Revisiting Classical Problems with Quantum Annealing

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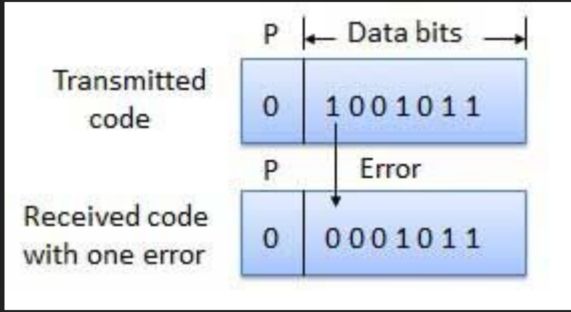
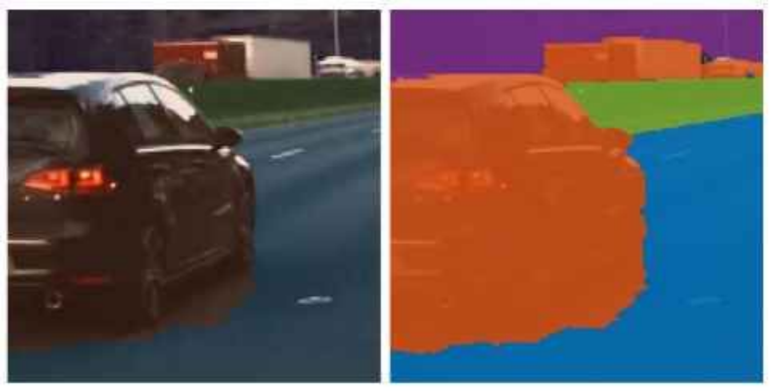
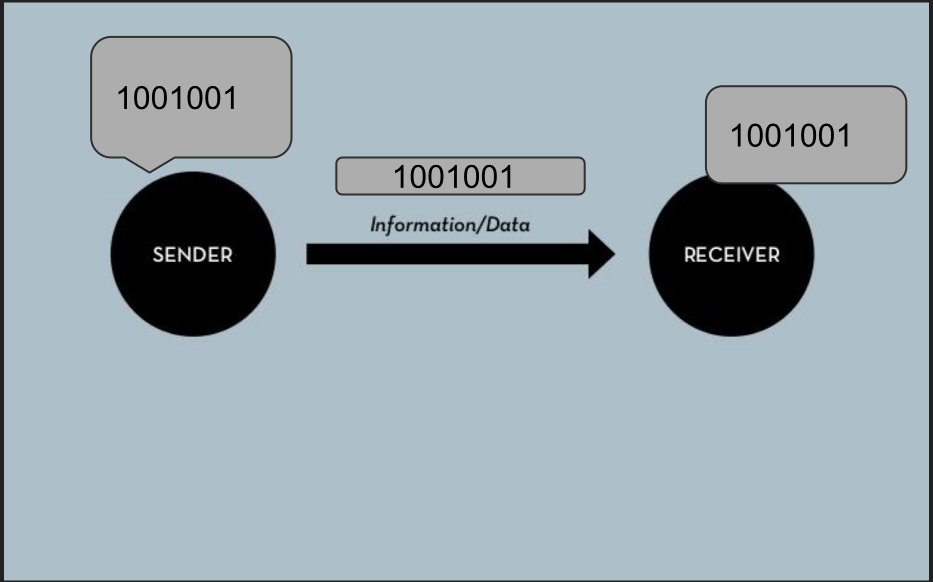
Problems

