QUESTION ANSWER BANK -UNIT I1-QUANTUM MECHANICS

<table>
<tr><td>1</td><td>

What is the Compton effect? On the basis of quantum theory, explain the existence of modified and unmodified components in Compton scattering.

Compton Effect:

- When a beam of monochromatic X-rays strikes the electron loosely bound to atom, X-rays are scattered in all possible directions with decrease in energy and electron recoils with gain in energy in other direction.
- This phenomenon is called as "Compton scattering" or "Compton effect" and the angle between direction of incident and scattered ray is called scattering angle (ϕ).
- The scattered X-rays consists of two components of wavelength-modified and unmodified.
- The difference between the modified and unmodified components of wavelength is called Compton Shift.

Existence of Modified component:

- When X-ray Photon collides with a free electron or loosely bound electron, the collision is between Photon and electron is an elastic collision.
- The photon transfers part of its energy and momentum to the electron at rest.
- The electron gains kinetic energy and recoils. The photon is thus, scattered with less energy as compared to that of incident photon.
- Since Energy is inversely proportional to wavelength, the wavelength of scattered photon is higher than that of the incident photon.
- **Therefore, when Photon collides with loosely bound electron it gives rise to modified component in scattered radiation.**

Existence of Unmodified component:

- When Photon collides with tightly bound electron, then the whole atom gets affected.
- Under such conditions, the rest mass 'm_O' must be replaced by 'M' the mass of the atom in the expression for Compton Shift.
- Thus, the Compton shift expression becomes,

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{Mc}(1-\cos\emptyset)$$

Since M >>>>m_O, $\Delta\lambda \approx 0$

$$\text{or } \lambda' - \lambda \approx 0$$

$$\lambda' \approx \lambda$$

- Hence the wavelength of scattered and incident photon is same.
- **Therefore, when Photon collides with tightly bound electron it gives rise to unmodified in scattered radiation.**

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<tr><td>2</td><td>

Describe the Davison and Germer experiment, which shows the existence of matter waves.

Davisson and Germer experiment proves the existence of the matter waves and confirms De-Broglie Hypothesis.

Experimental set up of Davisson and Germer experiment is as shown in figure 1.

- It consists of an electron gun (G), which produces collimated beam of electrons.
- An anode (A) connected to a variable voltage source to accelerate the electrons.
- The electrons are scattered by the atoms in the Nickel crystal.

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- The Intensity of beam of electrons scattered in different directions was measured with the help of a moving detector.
- The detector measures the intensity of scattered beam of electrons as function of scattering angles Φ for different values of accelerating potential 'V' applied between cathode and anode.
- A polar graph is plotted between intensity of scatted beam versus scattering angle Φ.as shown in figure 2.

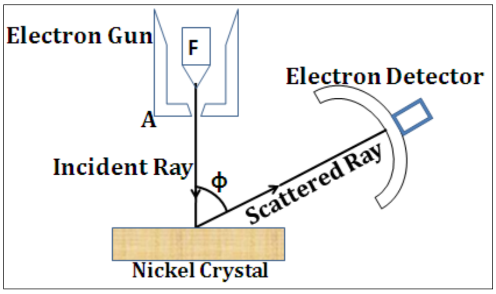


Fig.2.11. Davisson-Germer Experimental Set-up

Observations and Results of the experiment:

1. Observation: The intensity of the scattered electron beam exhibits distinct maxima and minima as shown in polar graph.
   Result: This shows that electron beam can be diffracted and confirms the wave nature of electrons.
2. Observation: At a potential difference of *54V*, intensity of electron beam I is found to be at its maximum at $\Phi = 50^o$ as shown in figure 2.
3. Result: This maximum is due to first order diffraction as no other maxima was obtained at lower values of angle.
4. It is due to constructive interference of electron waves scattered in this direction from parallel planes in the crystal like that of X-rays. Hence Bragg's law is applicable.

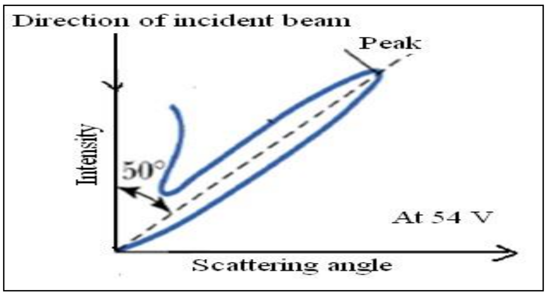


Fig.2. Polar Graph

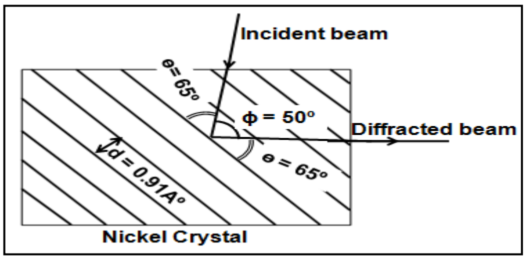


Fig.3. Diffraction of electron beam by (111) planes of Nickel crystal

Calculations: According to Bragg's law,

$$2d\sin\theta = n\lambda$$

For a given plane in Nickel crystal, d = 0.91Å and from figure 3, glancing angle is calculated as θ = 65˚

∴ Applying Bragg's law for n= 1,

We have, $\lambda = 2d\sin\theta$

$= 2\,(0.91\text{Å}) \times (\sin 65^\circ)$

$\therefore \lambda = 1.65\text{Å}$ …………………… Experimental Value

Thus, wavelength of electron wave from Bragg's law is 1.65Å

Now, wavelength of electron using de-Broglie equation is,

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.625 \times 10^{-34} JS}{\sqrt{2 \times 9.11 \times 10^{-31} kg \times 1.602 \times 10^{-19} C \times 54V}}$$

$\lambda = 1.66\text{Å}$ …………………Theoretical value

Conclusion: Therefore, the values of wavelength obtained experimentally using Bragg's law and theoretical value using de-Broglie hypothesis are in good agreement with each other which provides evidence for the existence of matter waves and hence verifies the de-Broglie hypothesis.

