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B.E. All Branches Second Semester (C.B.S.) / B.E. (Fire Engineering) Second Semester

Applied Mathematics - II

P. Pages : 3 **NJR/KS/18/4342/4998**

Time : Three Hours * 0 7 6 1 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) Show that if $n > -1$, **6**

$$\int_0^\infty x^n e^{-k^2x^2}\,dx = \frac{1}{2k^{n+1}}\left|\frac{n+1}{2}\right.$$

b) Evaluate : $\int_0^1 \frac{x^a - x^b}{\log x}\,dx,\ a>0,\ b>0$ **6**

by differentiating under integral sign.

OR

2. a) Evaluate : $\int_0^{2a} x\sqrt{2ax - x^2}\,dx$ **6**

b) Find the R.M.S. value for one complete period of the function $f(t) = \frac{1}{2} + \cos t$. Hence Show that peak value $= \sqrt{3}$ (R.M.S. Value) **6**

3. a) Trace the curve $ay^2 = x(x-a)^2$. **6**

b) Find the perimeter of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. **6**

OR

4. a) Find the area outside the circle r = 2a cos θ and inside the cardioid r = a (1+ cos θ). **6**

b) Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the X axis. **6**

5. a) Evaluate $\iint \frac{xy}{\sqrt{1-y^2}}\,dx\,dy$, over the positive quadrant of the circle $x^2 + y^2 = 1$. **6**

b) Evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$, by changing the order of integration. **6**

c) Change into polar coordinates and evaluate $\int_0^{\infty} \int_0^{\infty} e^{-\left(x^2+y^2\right)} dy\,dx$. **6**

OR

6. a) Evaluate $\int_1^3 \int_{1/x}^{1} \int_0^{\sqrt{xy}} xyz\,dz\,dy\,dx$ **6**

b) Find the mass of the area bounded by the curves $y = x^2$ and $x = y^2$, if the density at any point is $\rho = \lambda\left(x^2 + y^2\right)$. **6**

c) Find the area outside the circle r = a cos θ and inside the circle r = 2a cos θ. **6**

7. a) Show that vector $\bar{d}$ can be expressed in form **6**

$$\bar{d} = \frac{[\bar{d}\,\bar{b}\,\bar{c}]\,\bar{a} + [\bar{d}\,\bar{c}\,\bar{a}]\,\bar{b} + [\bar{d}\,\bar{a}\,\bar{b}]\,\bar{c}}{[\bar{a}\,\bar{b}\,\bar{c}]}$$

if $[\bar{a}\,\bar{b}\,\bar{c}] \neq 0$.

b) Find the directional derivative of $x^2 y^2 z^2$ at the point (1, 1, –1) in the direction of the tangent to the curve $x = e^t$, $y = \sin 2t + 1$, $z = 1 - \cos t$ at $t = 0$. **6**

c) Show that $\bar{F} = \left(2xy + z^3\right)i + x^2 j + 3xz^2 k$ is irrotational and hence find its scalar potential. **6**

OR

8. a) Show that **6**

i) curl grad $\phi = 0$

ii) div curl $\bar{A} = 0$

where $\bar{A} = A_{1i} + A_{2j} + A_{3k}$

b) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. Find the components of its velocity and acceleration at t = 1 in the direction of i –3j + 2k. **6**

c) Find the value of n for which vector $r^n \bar{r}$ will be solenoidal. **6**

9. Verify Green's theorem in the plane for **7**

$\oint\left[\left(3x^2-8y^2\right)dx+(4y-6xy)dy\right]$

where C is the boundary of the region bounded by $y=\sqrt{x}$ and $y=x$.

OR

10. If $\bar{A}=(y-2x)i+(3x+2y)j$, Find the circulation of $\bar{A}$ about a circle C in the XY-plane with centre at origin and radius 2 if C is traverse in the positive direction. **7**

11. a) Fit a curve $y=ax^b$ to the following data. **7**

x	1	2	3	4	5	6
y	2.98	4.26	5.21	6.10	6.80	7.5

b) Using Lagranges interpolation formula, find two missing terms from following table. **6**

x	1	3	4	8	10
y	8	–	11	32	–

OR

12. a) Solve $y_{n+2}+5y_{n+1}+6y_n=n+2^n$. **6**

b) Find the coefficient of correlation between the variables x and y hence find the regression lines. **7**

x	3	5	6	8	9	11
y	2	3	4	6	5	8

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