B.E. (Computer Science & Engineering (New) / Computer Technology)
Third Semester (C.B.S.)

Applied Mathematics – III Paper – I

P. Pages : 3 **TKN/KS/16/7320/7325**

Time : Three Hours Max. Marks : 80

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Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Illustrate your answers with the help of neat sketches.
10. Use of non-programmable calculator is permitted.

1. a) If $L\{f(t)\} = \bar{\lambda}(s)$, then prove that $L\left\{\int_o^t t(u)du\right\} = \frac{\bar{f}(s)}{s}$. Hence find the Laplace transform of $\int_o^t e^t \frac{\sin t}{t} dt$ **6**

b) Find $L^{-1}\left\{\log\left(1+\frac{1}{s^2}\right)\right\}$ and hence or otherwise show that **6**

$$L^{-1}\left\{\frac{1}{s}\log\left(1+\frac{1}{s^2}\right)\right\} = \int_o^t \frac{2}{x}(1-\cos x)\,dx$$

OR

2. a) Express $f(t) = \begin{cases} t^2, & o < t < 2 \\ 4t & t > 2 \end{cases}$ in terms of unit step function and find Laplace transform. **5**

b) Solve $\frac{dy}{dt} + 2y + \int_o^t y\,dt = \sin t$, given y(o) = 1 **7**

3. a) Using Fourier integral, show that $\int_o^t \frac{\sin \pi\lambda \sin \lambda x}{1-\lambda^2} d\lambda = \begin{cases} \frac{\pi}{2}\sin x, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$ **6**

b) Find the Fourier series for the function $f(x) = x^2 - 2$ for the internal (-2, 2). **6**

OR

4. a) Find the Fourier half-range. **6**
i) Sine series ii) Cosine series
for the function $f(x) = x^2$ in the internal (o, π).

b) Find the Fourier sine transform of $e^{-|x|}$ and hence show that

$$\int_{o}^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m} \quad ; m > 0$$ **6**

5. a) Prove that $Z\{n^p\} = -z \frac{d}{dz} Z\{n^{p-1}\}$ where p is any positive integer and hence find $z\{n\}$ and $Z\{n^2\}$. **6**

b) Find the Z-Transform inverse of $\frac{3z^2+2z+1}{z^2-3z+2}$. **6**

OR

6. a) Find the Z-Transform of sin (3n+5) and cos (3n+5) **6**

b) Solve using Z-Transform $y_{n+2} - 2\cos\alpha \ y_{n+1} + y_n = 0$ given $y_0 = 0$ and $y_1 = 1$. **6**

7. a) If f(z) is analytic function of z, then prove that **7**

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f^1(z)|^2$$

b) Evaluate using cauchy integral formula. $\int_c \frac{4-3z}{z(z-1)(z-2)} dz$. **7**

where c is the circle $|z| = 3/2$.

OR

8. a) Expand the function. **7**

$f(z) = \left[z^2 + 4z + 3\right]^{-1}$ by laurentz series valid for

i) $1 < |z| < 3$

ii) $0 < |z+1| < 2$

iii) $|z| < 1$

b) Evaluate by contour integration $\int_0^{2\pi} \frac{1}{1-2a\sin\theta + a^2} d\theta \quad o < a < 1$ **7**

9. a) Test the dependency of the vector **6**

$x_1 = (1, 1, -1, 1)$

$x_2 = (1, -1, 2, -1)$

$x_3 = (3, 1, 0, 1)$

and find the relation, if it exists.

b) Find the modal matrix B which reduces $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ to a diagonal form. **6**

c) Verify Cayley Hamilton Theorem and express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial of A, If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$. **6**

OR

10. a) Use Sylvester's theorem to show that. **6**

$$e^A = e^x \begin{bmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{bmatrix},$$

where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$

b) Solve $\dfrac{d^2y}{dt^2} + 4y = 0$, given $y = 8, \dfrac{dy}{dt} = 0$, when t = 0. **6**

c) Find the largest eigen value and corresponding eigen vector for the Matrix **6**

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}.$$

11. a) A random variable x has density function. **6**

$$f(x) = \begin{cases} kx^2, & 1 \le x \le 2 \\ kx, & 2 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find the constant K and the distribution function.

b) An aptitude test for selecting engineers in an industry is conducted on 100 candidates The average score is 42 and standard deviation of score is 2.4. Assuming normal. distribution for the score find. **6**

i) The number of candidates whose score is more than 60.

ii) The no. of candidates whose score lies between 30 and 60.

OR

12. a) Find The moment generating function for the random variable x having density function **6**

$$f(x) = \begin{cases} e^{-x}, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

and determine first four moment about origin.

b) let x be a random variable giving the number of aces in a random draw of four cards from a pack of 52 cards. Find the probability function and the distribution function for x. **6**

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