3 What is de-Broglie hypothesis? Show that de-Broglie wavelength of electron accelerating through potential V is given by $\lambda = \frac{12.26}{\sqrt{V}}$.

De-Broglie Hypothesis: The hypothesis that matter can behave like a wave is called De-Broglie hypothesis. Just as light shows dual nature, every material particle exhibits wave nature too. Every moving material particle has a wave associated with it. This wave is called as matter wave.

According to De-Broglie hypothesis, the wavelength 'λ' of the matter wave associated with a particle of mass 'm' moving with velocity 'v' is given as

$$\boldsymbol{\lambda = \frac{h}{m\vartheta} = \frac{h}{p}}$$

where 'p' is the momentum of the particle.

Consider an electron of charge 'e' and mass 'm' moving with velocity 'v' through a region of potential difference of 'V' volts. Therefore, the kinetic energy acquired by the electron is due to electrical energy,

∴ Kinetic energy = Electrical energy

$$\frac{1}{2}\mathrm{m}v^2 = eV$$

$$\text{or} \quad \mathrm{m}v^2 = 2eV$$

$$m^2v^2 = 2\mathrm{meV}$$

$$\therefore\ p^2 = 2\mathrm{meV}$$

$$\therefore\ \mathrm{p} = \sqrt{2meV} \quad \text{……………………….(1)}$$

By the definition of De Broglie wavelength, $\lambda = \frac{h}{p}$ ------------ (2) 3

Substituting the value of p from $eqn.$ (1), we get

	$$\lambda = \frac{h}{\sqrt{2meV}} \qquad \dots\dots\dots(3)$$ Equation (3) represents de Broglie wavelength 'λ' in terms of potential 'V'. Since 'e'=1.602 x 10^{-19}C, 'h'=6.623 x 10^{-34} Js and mass of electron 'm' = 9.1X10^{-31} kg. $$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.623 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.602 \times 10^{-19}}} \frac{1}{\sqrt{V}}$$ $$\therefore \lambda = \frac{12.26 \times 10^{-10}}{\sqrt{V}} \boldsymbol{m} = \frac{12.26}{\sqrt{V}} \text{ Å}$$	
4	State the Heisenberg Uncertainty Principle and explain the significance of the Heisenberg Uncertainty Principle in microscopic particle with examples. ***Heisenberg's Uncertainty Principle: It is not possible to make simultaneous measurement of both the position and the momentum of a microparticle with unlimited accuracy''.*** Mathematically, product of uncertainty in measurement of position (Δx) and momentum (Δp_x) of a microparticle is always greater than or equal to ħ. $$\mathbf{\Delta x.\ \Delta p_x \geq \hbar}$$ $$\text{where, } \hbar = \frac{h}{2\pi}$$ Heisenberg uncertainty principle is not significant in case of macro-bodies while it is significant in case of micro particle. This can be explained by considering an example, **CASE (I): Macro-body: Example:** let us consider the case of macro body such as football of mass 0.5 kg in flight. The uncertainty in the position of the ball is **say 1 mm**. The uncertainty in its velocity is, $$\Delta v \approx \frac{h}{2\pi\, m\, \Delta x} \approx \frac{6.623 \times 10^{-34}}{2 \times 3.14 \times 0.5 \times 10^{-3}} \approx 10^{-31}\, m/s$$ As the uncertainty in the velocity is negligibly small, we can say that uncertainty principle is not significant in case of macrobodies. Macroscopic bodies follow a definite path. **CASE (II): Micro-body: Example:** Consider an electron revolving around nucleus in a hydrogen atom, the uncertainty in its position **is $\underline{+}$ 1A^{o}**. Therefore, uncertainty in its velocity is, $$\Delta v \approx \frac{h}{2\pi m \Delta x} \approx \frac{6.623 \times 10^{-34}}{2 \times 3.14 \times 9.11 \times 10^{-31} \times 2 \times 10^{-10}} \approx 5 \times 10^{5}\, m/s$$ This value of uncertainty in velocity of an electron in the orbit is significant. Hence, we can say that uncertainty principle is significant in case of microbodies. Hence a microscopic particle like electron does not follow a definite path**.**	4
5	Show that the energy of an electron confined in 1-D potential well of length 'L' and finite depth is quantized. Is the electron trapped in the potential well allowed to take zero energy? Why? **PARTICLE IN ONE DIMENSIONAL POTENTIAL WELL OF INFINITE HEIGHT**	5

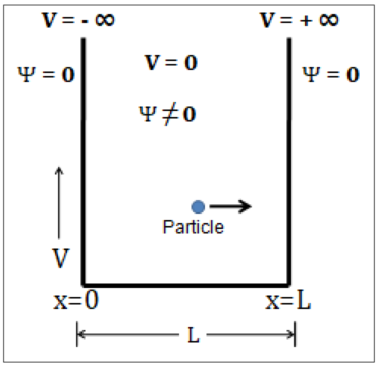


Fig. 3.7: Potential well of infinite height

A potential well is a potential energy function V(x) that has a minimum value.
Consider an electron confined to one-dimensional potential well of infinite height and width 'L'. The electron is in a **"trapped state"** or **"bounded state" .**
The electron is free to move in this small region and reflects back and forth between the walls.
The boundary condition are:
For potential energy:

$$V(x) = 0 \quad for \quad 0 < x < L \qquad and \qquad V(x) \to \infty \quad for \quad 0 \geq x \geq L$$

For wave function Ψ:

$$\Psi(x) = 0 \quad for \qquad 0 \geq x \geq L$$

To describe the motion of particle inside the potential well, we use one-dimensional Schrodinger's time independent equation, which is given by,

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0$$

Since V=0 inside the potential well, the above equation reduces to

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{8\pi^2 mE}{h^2}\Psi = 0 \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$\text{Substituting,} \quad k^2 = \frac{8\pi^2 mE}{h^2} \qquad (2)$$

$$\therefore \frac{\partial^2\Psi}{\partial x^2} + k^2\,\Psi = 0 \qquad (3)$$

General solution of such eqn. (3.31) can be written as,

$$\Psi(x) = A\sin kx + B\cos kx \qquad (4)$$

where 'A' and 'B' are constants whose values can be obtained by applying the boundary conditions.

