Faculty of Engineering & Technology

Fourth Semester B.E. (Computer Technology/C.S.E./ I.T./C.E.) (C.B.S.) Examination

DISCRETE MATHEMATICS AND GRAPH THEORY

Time—Three Hours] [Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Due credit will be given to neatness and adequate dimensions.

(3) Solve **SIX** questions as follows :

Que. No. 1 **OR** Que. No. 2

Que. No. 3 **OR** Que. No. 4

Que. No. 5 **OR** Que. No. 6

Que. No. 7 **OR** Que. No. 8

Que. No. 9 **OR** Que. No. 10

Que. No. 11 **OR** Que. No. 12

1. (a) Prove that :

(i) $(A \cap B) - C = (A - C) \cap (B - C)$

(ii) $(A \cap B)' = A' \cup B'$. 5

(b) Show that $((p \vee \sim q) \wedge (\sim p \vee \sim q)) \vee q$ is a tautology. 5

OR

2. (a) In a group of students, 70 have a personal computer, 120 have personal stereo and 41 have both. How many own at least one of these devices ? Draw an appropriate Venn diagram also. 4

(b) Show that :

$$1^2 + 2^2 + 3^2 + ...n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 1.$$

6

3. (a) Define equivalence relation. Is the relation R defined on the set of positive integers such that aRb iff $a \leq b$ is an equivalence relation ? Explain if not.

6

(b) Let A = {1, 2, 3, 4, 5, 6}. Define a relation R on A such that R = {(x, y) | x + y is divisor of 24}.

Find :

(i) Relation matrix of R

(ii) Relation matrix of R∘R

(iii) Draw digraph of R and R∘R. 6

(c) Let $f : A \to B$ and $g : B \to C$ such that $f(a) = a - 1$ and $g(b) = b^2$. Find :

(i) $f\circ g(x)$

(ii) $g\circ f(x)$

(iii) $g\circ g(x)$

(iv) $f\circ f(x)$.

OR

4. (a) The relation R defined on set A = {0, 1, 2, 3} R = {(0, 1), (1, 2) (2, 3)}. Find transitive closure of R. 6

(b) Let A = {1, 2, 3}, B = {a, b, c, d}. Let R and S be relations from A to B with relation matrices given by :

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}, \quad M_S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Find :

(i) M_{R-1} and M_{S-1}

(ii) Show that $M_{(R \cap S) \circ R^{-1}} = M_{R \circ R^{-1}} \wedge M_{S \circ R^{-1}}$. 6

(c) Let $f : A \to B$ defined by $f(x) = 2x^3 - 1$. Prove that f is one-to-one and onto. 6

5. (a) Define Grupoid, Semigroup, Monoid and Group with examples. 6

(b) Show that the set $H = \{a + ib \in C | a^2 + b^2 = 1\}$ is a subgroup of (C, *), where * = multiplication of complex numbers. 6

OR

6. (a) Define Co-sets and Normal subgroup. Prove that a subgroup H of a group G is normal iff

$$g^{-1}hg \in H \;\forall\; h \in H \text{ and } g \in G. \qquad 6$$

(b) If $*$ is a binary operation in Q^+, defined by $a * b = \frac{ab}{3}, \forall\, a, b \in Q^+$, show that $(Q^+, *)$ are form an abelian group. 6

7. (a) If R is a set of all real numbers, then prove that (R, +, .) is an integral domain. 6

