Faculty of Engineering & Technology

Fourth Semester B.E. (CT/CSE/IT) C.B.S. Examination

DISCRETE MATHEMATICS AND GRAPH THEORY

Time : Three Hours] [Maximum Marks : 80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Solve Question No. **1** **OR** Question No. **2**.

(3) Solve Question No. **3** **OR** Question No. **4**.

(4) Solve Question No. **5** **OR** Question No. **6**.

(5) Solve Question No. **7** **OR** Question No. **8**.

(6) Solve Question No. **9** **OR** Question No. **10**.

(7) Solve Question No. **11** **OR** Question No. **12**.

(8) Due credit will be given to neatness and adequate dimensions.

1. (a) Determine the validity of following argument by using truth table.

"If I try hard and I have a talent, then I will become a scientist. If I become scientist, then I will be happy. Therefore, if I will not be happy, then I did not try hard or I do not have talent." 5

(b) Prove that $\sqrt{3}$ is irrational. 5

OR

2. (a) Using Mathematical induction show that, for all positive integers n, $(n^3 - 4n + 6)$ is divisible by 3. 5

(b) Using laws of algebra of sets, prove that :

$$A - B = A \cap B^C$$

where A and B are subsets of universal set. 5

3. (a) If R is an equivalence relation on Set A, prove that R^{-1} is an equivalence relation. 6

(b) Let A = {1, 2, 3, 4} and let R = {(1,2), (2,3), (3,4), (2,1)} find the transitive closure of R. 6

(c) If $f : X \to Y$ and $g : Y \to Z$ and both f and g are one-one onto, then show that gof is also one-one onto and $(gof)^{-1} = f^{-1} o\, g^{-1}$. 6

OR

4. (a) Let A be the set of non-zero integers and let R be the relation on A × A defined by :

$$(a,b)\ R\ (c,d) \Leftrightarrow ad = bc$$

Show that R is an equivalence relation. 6

(b) Let A be given finite set and P(A) be its power set. Let "$\subseteq$" be the relation of P(A). Draw Hasse diagram of $(P(A),\subseteq)$ for (i) $A = \{a,b\}$, (ii) $A = \{a,b,c\}$. 6

(c) Let R and S be the relations on $\{1,2,3,4\}$ defined by :

$R = \{(1,1), (1,2), (3,4), (4,2)\}$ and

$S = \{(1,1), (2,1), (3,1), (4,4), (2,2)\}$.

Find RoS, M_{RoS}. Draw diagraph of RoS. 6

5. (a) Prove that the set {0, 1, 2, 3, 4} is a finite abelian group of order 5 under addition modulo 5 as a composition. 6

(b) Does the following table defined a semigroup or a monoid ?

*	a	b	c
a	c	b	a
b	b	c	b
c	a	b	c

6

OR

6. (a) Prove that $f : R \to R_+$ denoted by $f(x) = e^x$ an isomorphism of R onto R_+. Here R is the additive group of real numbers and R_+ is the multiplicative group of positive real numbers. 6

(b) Prove that any two right cosets of a subgroup H are either disjoint or identical. 6

7. (a) Show that $(z_6, +_6, \times_6)$ is a ring. Explain whether it is an integral domain. 7

(b) Show that the intersection of two subrings of a ring R is a subring. 5

OR

8. (a) Show that $(I, \oplus, \odot)$ is a commutative ring with identity where the operations $\oplus$ and $\odot$ are defined as :

for any $a, b \in I$, $a \oplus b = a + b - 1$

and $a \odot b = a + b - ab$

where I is set of integers. 7

(b) Construct a circuit for the Boolean expression $(A \cdot B) + [A' \cdot (A + B + B')]$. Simplify and construct the equivalent circuit. 5

. (a) Define :

(i) Complete Graph

(ii) Regular Graph

(iii) Isomorphic Graph. 6

(b) Draw the diagraph corresponding to the adjacency matrix :

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

find transitive closure of the diagraph. 6

(c) Draw a directed tree with 4 nodes at level 1, six nodes at level 2. Obtain the corresponding binary tree. 6

OR

10. (a) Define :

(i) Diagraph

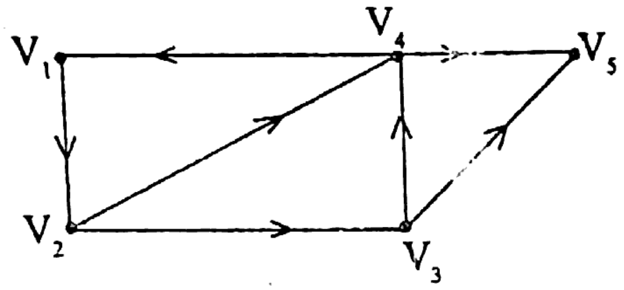
(ii) Indegree and outdegree of a node

(iii) Path and cycle. 6

(b) Draw diagraph corresponding to $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

and interpret AA^T, A^TA and A^2. 6

(c) Find all indegrees and outdegrees of the graph given below :



Give all elementary cycles of this graph. Is there any source or sink ? 6

11. (a) Solve the following recurrence relation :

$$a_n - 9a_{n-1} + 20a_{n-2} = 0, \ a_0 = -3, \ a_1 = -10. \quad 5$$

(b) Find the minimum number of elements that one needs to take from the set S = {1, 2, 3, ..., 9} to be sure that two of the numbers add up to 10. 5

OR

12. (a) Prove that $C(n + 1, r) = C(n, r) + C(n, r - 1)$. 5

(b) Find the generating function of the sequence $\{a_k\}$ if $a_k = 2 + 3k$. 5