According to first boundary condition on Ψ:

At x = 0 the wavefunction Ψ= 0,

Therefore, Equation no.4 becomes,

$$0 = A\sin 0 + B\cos 0$$

$$\therefore B = 0$$

$$\therefore Substituting\ B = o\ in\ eqn\ (4) we\ get$$

$$\Psi(x) = A\sin kx \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots (5)$$

According to second boundary condition on Ψ:,

At x = L , Ψ = 0,

$eqn. (5) becomes,$ $\qquad 0 = A\sin kx$

$$\because A \neq 0 \qquad \Rightarrow sinkL = 0$$

But we know that, $sinn\pi = 0$ where n = 1, 2, 3,……………

$$\therefore kL = n\pi$$

$$k = \frac{n\pi}{L}$$

$$\text{or } k^2 = \frac{n^2\pi^2}{L^2}$$

Substituting this value of k in eqn. (2) we get,

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{L^2}$$

$$E_n = \frac{n^2h^2}{8mL^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where n is called quantum number, given by n = 1, 2, 3, 4, ...

Equation 3.34 indicates that energy of particle confined in a certain region cannot take any arbitrary value, it can take only certain discrete set of values, i.e.

$$\therefore\ E_1 = \frac{h^2}{8mL^2}, \quad E_2 = \frac{h^2}{2mL^2}, \quad E_3 = \frac{9h^2}{8mL^2}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..$$

E_1, E_2, E_3..... E_n are the ***Eigen values*** or allowed values of the energy of the particle. Thus, the energy values for an electron confined in an infinite one-dimensional potential well are ***quantized.***

Electron cannot have zero value of Energy:

- If the energy of particle (electron) is zero then momentum of particle (electron) would also be zero (p=0).
- Since $p = \frac{h}{\lambda}$ therefore, p→0 gives rise λ→∞.
- Then by Heisenberg's Uncertainty principle its wavelength λ is "∞". Then, the particle (electron) associated with a wave of infinite wavelength cannot be confined to the well.
- Therefore particle (electron) must possess a certain minimum amount of kinetic energy.
- So, the exclusion of zero value of energy is a consequence of Heisenberg uncertainty principle.

6 Give the physical significance of wave function. What is the condition of normalization?

A mathematical function used in quantum mechanics to describe the propagation of the wave associated with a particle is called wave function denoted by Ψ. Ψ(x, y, z, t) is a function of the coordinates of *space and of time*.

Physical Significance of Wave function Ψ:

Max Born gave following interpretation for wave function Ψ:

- Wave function Ψ has no direct physical significance because it is not an observable quantity. Hence, wave function is a complex function.
- Mathematically Ψ represents the motion of the particle., but it is not possible to locate the exact position of particle at (x, y, z, t) using Ψ. There is only probability of finding the particle being at a specific point (x, y, z).
- To convert the complex wave function into real function we multiply it by its Complex conjugate Ψ* where Ψ × Ψ* = $|\Psi|^2$
- $|\Psi|^2$ is known as *probability density* which is defined as probability per unit volume.
- Thus, if P is the probability of finding the particle in a small volume 'dV', then P is directly proportional to $\int \Psi\Psi^* \, dV or \int |\Psi|^2 dV$.
- If $|\Psi|^2$ has large value, then there is strong probability of finding the particle.
- If $|\Psi|^2$ has small value, then there is little probability of finding the particle. 4
- If $|\Psi|^2$ is zero, then the probability of finding the particle is zero (particle is absent)

	Normalization condition on wave function Ψ: • If at all a particle exists, the probability of finding the particle somewhere in the universe must be unity. • Since P is the probability of finding the particle in small elemental volume dV , then P is directly proportional to $\int \|\Psi\|^2 dV$. • It is convenient to choose the constant of proportionality such that sum of all probabilities over all values of x, y and z is unity. Therefore, $\int \|\psi\|^2\, dV = \int_{x=-\infty}^{x=+\infty} \int_{y=-\infty}^{y=+\infty} \int_{z=-\infty}^{z=+\infty} \|\psi\|^2\ dxdydz = 1$ • The above condition is known as normalization condition and wave function Ψ satisfying this condition is said to be normalized wave function.	
7	In Compton Effect what happens when (i) Photon collides with free electron in scattering block. (ii) Photon collides with a bound electron in scattering block. **(i) Existence of Modified component**: • When X-ray Photon collides with a free electron or loosely bound electron, the collision is between Photon and electron is an elastic collision. • The photon transfers part of its energy and momentum to the electron at rest. • The electron gains kinetic energy and recoils. The photon is thus, scattered with less energy as compared to that of incident photon. • Since Energy is inversely proportional to wavelength, the wavelength of scattered photon is higher than that of the incident photon. • **Therefore, when Photon collides with loosely bound electron it gives rise to modified component in scattered radiation.** **(ii) Existence of Unmodified component:** • When Photon collides with tightly bound electron, then the whole atom gets affected. • Under such conditions, the rest mass ‘m_O’ must be replaced by ‘M’ the mass of the atom in the expression for Compton Shift. • Thus, the Compton shift expression becomes, $\Delta\lambda = \lambda' - \lambda = \frac{h}{Mc}\,(1 - \cos\emptyset)$ Since M >>>>m_O, $\Delta\lambda \approx 0$ or $\lambda' - \lambda \approx 0$ $\lambda' \approx \lambda$ • Hence the wavelength of scattered and incident photon is same. **Therefore, when Photon collides with tightly bound electron it gives rise to unmodified component in scattered radiation.**	4
8	State the Heisenberg’s Uncertainty principle? Why is it not significant for macroscopic bodies? Same answer as that given in question no.4	5
9	Show that the energy of a microparticle confined in an infinite one-dimensional potential well of length ‘L’ is quantized.	5