- Finding optimal coding for error detection
- Image segmentation





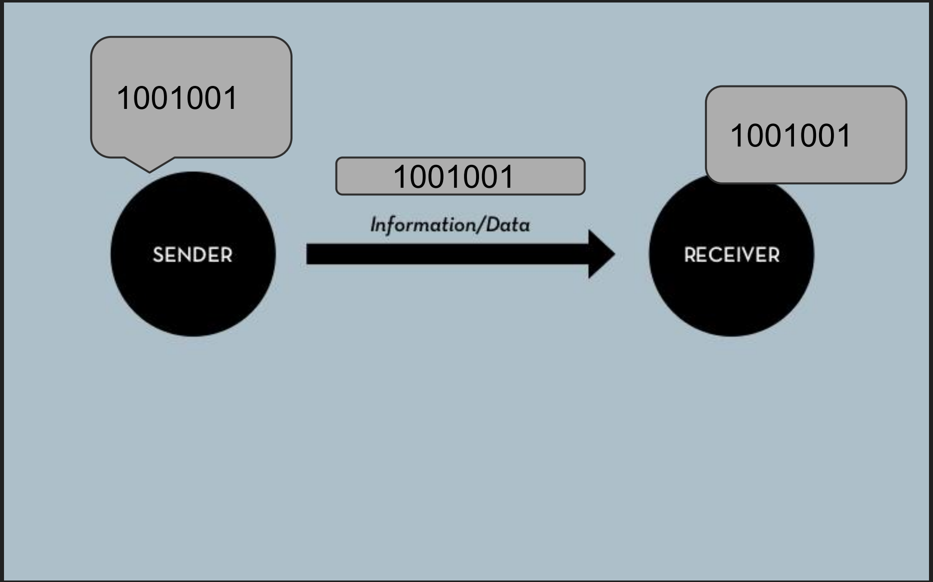
Problem



1001001
1001001
1001001
Information/Data
SENDER
RECEIVER

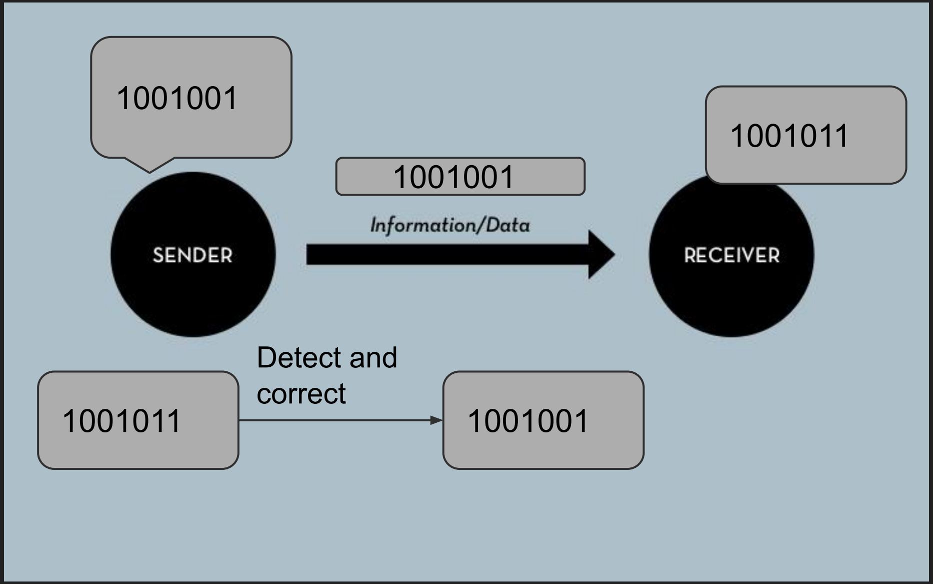
Problem

- Error E exist in our channel



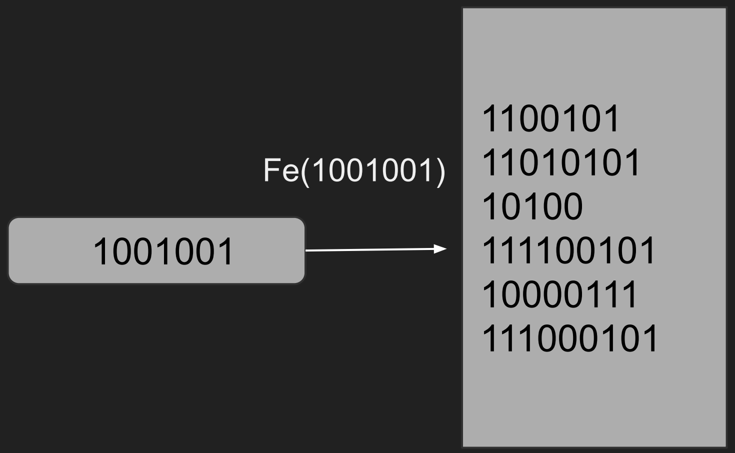
Problem

- Error E exist in our channel
- Receiver should be able to detect and correct the error



