in the paper [4]. This content-aware parameterization empowers the model to dynamically focus on relevant inputs while filtering out irrelevant ones, improving its ability to handle tasks requiring context sensitivity and reasoning. Such selective capabilities are particularly effective for tasks like selective copying and induction heads, where traditional SSMs face limitations.

1.1.2. Hardware-aware algorithm. The introduction of input-dependent parameters posed computational challenges, addressed by Mamba through a hardware-aware algorithm designed to optimize execution on modern GPUs. Instead of using convolutions, Mamba employs a parallel scan operation to compute recurrently while avoiding materializing large intermediate states in GPU memory. This approach incorporates kernel fusion to reduce memory IO, leverages faster memory levels (e.g., SRAM over HBM), and uses recomputation techniques to minimize memory requirements during backpropagation. These optimizations ensure that Mamba achieves true linear scaling in sequence length and demonstrates up to $3\times$ faster performance compared to previous SSM implementations on hardware such as the NVIDIA A100 GPU.

Together these two innovations create selective SSM which can be used like self-attention to create Mamba block shown in Figure 1.

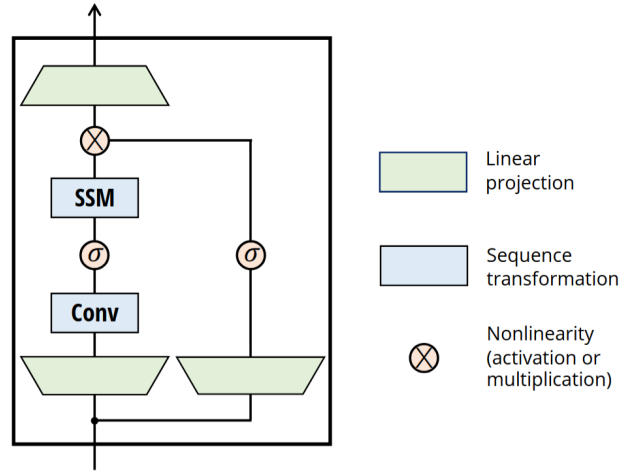


Figure 1. Mamba Architecture[3]

1.2. Future Improvements with Mamba

The Mamba architecture opens new avenues for enhancements by leveraging emerging techniques developed for Transformer-based models. Through the connection between State Space Models (SSMs) and attention mechanisms established in the SSD framework (Mamba-2) [2], researchers can adapt and extend Transformer innovations to Mamba, paving the way for significant advancements in its application and utility. Two key areas for future exploration include parameter-efficient fine-tuning and mitigation of catastrophic forgetting.

Parameter-Efficient Fine-Tuning. As large foundation models scale to billions of parameters, fine-tuning them for specific downstream tasks becomes computationally expensive and resource-intensive. Techniques like Low-Rank Adaptation (LoRA) and Adapters have emerged as efficient alternatives, enabling rapid and cost-effective fine-tuning by adjusting only a small subset of model parameters. Adopting these techniques for Mamba models could significantly broaden their usability across diverse domains. For instance, LoRA can facilitate efficient fine-tuning for SSD models, reducing computational overhead while maintaining performance. However, the integration of PEFT techniques into Mamba-based models remains an open area for further investigation.

Mitigation of Catastrophic Forgetting. Catastrophic forgetting—where a model loses performance on previously learned tasks when exposed to new ones—poses a challenge for foundation models, including Mamba. Addressing this issue is essential for ensuring Mamba's reliability across pretraining and downstream tasks. Emerging solutions, such as Reward Maximization and Distribution Matching strategies, as well as continual learning techniques developed for Transformers, offer promising approaches to mitigate forgetting in Mamba models. By connecting SSMs and attention, these techniques could be adapted for Mamba, but their application requires further exploration to realize their full potential.