	Same answer as that given in question no.5.	
10	Define wave function. Explain in brief the Mathematical conditions imposed on the Wave function to get well-behaved wave function. A mathematical function used in quantum mechanics to describe the propagation of the wave associated with a particle is called wave function denoted by Ψ. Ψ(x, y, z, t) is a function of the coordinates of space and of time. Mathematical conditions on wave function Ψ: To convert Ψ into a well- behaved wave function, it should satisfy the following conditions: ***(i) Ψ should be single valued function of space and time*** The wave function Ψ(x, y, z) should be single valued at any point. Because the function related to a physical quantity, it cannot have more than one value at the point. ***(ii) Ψ must be finite*** The wave function must be finite everywhere. Even if $x \rightarrow \infty\ or\ -\infty, y \rightarrow \infty\ or -\infty, z \rightarrow \infty\ or -\infty$, the wave function should not tend to infinity. It must remain finite for all values of x, y, z. Infinite probability has no meaning. ***(iii) Ψ and its derivatives must be continuous*** The wave function Ψ and its space derivatives $\frac{\partial\Psi}{\partial x}, \frac{\partial\Psi}{\partial y}$ and $\frac{\partial\Psi}{\partial z}$ should be continuous across any boundary. Since Ψ is related to a real particle, it cannot offer discontinuity at any point. **(iv) Wave function Ψ must be a normalized wave function.** The wave function Ψ which is ***single valued, finite and continuous is called*** **well behaved wave function.**	3
11	Why intensity of modified wavelength is higher than that of unmodified wavelength for low atomic number scatterer during Compton scattering. • From expression of Compton shift, $\Delta\lambda = \frac{h}{Mc}(1-\cos\varphi)$, it is seen that Δλ is inversely proportional to mass of the scattering particle. • Lower the mass of the scattering particle, higher will be Compton shift. • Hence to obtain significant Compton shift, scattering atom should be of low atomic number. • The intensity of modified and unmodified components depends on the target element considered i.e., whether it is a lower atomic number of element or higher atomic number element. Consider the following two cases: **CASE (I): HIGHER ATOMIC NUMBER ELEMENTS** • When X-rays collide with bounded electron, the target is not an electron but the whole atom to which it is bound. • Under such conditions there is no change in energy of incident photon and hence it is scattered as unmodified wavelength (λ). • In high atomic number element, the number of bounded electrons is more as compared to the free electrons, hence unmodified component is more intense. **CASE (II): LOWER ATOMIC NUMBER ELEMENTS** • When X-ray photon collides with free electron, then photon transfers some of its energy to that electron. Thus, photon scattered with less energy and greater	2

wavelength compared to that of incident photon. Hence modified component of wavelength appears in scattered radiation.

- In low atomic number element, the number of free electrons is more as compared to the bounded electrons; hence modified component is more intense.
- Scattering atom should be of low atomic number.

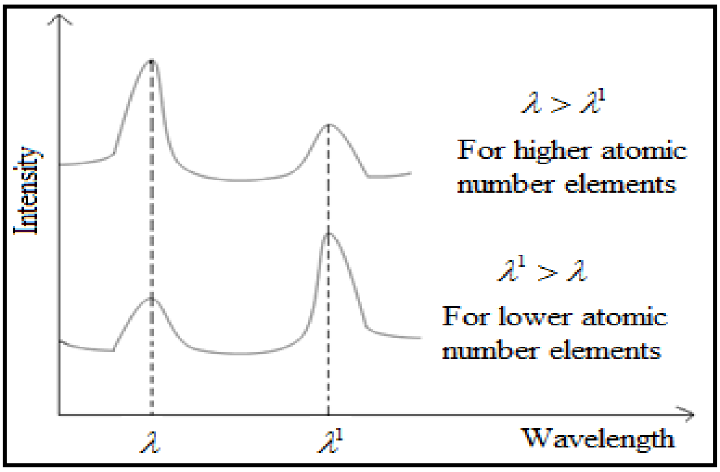


Fig.: Modified and unmodified components in various target elements.

12 Why wave nature of particle is not apparent in our daily observations?

According to De Broglie, matter waves are generated by every moving body, whether it is microscopic or macroscopic body. Let us calculate it for each case.

DE-BROGLIE WAVELENGTH OF MACRO BODIES:

The de-Broglie wavelength of matter wave is,

$$\lambda = \frac{h}{mv}$$

From this relation, it is seen that as mass increases, associated de-Broglie wavelength decreases. Thus, wavelength of macro bodies is insignificant in comparison to the size of body even at very low velocity.

For example: Consider a cricket ball of mass 500gm flying with velocity of 50 km/hr, its wavelength is calculated as follows:

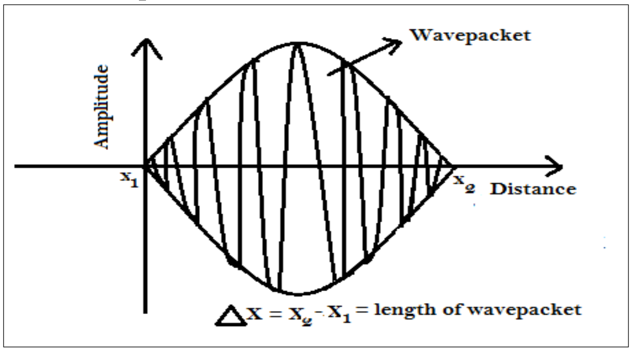
m = 500 gm = 500 x 10^{-3} kg

$$v = 50\ \mathrm{km/hr} = \frac{50 \times 10^3}{60 \times 60}\ \mathrm{m/s}$$

$$\lambda_{ball} = \frac{h}{mv} = \frac{6.623 \times 10^{-34}}{500 \times 10^{-3} \times \frac{50 \times 10^3}{60 \times 60}} = \frac{6.623 \times 10^{-34}}{500 \times 10^{-3} \times 13.9} = 0.95 \times 10^{-24} A^o$$

