



- Notes :
1. All questions carry marks as indicated.
 2. Solve Question 1 OR Questions No. 2.
 3. Solve Question 3 OR Questions No. 4.
 4. Solve Question 5 OR Questions No. 6.
 5. Solve Question 7 OR Questions No. 8.
 6. Solve Question 9 OR Questions No. 10.
 7. Solve Question 11 OR Questions No. 12.
 8. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = F(s)$ then show that 6

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s) ds$$

hence find $L\left\{\frac{\sin t}{t}\right\}$.

- b) Find $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$ by using convolution theorem. 6

OR

2. a) Express $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$ 6
in terms of unit step function and find Laplace transform.

- b) Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t} \sin t$ 6
given $y(0) = 0, y'(0) = 1$
by using Laplace transform method.

3. a) Find the Fourier series to represent 6
 $f(x) = x^2 - 2, -2 \leq x \leq 2$.

- b) Find Fourier sine transform of $\frac{e^{-ax}}{x}, a > 0$. 6

OR

4. a) Using the Fourier Cosine integral show that 6

$$\int_0^\infty \frac{\cos \lambda x}{1 + \lambda^2} d\lambda = \frac{\pi}{2} e^{-x}$$

- b) Find the half range cosine series for $\sin x$ when $0 < x < \pi$, hence deduce that 6

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}.$$

5. a) If $z \{f(n)\} = F(z)$ then show that 6

$$z \left\{ \frac{f(n)}{n+k} \right\} = z^k \int_z^\infty \frac{F(z)}{z^{k+1}} dz$$

hence find $z \left\{ \frac{1}{n+1} \right\}.$

- b) Prove that $\frac{1}{n!} * \frac{1}{n!} = \frac{2^n}{n!}$ 6

where $*$ is a convolution operation.

OR

6. a) Find Z-Transform of $\frac{(n+1)(n+2)}{2!} a^n.$ 6

- b) Solve $y_{n+2} - 2\cos\alpha \cdot y_{n+1} + y_n = 0$ given $y_0 = 0, y_1 = 1$ by using Z-Transform. 6

7. a) If $f(z)$ is analytic function with constant modulus. Show that $f(z)$ is constant. 7

- b) Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C is a circle $|z-i|=2$ by Cauchy Integral formula. 7

OR

8. a) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$ by using Contour Integration. 7

- b) Expand in Taylor's series $f(z) = \frac{z}{(z+1)(z+2)}$ about $Z = 2$. Also find the region of convergence. 7

9. a) Investigate the linear dependence of vectors 6
 $X_1 = (2, -1, 3, 2), X_2 = (1, 3, 4, 2), X_3 = (3, -5, 2, 2)$
 and if so find the relation.

- b) Find the modal matrix B corresponding to matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ and verify that $B^{-1}AB$ is diagonal form. 6

- c) By using Cayley Hamilton's theorem find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}.$ 6

OR

10. a) If $A = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$ verify $2\sin A = (\sin 2)A$ by Sylvester's theorem. 6
- b) Find the largest eigen value and corresponding eigen vector for the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ by iteration method. 6
- c) Solve $\frac{d^2x}{dt^2} + 4x = 0$, $x(0) = 1$, $x'(0) = 0$ by matrix method. 6
11. a) Each of the three identical Jewellery boxes has two drawers. In each drawer of the first box there is a gold watch. In each drawer of the second box there is a silver watch. In one of the drawer of the third box there is a gold watch while in the other there is silver watch. If we select a box at random, open one of the drawer and find it to contain a silver watch. What is the probability that the other drawer has gold watch. 6
- b) The distribution function of a random variable X is given by 6
- $$F(x) = \begin{cases} cx^3, & 0 \leq x < 3 \\ 1, & x \geq 3 \\ 0, & x < 0 \end{cases}$$
- Find :
- Probability density function
 - C
 - $p(x > 1)$

OR

12. a) A random variable X can assume the value 1 and -1 with probability $\frac{1}{2}$ each. 6
- Find (i) moment generating function (ii) first two moments about origin and about mean.
- b) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused. 6