1.3. MambaVision

Mamba is excellent for sequence modeling tasks such as language processing, audio, and genomics, however it faces challenges when applied to vision tasks. This is due to its autoregressive nature, which limits its ability to efficiently capture global spatial dependencies. Vision tasks require simultaneous understanding of both local pixel-level interactions and global spatial relationships, which are not inherently modeled in Mamba's sequential SSM framework.

To address these limitations, MambaVision [6] introduces a hybrid architecture that combines the efficiency of Mamba's SSMs with the global modeling capabilities of Transformers. By retaining Mamba's core principles and incorporating key modifications, such as convolutional layers for local feature extraction and Transformer blocks for capturing long-range spatial dependencies, MambaVision is specifically designed to meet the demands of vision applications effectively.

Key innovations in MambaVision are:

Hybrid Architecture.

- MambaVision employs a multi-resolution hierarchical structure with four stages:
 - Stages 1 and 2 use convolutional layers for fast local feature extraction at high resolutions.
 - Stages 3 and 4 combine MambaVision Mixer blocks (based on SSMs) with Transformer blocks to capture long-range dependencies.
- This design ensures both local and global spatial relationships are effectively modeled.

Redesigned Mamba Blocks.

- MambaVision replaces causal convolutions with regular convolutions to remove directional constraints unsuitable for spatial data.
- A symmetric branch without SSMs is added to enhance global context modeling, with outputs from both branches concatenated for richer feature representation.

Transformer Integration.

- Self-attention blocks are incorporated in the final layers of the architecture, enabling the model to recover lost global context and better capture long-range spatial dependencies.

The resultant MambaVision architecture is depicted in Figure 2.

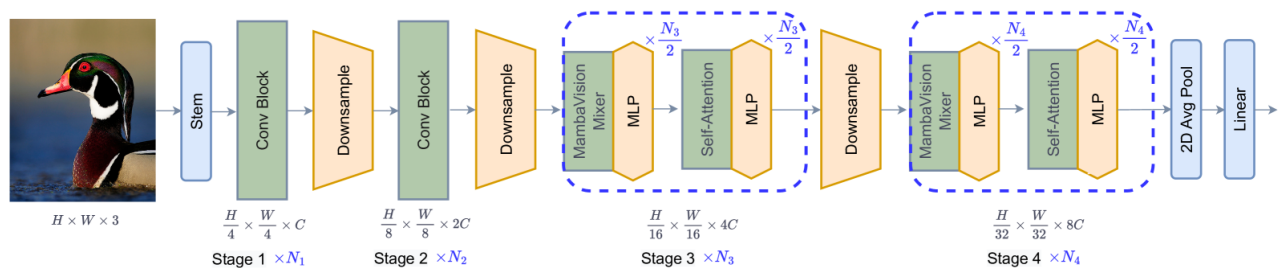


Figure 2. Mambavision Architecture[6]

MambaVision achieves state-of-the-art performance across various vision tasks by effectively balancing accuracy and computational efficiency. In image classification, it achieves optimum tradeoff for Top-1 accuracy and throughput on the ImageNet-1K dataset, outperforming both Transformer-based and convolutional backbones. Additionally, in object detection and segmentation tasks, MambaVision surpasses comparably sized models on benchmarks such as MS COCO and ADE20K, demonstrating superior performance in instance segmentation and semantic segmentation, thereby validating its versatility and efficiency as a vision backbone.

1.3.1. Experiment.

To evaluate the adaptability and performance of MambaVision in vision tasks, an experiment was conducted by fine-tuning the MambaVision model and a standard Vision Transformer (ViT) on the CIFAR-100 dataset [9] shown in Figure 3. This dataset, comprising 100 classes of diverse object categories, serves as an ideal benchmark for assessing a model's capability to handle fine-grained image classification. The experiment aimed to compare the effectiveness of the hybrid MambaVision architecture against the established ViT framework in terms of classification accuracy and computational efficiency, providing insights into their relative strengths in processing high-dimensional visual data.