$$\approx \mathbf{10^{-24} A^o}$$

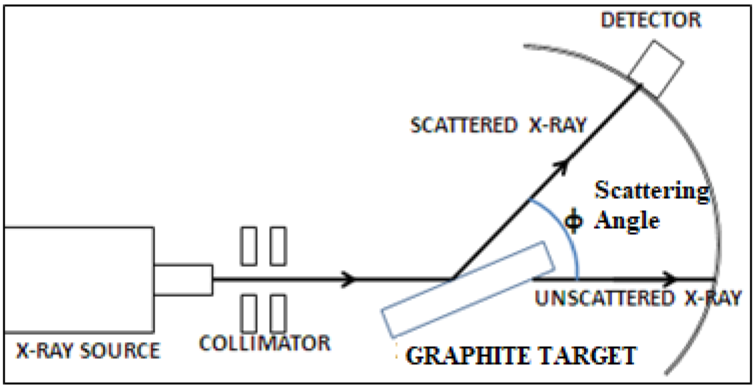
The value of wavelength ($10^{-24} A^o$) is insignificant as compared to size of ball. Hence, *macro bodies do not exhibit wave nature.* 3

	DE-BROGLIE WAVELENGTH OF MICROCSOPIC PARTICLES: Consider an electron accelerated through potential difference of 100 V. The de Broglie wavelength associated with the electron can be calculated by the relation, $$\lambda = \frac{h}{\sqrt{2meV}}$$ By putting this value of m_o, e and h $(m_o = 9.11 \times 10^{-31} kg, \quad e = 1.6 \times 10^{-19} C \quad and\ h = 6.625 \times 10^{-34} Js)$ in above equation We get, $\lambda = \frac{12.26}{\sqrt{V}} A^o = \frac{12.26}{\sqrt{100}} = 1.226 A^o$ This wavelength is measurable. This is in the order of wavelength of x-rays hence electron wave exist. Hence it proves that, the de-Broglie wavelength is significant in case of microscopic particles only. As we always come across macroscopic bodies in our daily life, the wave nature is not apparent.	
13	What is a wave packet? Why cannot a single monochromatic wave represent a particle? • A single monochromatic wave can be represented by equation Y=A sin (ωt - kx) where 'Y' represents the displacement of the particle with respect to position 'x' and time 't', 'k' is the propagation constant, 'ω' is the angular frequency and 'A' is its amplitude. • Such a wave extends infinitely in space. • It has no beginning and no end. It is completely non–localized. • Such a single monochromatic wave cannot represent a particle since the particle is localized in space. CONCEPT OF WAVEPACKET • Wave packet is a superposition of group of harmonic waves with slightly different frequencies. • These waves interfere constructively over a very small region and cancel each other everywhere except that small region. • Such a wave packet can represent a matter wave associated with a particle.  • The wave packet has characteristics of both wave as well as particle. • The regular separation between two successive maxima in a wave packet is one of the characteristics of a wave. • The wave packet is localized like a particle in space.	4
14	Write down Schrodinger time-dependent and time-independent wave equations in one dimension. **Time-dependent Schrodinger's wave equation**	3

	One Dimensional $$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V(x)\Psi=i\hbar\frac{\partial\Psi}{\partial t}$$ **Time-independent Schrodinger's wave equation** I One Dimensional $$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V(x)\Psi=E(x)\Psi$$ where, $\nabla^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)-$ represents differential operator, $V-$ is the applied potential and $\Psi-$ is the wave function associated with the particle of mass 'm' $E-$ is the total energy possessed by the particle. These equations describe behavior of matter waves.	
15	Why visible light cannot cause Compton effect? Explain. Visible light cannot cause Compton effect. It can be explained as follows: (a) Compton effect can be observed only when the striking photon is energetic enough to share some of its energy with an electron of the target. Binding energy of electron is comparable with the visible light photon energy. Hence visible light having low energy photons cannot cause Compton effect. Whereas X-ray photons are energetic enough for Compton effect to occur. (b)The maximum value of Compton shift is 0.0484Å for $\varphi=180^o$. The statement that Compton shift cannot be experimentally observed in visible light can be proved by calculating the percentage Compton shift observed in X-rays and visible radiation as follows: **CASE (I):** Compton shift in visible light: $$(\%\text{Shift})_{\text{vis}}=\frac{\Delta\lambda}{\lambda_{vis}}=\frac{0.0484}{5500\text{Å}}=8.82X10^{-6}\approx\text{negligible}$$ % shift is negligible and hence Compton shift cannot be detected. **CASE (II):** Compton shift in X-rays: $$(\%\text{Shift})_{\text{X-rays}}=\frac{\Delta\lambda}{\lambda_{X-rays}}=\frac{0.0484}{1\text{Å}}=0.0484\text{ Å}$$ % shift is appreciable and hence Compton shift can be detected. Thus, Compton shift cannot be observed experimentally in visible light, but observed with X-rays.	2
16	Show that the wave function for a particle in a one- dimensional potential well of length L & infinite depth is given by ψ n (x) = A sin(nΠx/L) where A is given by √2/L.	4
17	What is Compton effect? Write the expression for Compton shift.	3

Compton Effect:

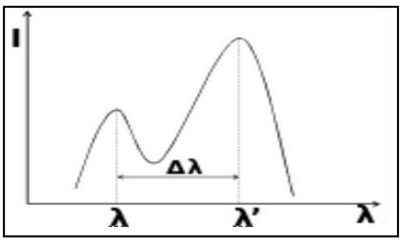
- When a beam of monochromatic X-rays strikes the electron loosely bound to atom, X-rays are scattered in all possible directions with decrease in energy and electron recoils with gain in energy in other direction.
- This phenomenon is called as “Compton scattering” or “Compton effect” and the angle between direction of incident and scattered ray is called scattering angle (ϕ).
- The scattered X-rays consists of two components of wavelength-modified and unmodified.
- The modified component has higher wavelength than that of incident one and unmodified component has same wavelength as that of incident wavelength.
- The difference between the modified and unmodified components of wavelength is called Compton Shift.



