B.E. (Computer Science Engineering / Computer Technology / Computer Engineering / Information Technology) Fourth Semester (C.B.S.)

Discrete Mathematics & Graph Theory

P. Pages : 3 NJR/KS/18/4428/4433/4438/4443

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Illustrate your answers whenever necessary with the help of neat sketches.

1. a) Prove that **4**

$A-(B\cap C)=(A-B)\cup(A-C).$

b) Write inverse, contrapositive and converse of the statement : **3**
"Oxygen in air decreases as you go up"

c) Write the negation of the following statements: **3**
i) He is tall but handsome.
ii) If she work, she will earn money and
iii) If he studies, he will go to college or to art school.

OR

2. a) Prove by method of induction. **3**

$1+2^n<3^n$, for $n\geq 2$.

b) Test the validity of the following statement: **4**
"If I like discrete mathematics, then I will study. Either I study discrete mathematics or I failed the course. Therefore, if I fail the course, then I don't like Discrete mathematics".

c) Prove that **3**

$A\times(B\cap C)=(A\times B)\cap(A\times C).$

3. a) Let A be the set of non-zero integers and let R be the relation on A x A defined by **6**
$(a,b)R(c,d)\Leftrightarrow ad=bc.$
Show that R is an equivalence relation.

b) Let $A=\{a,b,c\}$ and P (A) be its power set. Let "$\subseteq$" be the relation defined on P (A). Draw Hasse diagram of the Poset $(P(A),\subseteq)$. **6**

c) If $f:X\to Y$ and $g:Y\to Z$ are one-one and onto functions, there show that gof is also one-one and onto and $(gof)^{-1}=f^{-1}og^{-1}$. **6**

OR

4\. a) Let X = {ball, bed, dog, egg, let} and **6**
R = {(x,y) | x, y $\in$ X, x R y, if x and y contain some common letter}.
Write M_R. Draw the graph of relation R prove that R is compatible but not transitive.

b) Prove that **6**
i) $f_{A \cap B} = f_A \cdot f_B$ and
ii) $f_{A \cup B} = f_A + f_B - f_A \cdot f_B$,
where f is the characteristic function.

c) List all possible functions from the set $X = \{a, b, c\}$ to the set $Y = \{0,1\}$. Indicate in each case whether the function is one-one or onto or both. **6**

5\. a) Show that the fourth roots of unity forms an abelian group with respect to multiplication. **6**

b) Determine whether the set of even integers with binary operation $*$ defined by $a * b = \frac{ab}{2}$ is semigroup or monoid. Show whether it is commutative. **6**

OR

6\. a) Show that the intersection of any two normal subgroups of a group G is a normal subgroup of G. **6**

b) Let T be the set of all even integers. Show that the semigroup (Z, +) and (T, +) are isomorphic. **6**

7\. a) Prove that the set $S = \{0, 1, 2, 3, 4\}$ is a ring w.r.t. the operations of addition and multiplication modulo 5. **7**

b) Construct the switching circuit for the Boolean expression. **5**
$(A \cdot B) + C + (A' \cdot C')$.
Simplify this and construct an equivalent simplified circuit.

OR

8\. a) Show that $S = \{a + b\sqrt{2} \mid a, b \in Z\}$ is an integral domain w.r.t. the addition "+" and multiplication "$\times$". **7**

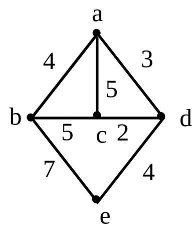
b) Define a lattice. Draw Hasse diagram of the lattices D_{20} and D_{30}. **5**

9\. a) Draw the digraph corresponding to the matrix. **6**

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

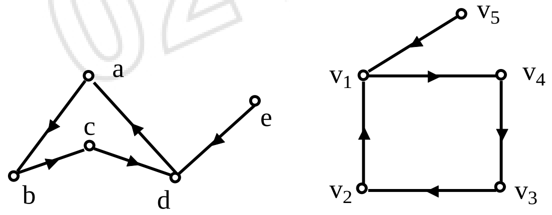
and interpret AA^T, A^TA and A^2.

b) Apply Kruskal's algorithm to construct a minimal spanning tree for the weighted graph given below : **6**



Also find the minimum weight of this spanning tree.

c) Define isomorphic graphs. Show that following two graphs are isomorphic. **6**

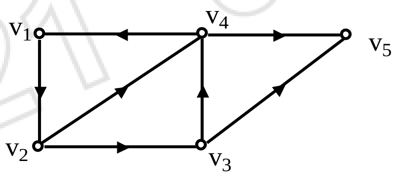


OR

10. a) Define. **4**

i) Weighted graph, ii) Euler's path
iii) Complete Binary tree and iv) Complete graph.

b) Find in-degree and out-degree of each node of the graph given below and give all elementary cycles of this graph. **6**



c) Construct the tree for the following expression: **8**

$(5(1-x)\div(5-(y+3)))\bullet(7+(x+y))$.

Also, draw the corresponding binary tree.

11. a) State extended pigeonhole principle. Show that if any 30 people are selected, then we may choose a subset of 5 so that all 5 were born on the same day of the week. **5**

b) Solve the following recurrence relation: **5**

$a_r = 3a_{r-1} + 2,$ given $a_o = 1.$

OR

12. a) Prove that **5**

$C(n,r) = C(n-1, r-1) + C(n-1, r).$

b) Find the generating function for the sequence: **5**

$1, a, a^2, ----,$ where a is a fixed constant.

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