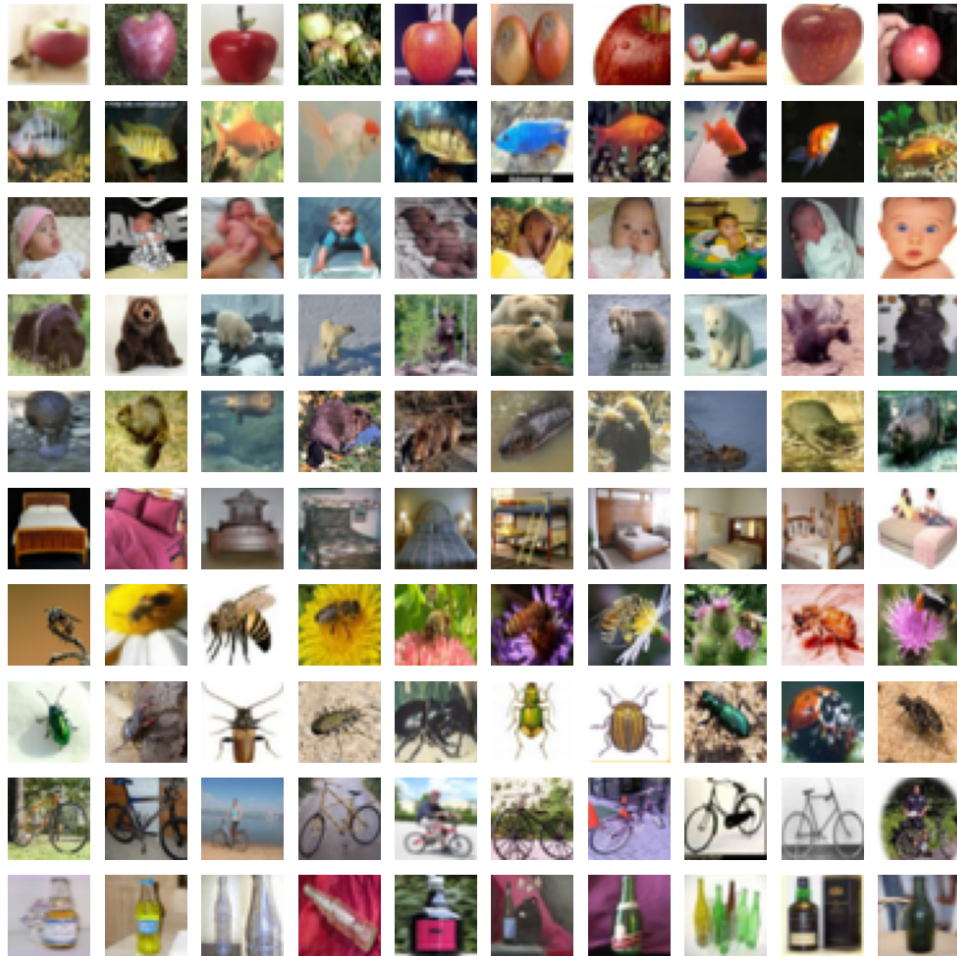


Figure 3. Sample of CIFAR100 Dataset[9]

For this experiment, the **google/vit-base-patch16-224** [12] model with 86.6 million parameters and the **nvidia/MambaVision-B-1K** [6] model with 97.7 million parameters were fine-tuned for the selected dataset. Since both models were originally trained on ImageNet-1k data with 1000 image classes, slight adaptations were made to fine-tune them on CIFAR-100, which contains only 100 image classes. The model selection ensures a fair comparison, given their similar scale. The fine-tuning code can be found in the supplementary GitHub repository.

