B.E. (Computer Science Engineering / Computer Technology / Computer Engineering / Information Technology) Fourth Semester (C.B.S.)

Discrete Mathematics & Graph Theory

P. Pages : 3 **NIR/KW/18/3373/3378/3383/3388**

Time : Three Hours *1310* Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Illustrate your answers whenever necessary with the help of neat sketches.
10. Use of non programmable calculator is permitted.

1. a) State and prove De Morgan's laws. **5**

b) Prove that $A \cap (B - C) = (A \cap B) - (A \cap C)$. **5**

OR

2. a) Test the validity of the following argument If I study, then I will not fail in Mathematics. **5**
If I do not play basketball, then I will study.
But I failed in Mathematics.
Therefore, I must have played basketball.

b) Prove by mathematical induction that **5**
$3^{4n+2} + 5^{2n+1}$ is a multiple of 14.

3. a) Draw the tree diagram of $(A \times B \times C)$ **6**
where $A = \{2, 3\}, B = \{1, 3, 5\}, C = \{3, 4\}$ and hence find $(A \times B \times C)$.

b) Let A be a given set and P (A) be its power set. Let $\le$ be the inclusion relation on P (A). Draw the Hasse diagram of $(P(A), \le)$ for $A = \{a, b\}$ and $A = \{a, b, c\}$. **6**

c) Using the properties of characteristic function, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **6**

OR

4. a) For the relation $R = \{(1,2), (2,1), (2,3), (3,4)\}$ find the relation matrix and draw the graph. Also find the transitive closure of R. **6**

b) Let X = {ball, bed, egg, dog, let} and $R = \{(x, y) \mid x \in X, x R y$ if x and y contain some common letter}. Draw the graph of R. Prove that R is compatible but not transitive. **6**

c) Let $f : A \to B$ and $g : B \to C$ such that f (a) = a – 1 and $g(b) = b^2$. Find (i) fog (x) (ii) gof (x) (iii) gog (x) (iv) fof (x). **6**

5. a) Prove that fourth roots of unity forms a group under multiplication. **6**

b) Prove that the order of each subgroup of a finite group is a divisor of the order of the group. **6**

OR

6. a) Prove that monoid homomorphism preserves associativity, identity, commutativity and invertibility. **6**

b) If f is a homomorphism of G into G' with kernel k, then prove that k is a normal subgroup of G. **6**

7. a) If R is a ring, then show that for all $a, b \in R$. **6**

i) $a \cdot 0 = 0 = 0 \cdot a$ ii) $a \cdot (-b) = (-a) \cdot b = -(a \cdot b)$.

iii) $(-a) \cdot (-b) = a \cdot b$.

b) Prove that every chain is a distributive lattice. **6**

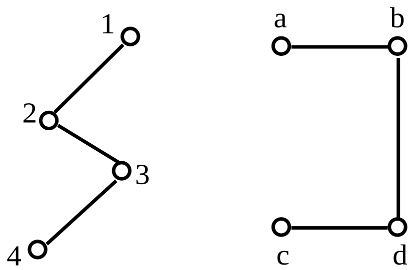
OR

8. a) If $f : R \to R'$ is a ring homomorphism, then prove that **6**

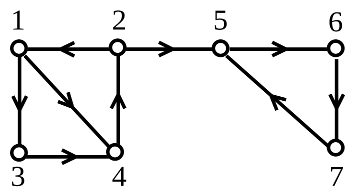
i) $f(0) = 0$ ii) $f(-a) = -f(a)$, for all $a \in R$.

b) Construct the switching circuit for the Boolean expression $(A \cdot B) + [A' \cdot (A + B + B')]$. Simplify this expression and also draw the equivalent switching circuit. **6**

9. a) Show that the following graphs are isomorphic. **6**



b) Find the node base of the following digraph. **6**



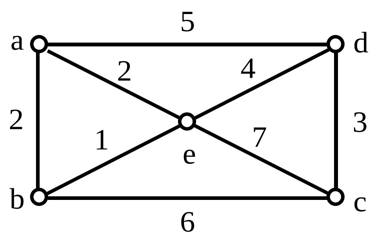
c) Draw a directed tree with two nodes at level 1, five nodes at level 2, three nodes at level 3. Obtain the corresponding binary tree. **6**

OR

10. a) Draw a digraph corresponding to the following adjacency matrix and interprete the results AA^T, A^TA, A^2. **6**

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

b) Find the minimal spanning tree of the following graph using Prim's algorithm. **6**



c) Represent the following algebraic expressions using binary tree. **6**

i) $(x+(y+z))-(a\,x(b+c))$.

ii) $(2x+(3-4x))+(x-(3x11))$.

11. a) Show that if five integers are chosen from 1 to 8, then two of them will have a sum 9. **5**

b) Solve the following recurrence relation using generating function $a_n = 3a_{n-1} + 2$ with $a_0 = 1$. **5**

OR

12. a) Find the generating function of $n^2, n \geq 0$. **5**

b) Prove that $C(n+1, r) = C(n, r) + C(n, r-1)$. **5**

****