Compton Shift $\Delta\lambda$ is given by expression,

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\Phi)$$

where λ' = wavelength of scattered X − ray photon = modified wavelength
and λ = wavelength of incident X − ray photon = unmodified wavelength
m_o = Rest mass of electron
c = velocity of light
h = Planck’s constant



18 In Compton Effect, write down the equations of energy and momentum conservation, considering elastic collision between electron and photon. 3

CONSERVATION OF ENERGY:

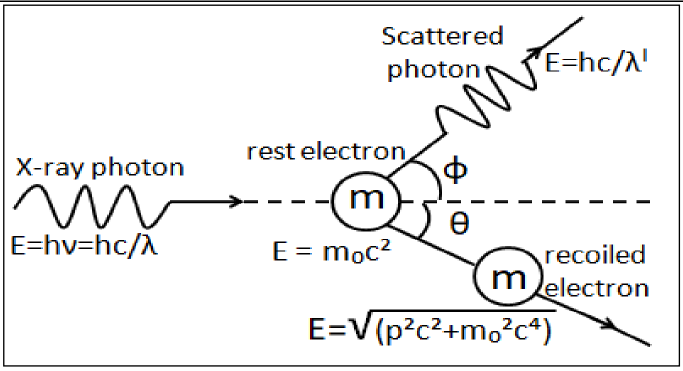


Fig1: Energy values before and after collision of photon with electron

By principle of conservation of energy,

Before collision ***After collision***

$$\begin{bmatrix} \text{energy of} \\ \text{incident photon} \end{bmatrix} + \begin{bmatrix} \text{energy of} \\ \text{rest elecron} \end{bmatrix} = \begin{bmatrix} \text{energy of} \\ \text{scattered photon} \end{bmatrix} + \begin{bmatrix} \text{energy of} \\ \text{recoiled electron} \end{bmatrix}$$

$$\boldsymbol{\frac{hc}{\lambda} + m_0c^2 = \frac{hc}{\lambda'} + \sqrt{m_0{}^2c^4 + p^2c^2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

CONSERVATION OF MOMENTUM:

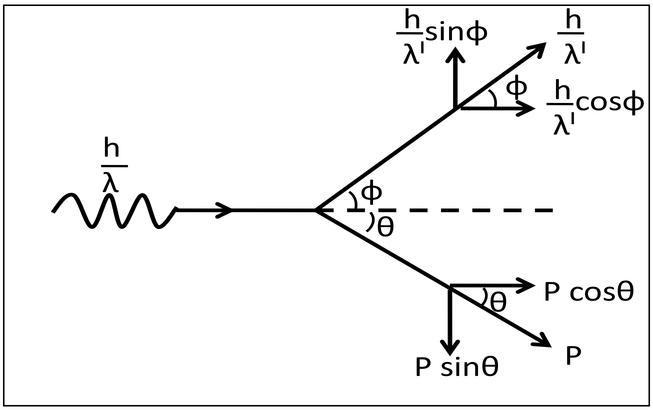


Fig. 2: Momentum values before and after collision of photon with electron

<u>Along X-axis</u>: -

Before collision ***After collision***

$$\begin{bmatrix} \text{momentum of} \\ \text{incident photon} \end{bmatrix} + \begin{bmatrix} \text{momentum of} \\ \text{rest elecron} \end{bmatrix} = \begin{bmatrix} \text{momentum of} \\ \text{scattered photon} \end{bmatrix} + \begin{bmatrix} \text{momentum of} \\ \text{recoiled electron} \end{bmatrix}$$

$$\boldsymbol{\frac{h}{\lambda} + 0 = \frac{h}{\lambda'}cos\Phi + Pcos\theta} \qquad \textbf{------------- (2)}$$

<u>Along Y-axis</u>: - $0 + 0 = \frac{h}{\lambda'}sin\Phi + (-Psin\theta)$

or $Psin\theta = \frac{h}{\lambda'}sin\Phi$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$

19 Prove that electron does not exist in the nucleus.

Let us assume that electron exists inside the nucleus. The radius of nucleus of an atom is nearly 10^{-14} m. 3

∴ Diameter is 2 x 10^{-14} m.

If an electron lies inside the nucleus then the maximum uncertainty in the position of an electron will be the diameter of nucleus.

$\therefore$ $\Delta x = 2 \times 10^{-14}$ m

According to Heisenberg uncertainty principle,

$$\Delta x \cdot \Delta p_x = \frac{h}{2\pi}$$

$$\Delta p_x = \frac{h}{2\pi \Delta x} = \frac{6.626 \times 10^{-34}}{2 \times 3.14 \times 2 \times 10^{-14}}$$

$$= 0.527 \times 10^{-20} \text{ kg m/s}$$

Thus, if electron is within nucleus then its momentum must be at least equal to Δp_x.

$$\therefore p_{min} \cong \Delta p_x = 0.5 \times 10^{-20} \text{ kg m/s} \quad \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$\therefore$ Total relativistic energy of electron is,

$$E = \sqrt{p^2c^2 + {m_0}^2c^4}$$

$$\therefore E^2 = p^2c^2 + {m_0}^2c^4$$

$$\because {m_0}^2c^4 \ll p^2c^2$$

$$E_{min} \approx P_{min} \cdot c \qquad \{from\ eqn\ (1)\}$$

$$E_{min} \approx 0.5 \times 10^{-20} \times 3 \times 10^8$$

$$\approx 1.58 \times 10^{-12} J$$

$$\approx \frac{1.58 \times 10^{-12}}{1.60210^{-19}} eV$$

$$\approx 9.875 \times 10^6 eV$$

$$E_{min} \approx 10 MeV$$

That means if free electron exists within nucleus, then its minimum kinetic energy will be equal to **10 MeV.**

But, maximum K.E. of β particles (electrons) emitted by radioactive nucleus is of the order of **4 MeV.** Hence our assumption is wrong. Therefore, free electron cannot reside within nucleus.