Figure 4 shows the training and test accuracies for the fine-tuning stages of both models. ViT slightly outperforms in terms of both training and test accuracy, reflecting its status as a leading solution for image classification problems due to its Transformer architecture. However, MambaVision’s accuracy is very close, indicating strong performance as well

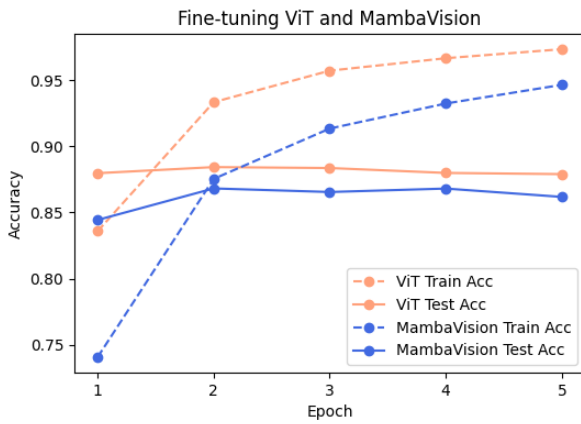


Figure 4. Fine-tuning ViT and MambaVision for CIFAR100

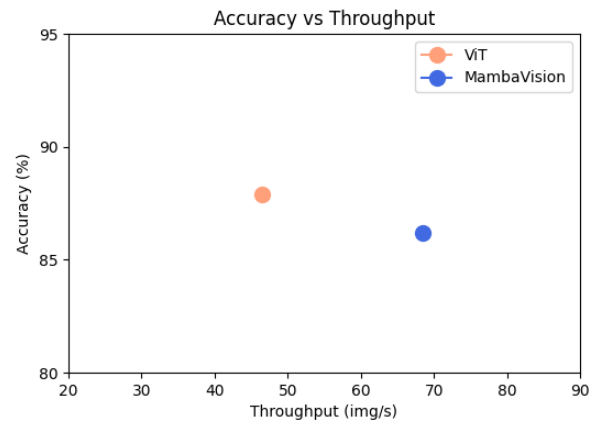


Figure 5. Inference Accuracy and Throughput for fine-tuned models

However, the advantages of MambaVision become apparent when examining the results during the inference stage. For this purpose, inference was profiled using the CIFAR-100 test dataset. Figure 5 compares the accuracy and image throughput during inference. As shown in the figure, MambaVision achieves approximately 85% test accuracy while maintaining a throughput of 68 images/sec. In contrast, ViT achieves around 87% test accuracy but has a lower image throughput of only 45 images/sec. The tests were conducted using an Nvidia 4090 GPU.

Despite the slightly larger size of the MambaVision base model compared to the ViT base model, it achieves higher throughput while maintaining comparable test accuracies, highlighting the advantages of the Mamba Architecture.

1.4. Parameter-Efficient Fine Tuning for Mamba - LoRa

LoRA (Low-Rank Adaptation) is a technique for parameter-efficient fine-tuning (PEFT) of pre-trained models, originally designed for Transformers [7]. Instead of updating all parameters during fine-tuning, LoRA inserts low-rank matrices into the model, which are specifically trained for the target task while keeping the original model parameters frozen. This reduces the number of trainable parameters, making the fine-tuning process efficient in terms of both memory and computation. Related work such as MambaPEFT: Exploring Parameter-Efficient Fine-Tuning for Mamba [13] and Mamba State-Space Models Can Be Strong Downstream Learners [5] have demonstrated how parameter-efficient fine-tuning methods can be adapted for Mamba's state-space models, offering insights into effective strategies for leveraging these architectures in downstream tasks.

LoRA modifies the linear layers of the model by introducing additional low-rank decomposition matrices. For a weight matrix W in a linear layer, LoRA reparameterizes it as:

$$W' = W + BA \quad (1)$$

Here:

- A and B are learnable low-rank matrices (e.g., $A \in \mathbb{R}^{d \times r}$, $B \in \mathbb{R}^{r \times d}$, where r is much smaller than d).
- W remains frozen during fine-tuning, ensuring efficient adaptation with minimal task-specific parameters.

Following modifications to different layers of Mamba can be made to implement LoRA:

State Transition Layers.