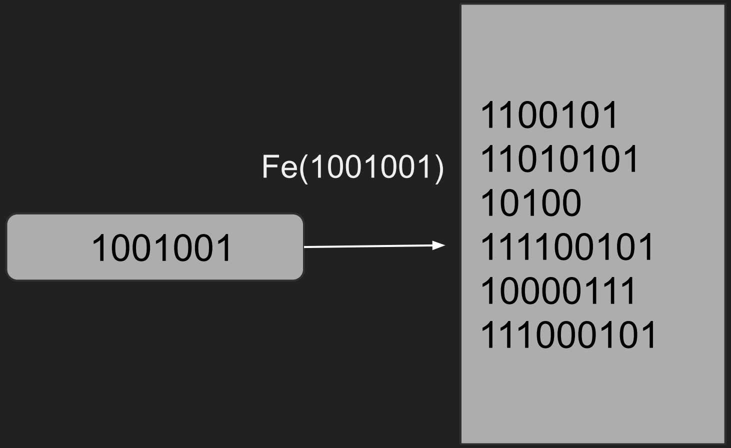
Problem

- We can look at error E as a function Fe(x) which map each code to set of codes.



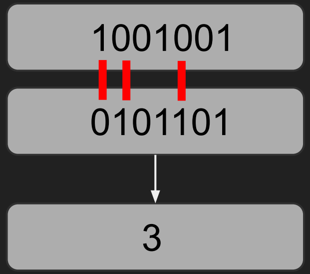
Problem

- We can look at error E as a function Fe(x) which map each code to set of codes.
- In this project we consider E as all the codes with max hamming distance of 1.
- $Fe(x) = H_1(x)$



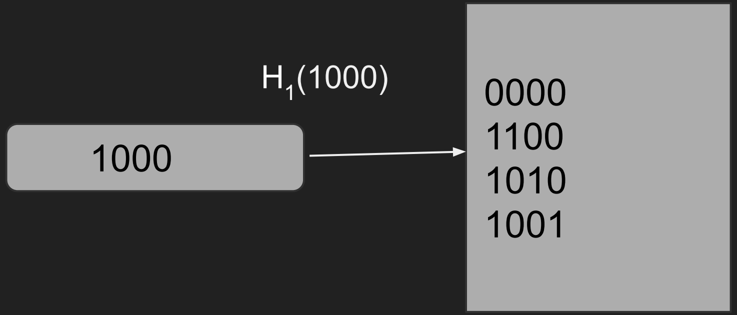
Problem

- $Fe(x) = H_1(x)$
- Hamming distance of x_1, x_2 is sum of all bits that x_1 and x_2 are different.
- $H(x_1, x_2) =$ count_ones(XOR(x_1, x_2))