20 What is a de-Broglie hypothesis? Show how the quantization of angular momentum follows the concept of matter waves.

De-Broglie Hypothesis: The hypothesis that matter can behave like a wave is called De-Broglie hypothesis. Just as light shows dual nature, every material particle exhibits wave nature too. Every moving material particle has a wave associated with it. This wave is called as matter wave.

The Bohr's postulate on quantization of Angular momentum can be justified using de-Broglie hypothesis.

- According to de-Broglie hypothesis, electron revolving around the nucleus has a matter wave associated with it.
- As the electron revolves round in one of its circular orbits, the associated matter waves propagate along the circumference again & again.
- A wave must meet itself after going once around the nucleus. 3

- If it did not meet, then the wave would be out of phase with itself after one orbit (figure 1) and after many revolutions, the wave would be destroyed due to destructive interference.
- Thus, while moving in circular orbit the wave should form ***standing wave pattern***.
- A stationary wave pattern can form in the orbit only if *an integral number of wavelengths fit into the orbit.*

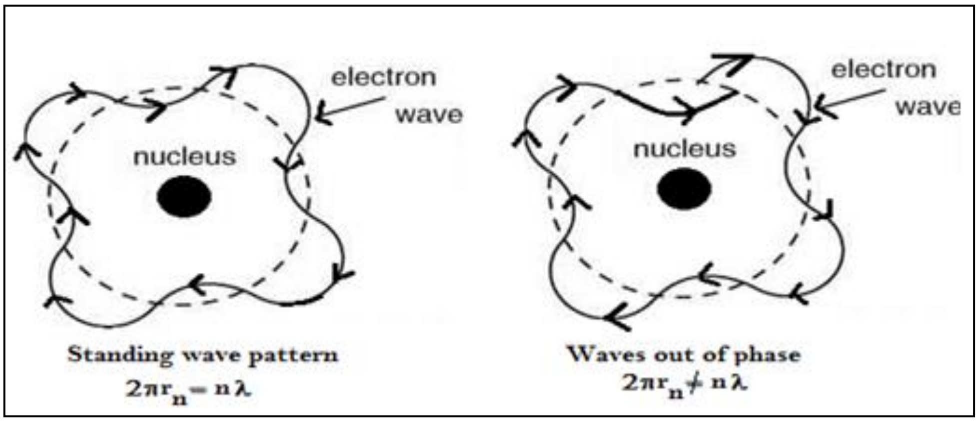


Fig. 2.10: Motion of wave associated with electron

- Thus, condition for formation of standing waves in an orbit is,

$$2\pi r_n = n\lambda$$

$$\therefore 2\pi r_n = n\frac{h}{p} \qquad \text{(using De} - \text{Broglie Hypothesis,} \lambda = \frac{h}{p}\text{)}$$

$$\text{or } 2\pi r_n = \frac{nh}{mv}$$

$$mvr_n = \frac{nh}{2\pi}$$

$$\therefore \boldsymbol{L_n} = \boldsymbol{n}\hbar \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.22)$$

where, L_n is angular momentum in nth orbit.

Thus, angular momentum is quantized. The quantization of angular momentum on the basis of De-Broglie hypothesis is a direct consequence of wave nature of electron.

21 An electron is confined to move between two rigid walls separated by 1nm. Find the first two allowed energy levels. 3

22 Calculate the wavelength associated with a stone of mass 50 gms moving with the speed of 50m/s and an electron accelerated through a potential difference of 100 Volts.

Given: For electron:

$h = 6.63 \times 10^{-34}$ Js

$m = 9.1 \times 10^{-31}$Kg

$V = 100$V

Solution: $\lambda_{\text{electron}} = \dfrac{h}{\sqrt{2meV}} = \dfrac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.602 \times 10^{-19} \times 100}}$

$= 1.228 \times 10^{-10}$m

Given: For stone:

$h = 6.63 \times 10^{-34}$ Js

$m = 50 \text{ gm} = 50 \times 10^{-3}$Kg 3

$v = 50$ m/s

	$\lambda = ?$ **Solution**: $\lambda_{\text{stone}} = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{50 \times 10^{-3} \times 50} = 2.652 \times 10^{-34}\ m$	
23	Calculate the lowest three permissible energies of an electron if it is bound by an infinite potential well of width 2.5 x10^{-10} m. **Given:** $\text{L} = 2.5 \times 10^{-10}\text{m}$ $\text{h} = 6.63 \times 10^{-34}\text{Js}$ Mass of electron 'm' $= 9.1 \times 10^{-31}$ Kg $\text{E}_1 = ? \quad \text{E}_2 = ? \quad \text{E}_3 = ?$ **Solution**: $\text{E}_\text{n} = \frac{\text{n}^2\text{h}^2}{8\text{mL}^2}$ $\text{E}_1 = \frac{\text{h}^2}{8\text{mL}^2} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2} = 9.66 \times 10^{-19}\ \text{J} = \frac{9.66 \times 10^{-19}}{1.602 \times 10^{-19}} = 6.029\ \text{eV}$ $\text{E}_2 = 4 \times \text{E}_1 = 4 \times 9.66 \times 10^{-19}\ J = 38.64 \times 10^{-19}\ J = 24.11\ \text{eV}$ $\text{E}_3 = 9 \times \text{E}_1 = 9 \times 9.66 \times 10^{-19}\ J = 86.94 \times 10^{-19}\ J = 54.26\ \text{eV}$	3
24	An electron and a 150gm baseball are travelling 220 m/sec, measured to an accuracy of 0.005%. Calculate & compare uncertainty in the position of each. **Ans.: Given: For an electron** $m = 9.1\text{x}10^{-31}$ kg $v_x = 220$ m/sec $\Delta v_x = v_x \times accuracy = v_x \times 0.005\%$ $\Delta x = ?$ **For base ball** $\text{m} = 150\ \text{gm} = 0.15\ \text{kg}$ $v_x = 220\ \text{m/sec}$ $\Delta v_x = v_x \times accuracy = v_x \times$ 0.005% $\Delta x = ?$ **Solution:** **For an electron**: $\Delta v_x = v_x \times 0.005\% = 220 \times \frac{0.005}{100} = 0.011$ $\Delta x = \frac{\text{h}}{2\pi.\ m\Delta v_x} = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 0.011} = 0.0105\ m$ **For base ball**: $\Delta v_x = v_x \times 0.005\% = 220 \times \frac{0.005}{100} = 0.011$ $\Delta x = \frac{\text{h}}{2\pi.\ m\Delta v_x} = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 0.15 \times 0.011} = 6.39 \times 10^{-32}\ m$ Thus, the uncertainty in finding the position of microscopic bodies like electron is very large compared to its size and that in macroscopic bodies like baseball it is negligibly small compared to its size.. Thus, Heisenberg's uncertainty principle is insignificant in macroscopic bodies while it is significant in case of microscopic bodies.	3
25	Find the lowest three energy of an electron confined to move in a one- dimensional potential well of length $1\mathring{A}$. Express the result in eV. **Given:** $\text{L} = 1\text{A} = 10^{-10}\text{m}$	3

	$h = 6.63 \times 10^{-34}$ Js Mass of electron 'm' $= 9.1 \times 10^{-31}$ Kg $E_1 = ?$ $p = ?$ **Solution**: $E_n = \frac{n^2h^2}{8mL^2}$ For minimum energy, n=1, $\therefore \quad E_1 = \frac{h^2}{8mL^2} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2} = 60.1 \times 10^{-19}\text{ J} = \frac{60.1 \times 10^{-19}}{1.602 \times 10^{-19}} = 37.5\text{eV}$ $E_2 = 4 \times E_1 = 4 \times 60.1 \times 10^{-19} J = 2.404 \times 10^{-17} J = 150.06\text{eV}$ $E_3 = 9 \times E_1 = 9 \times 60.1 \times 10^{-19} J = 5.409 \times 10^{-17} J = 337.5\text{ eV}$	
26	An X-ray of wavelength $0.3Å$ is scattered through an angle 45°by a loosely bound electron. Find the wavelength of the scattered photon. **Given:** $\lambda = 0.3\ A^0$ $\phi = 45^0$ $m_o = 9.1X10^{-31}kg$ $h = 6.63X10^{-34}$ Js $c = 3X10^8$m/s Find λ'= wavelength of scattered photon **Solution**: Compton shift $= \Delta\lambda = \lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\emptyset)$ $\Delta\lambda = \frac{h}{m_o c}(1 - \cos\emptyset) = \frac{6.63 \times 10^{-34}}{9.1X10^{-31} \times 3X10^8}(1 - cos45°) = 0.0242Å(1 - 0.707)$ $= 7.0906 \times 10^{-3}Å \quad = 7.0906 \times 10^{-13}m$ Since Wavelength of scattered photon $= \lambda' = \lambda + \Delta\lambda$ $= 0.3 \times 10^{-10}\text{m} + 7.0906 \times 10^{-13}m$ $= 3.0709 \times 10^{-11}m$ = 0.30709 Å .	3
27	A beam of X-rays is scattered by loosely bound electrons at 45° from the direction of beam. The wavelength of the scattered X-rays is 0.22 $Å$. What is the wavelength of the incident X-rays? **Given:** $\lambda' = 0.22$ A° $\phi = 45^o$ $m_o = 9.1 \times 10^{-31}kg$ $h = 6.63 \times 10^{-34}$ Js $c = 3 \times 10^8$m/s $\lambda = ?$ **Solution**: Wavelength of incident X – Ray $\lambda = \boldsymbol{\lambda}' - \frac{h}{m_o c}(1 - \cos\emptyset)$ $\lambda = 0.22Å - \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8}(1 - cos45°)$ $= 0.22Å - 0.0248\ Å\ (1 - cos45°)$	3

	$= 0.212$ Å .	
28	Calculate the de-Broglie wavelength of the orbital electron of hydrogen atom given that its energy is 13.6 eV and a bullet of mass 2 gm travelling with velocity 5x 10^5m/s. **For Electron:** **Given:** $\mathrm{E} = 13.6\ \mathrm{eV} = 13.6 \times 1.602 \times 10^{-19}\mathrm{J} = 2.178 \times 10^{-18}\ \mathrm{J}$ $\mathrm{m} = 9.1 \times 10^{-31}\mathrm{Kg}$ $\mathrm{h} = 6.63 \times 10^{-34}\ \mathrm{Js}$ $\lambda = ?$ $\textbf{Solution}: \lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2.178 \times 10^{-18}}} = \frac{6.63 \times 10^{-34}}{1.99 \times 10^{-24}}$ $= 3.331 \times 10^{-10}\mathrm{m} = 3.331$Å For bullet: $\mathrm{h} = 6.63 \times 10^{-34}\ \mathrm{Js}$ $\mathrm{m} = 2\ \mathrm{gm} = 2 \times 10^{-3}\mathrm{Kg}$ $\mathrm{v} = 5\mathrm{x}\ 10^5\mathrm{m/s}$ $\lambda = ?$ $\textbf{Solution}: \lambda_{\mathrm{bullet}} = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{2 \times 10^{-3} \times 5\mathrm{x}\ 10^5} = 6.63 \times 10^{-37}m$	