- Mamba's state transition layers manage the flow of information between time steps, applying linear transformations to evolve the hidden states. LoRA can be used here to modify the transition matrix without needing to retrain the entire model. This allows the model to adapt its dynamics to specific tasks such as time-series prediction or sequence-to-sequence translation
- *Example* The transition matrix W can reparameterized as per Equation 1.

Projection Layers.

- Projection layers in Mamba connect the internal state space to the output layer, effectively mapping the model's state to predictions. These layers can benefit from LoRA by adapting their transformation parameters. For example, LoRA can be applied to projectors to adjust feature representations for visual tasks (like MambaVision) or to improve prediction quality in text-based applications.
- *Example* LoRA can introduce task-specific adaptivity by reparameterizing the projection matrices used for feature extraction and classification, such as adjusting convolutional or linear layers to enhance performance in object detection or segmentation.

Embedding Layers.

- If Mamba uses embedding layers (e.g., for sequence input), LoRA can be applied to these as well, allowing for domain-specific fine-tuning. This is particularly useful for text tasks where

vocabulary sizes vary or where specific embeddings need to be fine-tuned for the target domain (e.g., medical, financial, or technical language).

- *Example* LoRA can modify embedding matrices to allow for vocabulary adaptation, for instance, mapping the state space representations to embeddings that better capture the semantics of domain-specific data.

Figure 6 shows how Mamba can be modified for LoRA based on the proposed changes, incorporating low-rank matrices into key components to enhance task-specific fine-tuning efficiency,

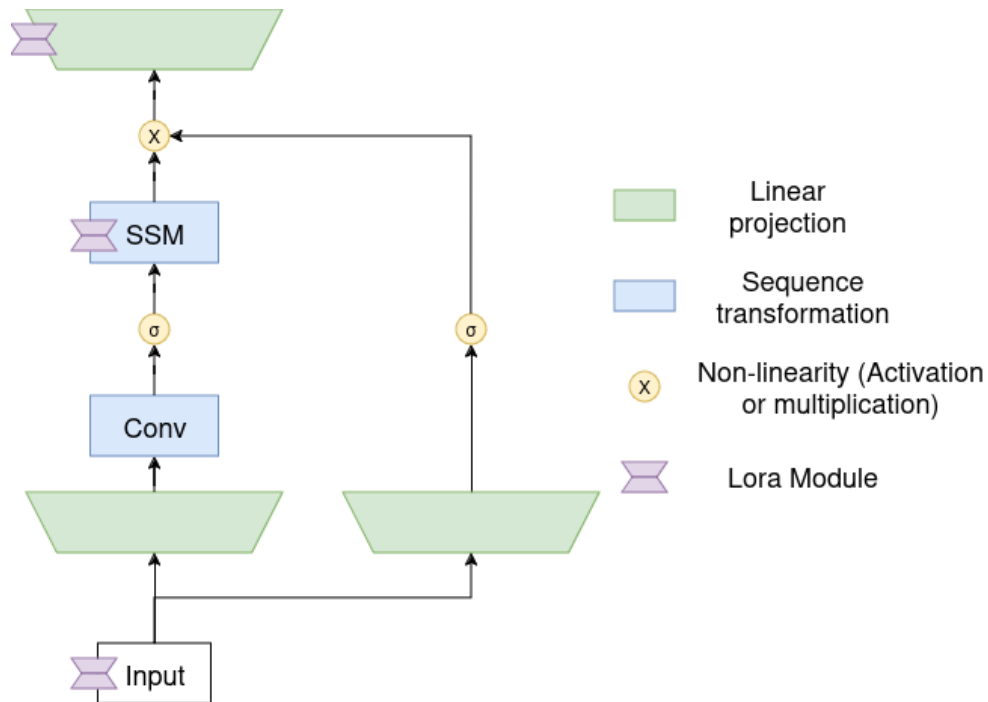


Figure 6. Proposed Parameter-Efficient Fine Tuning for Mamba using LoRA

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