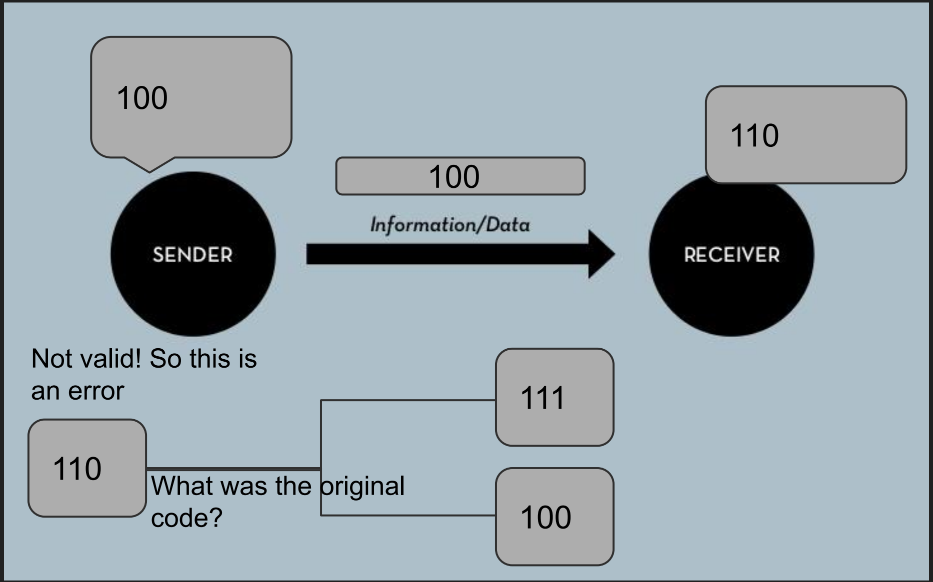
Problem

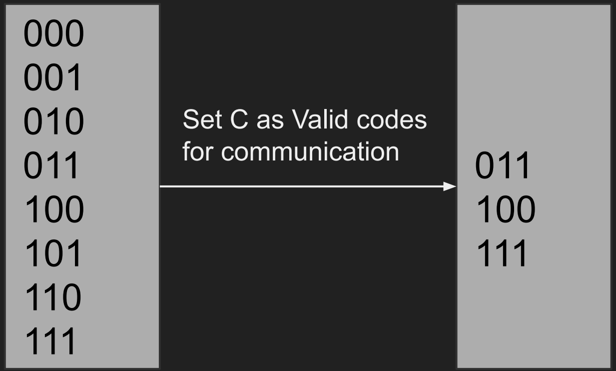
- $Fe(x) = H_1(x)$
- E is all codes with max hamming distance of 1.
- OR 1 bit can change during transmission.



Problem

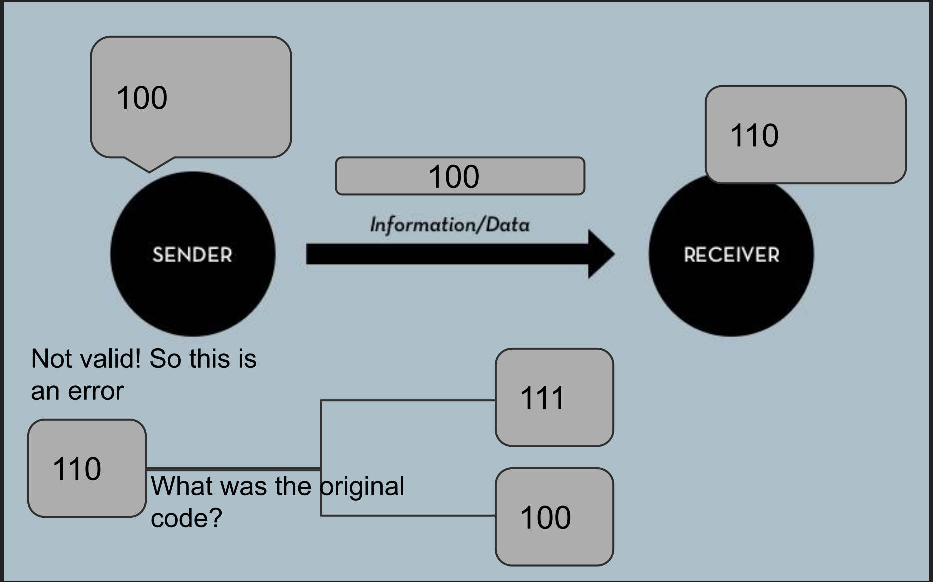
- We can't use all codes for communication. Why?