(b) Find the complements of every element of the lattice $(D_{20}, \leq)$. 6

OR

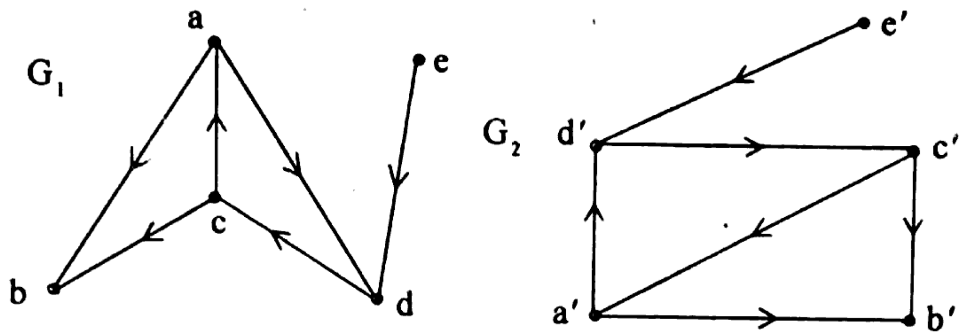
8. (a) Let $R = \left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix} : a, b \in z\right\}$ is a ring. If f be the mapping that takes $\begin{bmatrix} a & b \\ b & a \end{bmatrix} \to a - b$. Show that f is homomorphism. Also determine the Kernel of f. 6

(b) Draw the switching system circuit represented by the Boolean Expression :

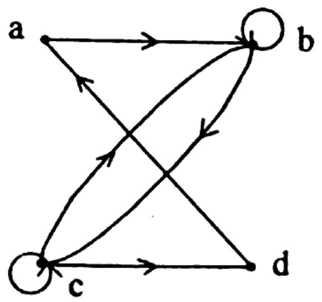
$$F = (A + B)(\overline{B} + C) + (\overline{C} + A)(C + B)$$

Simplify F and draw the simplified equivalent circuit. 6

9. (a) Define isomorphic graphs. Show that the following graphs are isomorphic. 6



(b) Find the adjacency matrix of the following graphs and its complement :



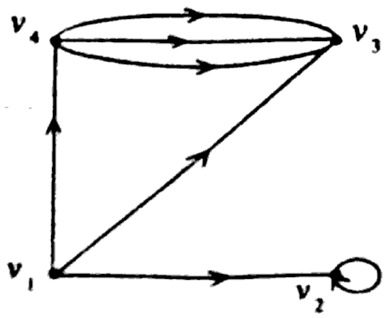
6

(c) Draw the tree for the algebraic expression :

$$(a + 5) * [\{(7 * b) + c\} / (g + d)].$$ 6

OR

10. (a) Define in-degree and out-degree of the graph. Find the in-degree and out-degree of each vertex of graph :



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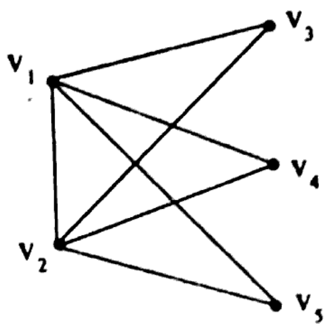
(b) Define Eulerian path and Eulerian circuit. Show that the graph given below is an Eulerian graph and circuit.



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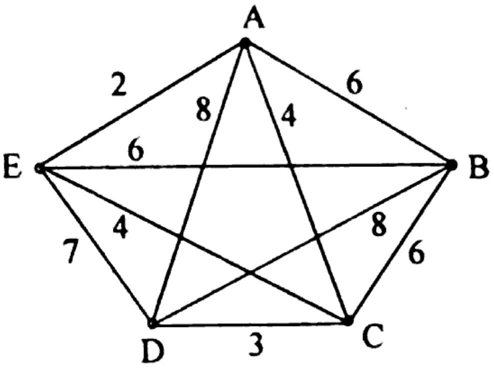
(c) Use Prim's algorithm to find minimal spanning tree for the graph :



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11. (a) Find the general solution of the recurrence relation :

$$2a_n - 7a_{n-1} + 3a_{n-2} = 2^n. \qquad 5$$

(b) Show that if any 30 people are selected, then one may choose a subset of 5 so that all five were born on the same day of the week. 5

OR

12. (a) Apply the generating function technique to solve the recurrence relation

$$a_{n+2} - 5a_{n+1} + 6a_n = 2, \quad a_0 = 1, \ a_1 = 2.$$ 5

(b) Define Pigeon–hole Principle. Show that if 9 books are to be kept in 4 shelves, there must be at least one shelf which contains at least 3 books. 5