Faculty of Engineering & Technology

Fourth Semester B.E. (Computer Technology)/C.S.E./ I.T./C.E. (C.B.S.) Examination

DISCRETE MATHEMATICS GRAPH THEORY

Time—Three Hours] [Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Solve **SIX** questions as follows :

Que. No. – 1 **OR** Que. No. – 2

Que. No. – 3 **OR** Que. No. – 4

Que. No. – 5 **OR** Que. No. – 6

Que. No. – 7 **OR** Que. No. – 8

Que. No. – 9 **OR** Que. No. – 10

Que. No. – 11 **OR** Que. No. – 12

(3) Use of Non-programmable calculator is permitted.

(4) Illustrate your answers wherever necessary with the help of neat sketches.

1. (a) Prove by mathematical induction method that the sum of the cubes of three consecutive integers are divisible by nine. 4

(b) Prove the logical equivalence by using algebra of proposition :

(i) $p \wedge (\sim p \vee q) \equiv p \wedge q$

(ii) $\sim (p \rightarrow q) \equiv p \wedge \sim q$ 3

(c) Prove that $A-(B \cup C) = (A - B) \cap (A - C)$. 3

OR

2. (a) Is the following argument valid ?

If I study, then I will not fail in mathematics.

If I do not play basketball, then I will study.

But if failed in mathematics

Therefore I played basketball. 6

(b) Write the contrapositive of each of the following statement :

(i) If I have enough money, then I will buy a car and I will buy a house.

(ii) If I have time and I am not too tired, then I will go to the store. 4

3. (a) If R be a relation in the set of integers Z defined by $R = \{(x, y) : x \in z, y \in z, (x - y)$ is divisible by $6\}$, then prove that R is an equivalence relation. 6

(b) Define transitive closure of a relation.

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3), (3, 4)\}$ be a relation on A. Find transitive closure of R and draw its digraph. 6

(c) List all possible functions from set $X = \{a, b, c\}$ to $Y = \{0, 1\}$. Indicate in each case whether the function is 1–1, onto and 1–1 onto. 6

OR

4. (a) Define characteristic function. Using properties of characteristic function prove that :

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $A \cup (B \cup C) = (A \cup B) \cup C$

(iii) $(A^c)^c = A$ 6

(b) Let $A = \{a, b, c, d\}$ and e(A) be its power set. Let $\subseteq$ be the inclusion relation on the elements of e (A). Draw Hasse diagram of $(e(A), \subseteq)$. 6

(c) Draw tree diagram of $A \times B \times C$

where $A = \{2, 3\}$, $B = \{1, 3; 5\}$ and $C = \{3, 4\}$ and hence find $A \times B \times C$. 6

5. (a) Determine whether the set of integers Z, (Z, *) defined by $a * b = a + b - ab$ is a monoid. 6

(b) Show that the set $\{1, w, w^2\}$, where w is cube root of unity forms a group under multiplication. 6

OR

6. (a) Prove that : Iff is a homomorphism of a group G into a group G' with Kernel K, then K is a normal subgroup of G. 6

(b) Let G be the group of integers under addition and let H be the set of all integral multiples of 3. Prove that H is a subgroup of G and determine all cosets of H in G. 6

7. (a) If R is a ring such that $a^2 = a \forall a \in R$. Prove that :

(i) $a + a = 0, \forall a \in R$

(ii) $a + b = 0 \Rightarrow a = b$

(iii) R is a Commutative ring. 6

(b) Show that the ring of real numbers (R, +, .) is an integral domain. 6

OR

8. (a) Show that there are 5 partitions of a set of three elements. Draw the diagram of the corresponding lattice. 6

(b) Construct switching circuit for the Boolean expression $(A.B) + [A'.(A + B + B')]$. Simplify this and construct an equivalent simplified circuit. 6

9. (a) Define :

(i) Directed graph

(ii) Simple path and elementary path

(iii) Euler path

(iv) Connected graph

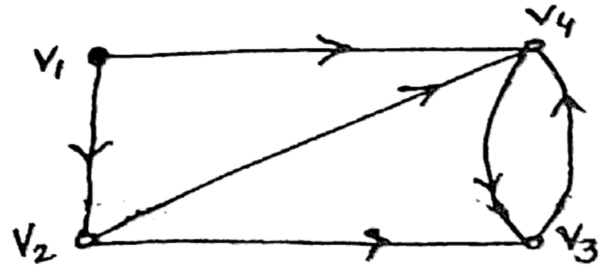
(v) Diameter of graph. 6

(b) Draw the diagraphs corresponding to adjacency matrices :

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

and prove that diagraphs of A and B are isomorphic. 6

(c) Give three different elementary paths from V_1 to V_3 for the diagraph given below. What is the shortest distance between V_1 and V_3 ? Is there any cycle in the graph ? Is the diagraph transitive ? 6



OR

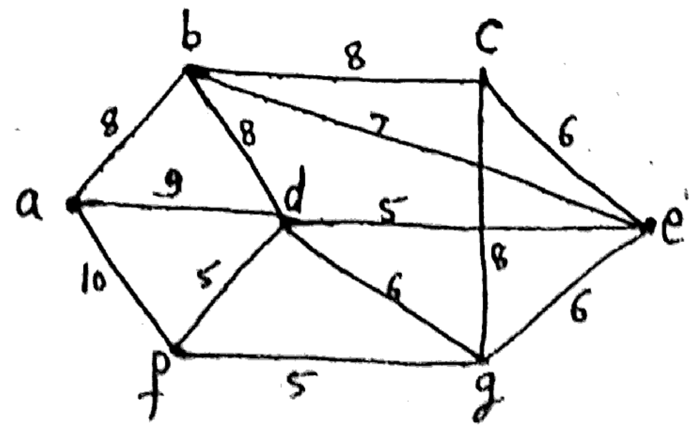
10. (a) Draw tree representation for the tree given by :

$A = \{1, 2, 3, 4, 5, 6, 7\}$;

$R = \{(1, 2), (1, 3), (1, 4), (2, 5), (4, 6), (4, 7)\}$

and draw corresponding binary tree. 6

(b) Apply Prim's algorithm to construct a minimal spanning tree for the weighted graph given below : 6

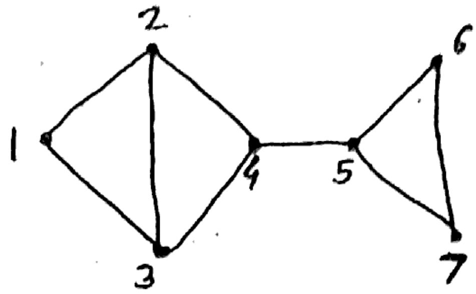


(c) Find :

(i) Diameter

(ii) Radius and

(iii) Centre of the graph given below :



Is there any bridge ? 6

11. (a) Find a general solution for :

$$a_r + a_{r-1} = (3r)2^r.$$ 5

(b) Show that if any five numbers from 1 to 8 are chosen then two of them add up to 9 using pigeon hole principle. 5

OR

12. (a) Find the generating function for the sequence :

$$1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n+1}, \ldots\ldots$$ 5

(b) Prove that :

$$C(n, r) = C(n-1, r-1) + C(n-1, r)$$ 5