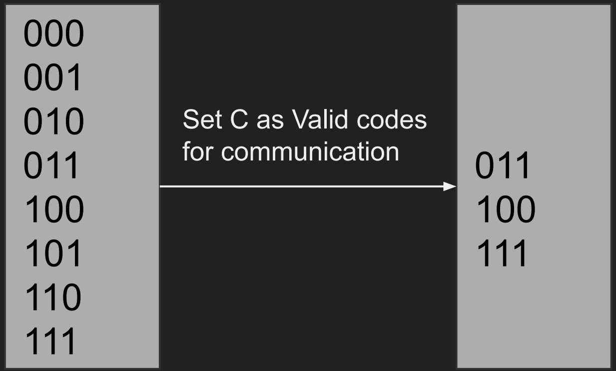




Problem

- We can’t use all codes for communication. Why?
- We need to find maximum number of codes we can use for communication.





Problem

- Finding optimal coding for error detection
- We have B as all binary codes with length n
- Select maximum number of codes of set B such that receiver can detect and correct all error codes.
- e can be any kind of error like deleting bits, flipping bits, ...

Method

- For each code in set B we consider a qubit in QUBO model.
- We select code c_i if and only if $q_i = 1$

Method

- For every two codes like x_1, x_2 -> intersection of $Fe(x_1)$ and $Fe(x_2)$ should be empty
- We should prevent solver to select both x_1, x_2 if $Fe(x_1)$ have intersection with $Fe(x_2)$
- For all x_1, x_2 which intersection of $Fe(x_1)$ and $Fe(x_1)$ is not empty, we set $J_{x1,x2}$ = |B|
- All h_i coefficients should be -1 because we want to find maximum set.

$$\forall x_1, x_2$$

$$F_e(x_1) \cap F_e(x_2) \neq \emptyset \Rightarrow$$

$$J_{x_1,x_2} = |B|$$

Results

$n = 3:$

Solver	Result	Energy
Exact Solver	0:1,1:0,2:0,3:0,4:0,5:0, 6:0,7:1	-2
Tabu	0:1,1:0,2:0,3:0,4:0,5:0, 6:0,7:1	-2
Simulated Annealing	0:1,1:0,2:0,3:0,4:0,5:0, 6:0,7:1	-2
Dwave physical	0:1,1:0,2:0,3:0,4:0,5:0, 6:0,7:1	-2

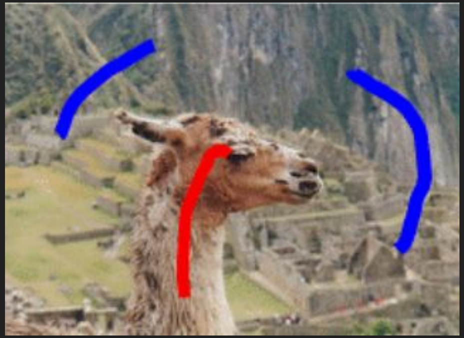
$n = 5:$

Solver	Result(set C)	Energy
Exact Solver	FAILED	-
Tabu	0,14,19,29	-4
Simulated Annealing	6,13,19,24	-4
Dwave physical	0,7,26,29	-4

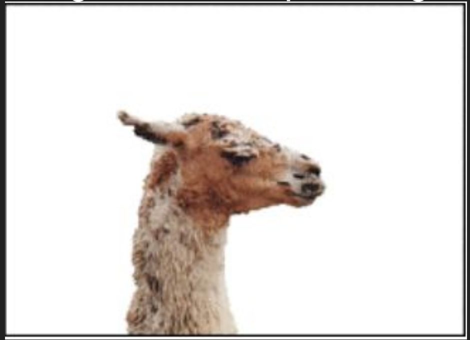
Image Segmentation with Graph-Cut

- This segmentation technique was proposed by **Boycov and Jolli** in their paper titled "**Interactive Graph Cuts for Optimal Boundary & Region Segmentation of Objects in N-D Images**".
- Image segmentation is one problem in Computer Vision that can and has been formulated in terms of energy minimization.
- Quantum Annealing is the best way to solve energy minimization problems
- Energy minimization is approximated by solving maximum flow problem in graph.

