Faculty of Engineering & Technology

Second Semester B.E. Examination

APPLIED MATHEMATICS

Paper – II

Time—Three Hours] [Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) Attempt Q.1 OR Q.2, Q.3 OR Q.4, Q.5 OR Q.6, Q.7 OR Q.8, Q.9 OR Q.10, Q.11 OR Q.12.

(2) Retain the construction lines.

(3) Figures to the right indicate full marks.

(4) Use of non-programmable calculator is permissible.

1. (a) Evaluate

$$\int_0^{\pi} \sin^5 x(1-\cos x)^3\, dx.$$ 6

(b) Show that the mean value of kx $(l - x)$ between $x = 0$ to $x = l$ is two-third of its maximum value. 6

OR

2. (a) By successive differentiation of

$$\int_0^1 x^m\, dx = \frac{1}{m+1}$$ w.r.t. m, evaluate

$$\int_0^1 x^m (\log x)^n\, dx.$$

6

(b) Show that

$$\int_0^{\infty} e^{-k^2x^2} x^n\, dx = \frac{1}{2k^{n+1}} \left\lceil\left(\frac{n+1}{2}\right)\right.$$

Hence find

$$\int_0^{\infty} e^{-x^2}\, dx.$$ 6

3. (a) Trace the curve $ay^2 = x^2 (a - x)$. 6

(b) Find the area enclosed between the curve $y^2 (2a - x) = x^3$ and its asymptote. 6

OR

4. (a) Find the perimeter of the curve

$r = a (1 + \cos \theta)$. 6

(b) A loop of the curve $y^2 = x^2(1 - x^2)$ is rotated about the x-axis. Find the volume generated. 6

5. (a) Evaluate $\iint (x^2 + y^2)\, dxdy$ over the region in the positive quadrant for which $x + y \le 1$. 6

(b) Change the order of integration and hence evaluate

$$\int_0^{\infty}\int_0^{x} x e^{\frac{-x^2}{y}}\, dy\, dx.$$ 6

(c) Evaluate :

$$\int_0^{1}\int_{y^2}^{1}\int_0^{1-x} x\, dz\, dx\, dy.$$ 6

OR

6. (a) Evaluate

$$\iint \frac{r}{\sqrt{a^2+r^2}}\, dr\, d\theta$$

over one loop of lemniscate $r^2 = a^2 \cos 2\theta$. 6

(b) Evaluate

$$\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{3/2}}\, dx\, dy$$

by changing into Polar Coordinates. 6

(c) Find the mass of the area bounded by the curves $y = x^2$ and $x = y^2$, if the density at any point is given by $\rho = K\,(x^2 + y^2)$. 6

7. (a) If the vector $\bar{x}$ and scalar λ satisfies the equation

$\bar{a} \times \bar{x} = \lambda \bar{a} + \bar{b}$ and $\bar{a} \cdot \bar{x} = 1$. Find the value of λ and $\bar{x}$

in terms of $\bar{a}$ and $\bar{b}$. 6

(b) Find the directional derivatives of $\phi = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the vector $i + 2j + 2k$. 6

(c) Show that $\bar{F} = (4xy - z^3)i + 2x^2 j - 3xz^2 k$

is conservative force field. Find the scalar potential function ϕ. 6

OR

8. (a) Prove that

$$(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d}) = \begin{vmatrix} \bar{a}\cdot\bar{c} & \bar{b}\cdot\bar{c} \\ \bar{a}\cdot\bar{d} & \bar{b}\cdot\bar{d} \end{vmatrix}.$$ 5

(b) Find the angle between the tangents to the curve

$\bar{r} = t^2 i + 2tj - t^3 k$ at the points $t = \pm 1$. 6

(c) A particle moves along a curve

$$\bar{r} = (t^3 - 4t)\,i + (t^4 + 4t)\,j + (8t^2 - 3t^3)\,k$$

where t is the time. Find the magnitude of tangential and normal components of its acceleration at $t = 2$. 7

9. Evaluate $\iint_S \bar{A} \cdot \hat{n}\, ds$, where

$\bar{A} = (x + y^2)\,i - 2xj + 2yz\,k$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant. 7

OR

10. Evaluate $\oint_C [(x^2 - \cosh y)\,dx\ (y + \sin x)\,dy]$ by Green's Theorem, where C is the rectangle with the vertices $(0, 0)$, $(\pi, 0)$, $(\pi, 1)$ and $(0, 1)$. 7

11. (a) Using method of least squares, fit a curve $y = ab^x$ to the following data :

x	2	3	4	5	6
y	145	175	210	250	300

7

(b) Apply Lagrange's interpolation formula to find f (x) from the following data :

x	0	1	2	5
f (x)	2	3	12	147

6

OR

12. (a) Calculate the coefficient of correlation and hence the equations of lines of regression for the following data : 7

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

(b) Solve $y_{x+2} - 4y_x = 9x^2$. 6