Scribbled Input Image

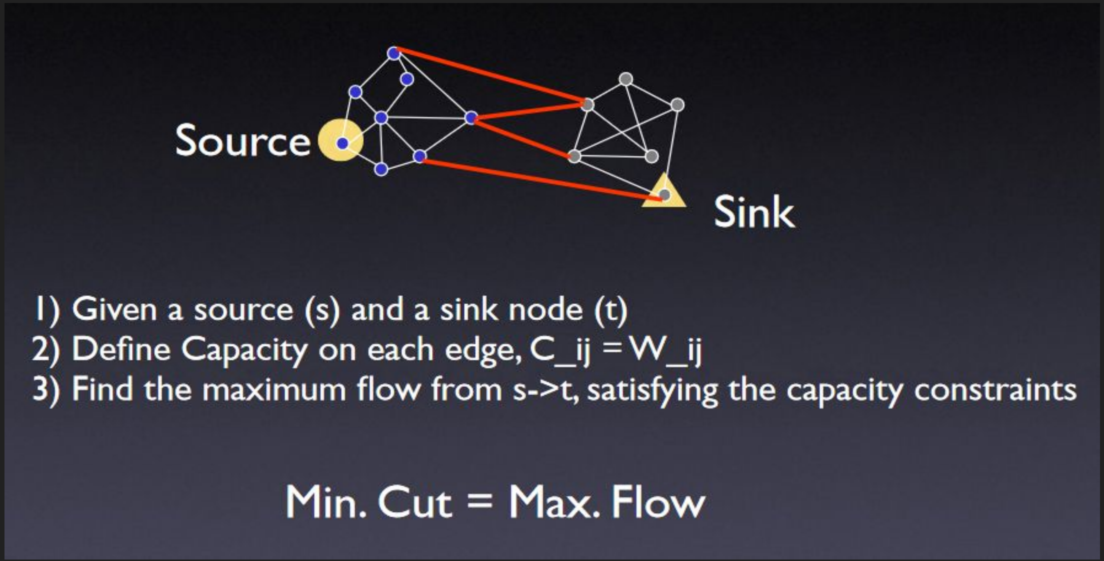


Segmented Output Image



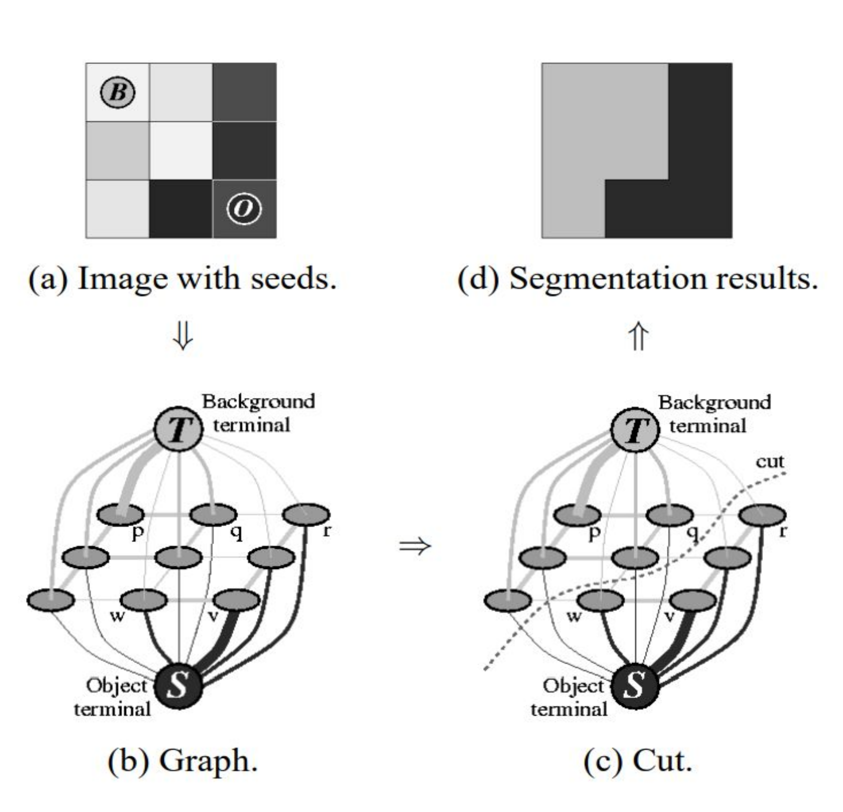
Graph-Cut Algorithm

- Minimization is done using a standard minimum cut algorithm.
- The Max Flow problem consists of a directed graph with edges labeled with capacities, and there are two distinct nodes: the source and the sink.



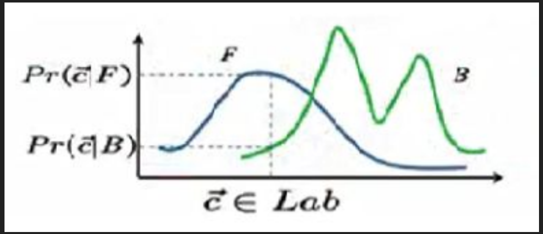
Quantum Annealing and Max-Flow/Min-Cut

- Objective function of Max-cut, the total edge weights between the two parts.
- Converting to mincut algorithm: just reverse the signs of the weights in the graph
- $H=-\sum_{(i,j)\text{ is an edge}} d_{i,j}(x_i-x_j)^2$
- We will have two sets of qubits assigns with value 1 or -1 which basically splits the total vertex in two sets foreground and background



Metrics for the Quantum Annealing Model

1. Estimation of Feature Distribution
   a. The color distribution of respective pixels in the foreground and background are calculated
   b. Each pixel is then assigned a probability of belonging to either the foreground or background from respective distributions
2. Inter-pixel edge weights for the graph are set by the intensity values as given beside.
3. To convert the maxcut to the mincut problem, it suffices to make the original edge weights negative.



$$w(a,s) = \lambda \log(Pr(I_a|\mathcal{O})),$$

$$w(a,b) = \exp\left(-\frac{(I_a - I_b)^2}{2\sigma^2}\right)$$

Interpreting the Annealing result

- Once Quantum Annealing is done, we get the state of each qubit, i.e. pixel as either -1 or 1.
- That means a pixel either belongs to the background or foreground.
- This allows us to map -1 to 0 and 1 to 1, for creating a mask that separates the foreground and background.

Calculate PDFs

Perform QA

Qubit results reshaped to image dimensions

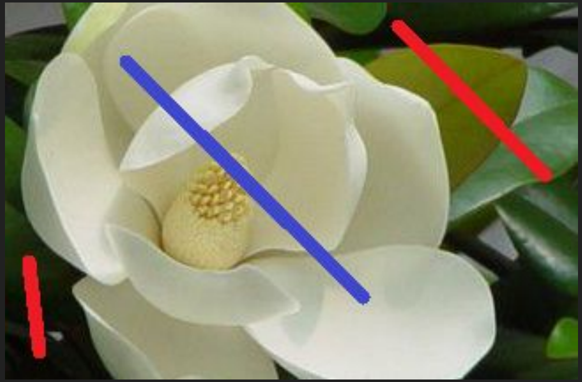
```
[[-1, -1, -1, ..., -1, -1, -1],
 [-1, -1, -1, ..., -1, -1, -1],
 [-1, -1, -1, ..., -1, -1, -1],
 ...,
 [-1, -1, -1, ..., -1, -1, -1],
 [-1, -1, -1, ..., -1, -1, -1],
 [-1, -1, -1, ..., -1, -1, -1]]
```

Mapping qubit state to grayscale

Image Segmentation mask

Applying The Mask

Results



Results/Conclusion

- Larger images don’t scale inversely to image segmentation relative to classical computing.
- Quantum hardware can handle small images but needs help with mitigating noise.

Challenges

- Reading the right pixel values and finding the scribbled vs non-scribbled pixels to map to the graph-cut algorithm
- Extracting pixel values from the Gaussian Mixture Model.
- Experimenting with the relevancy of Sink and Source nodes.
- Calculating the correct coupling between pixels in their qubit form for J.

Future Scope/Work

- Creating an interactive user input.
- Multi qubit error correction. Normalized Graph Cut.
- Building an API to provide this as a service if future quantum hardware is capable of accommodating larger images.

Thank You!

Open to Questions on Piazza and here.
References will be on Piazza post.