

Faculty of Engineering & Technology
Second Semester B.E. (C.B.S.) Examination
APPLIED MATHEMATICS—II
Paper—I

Time—Three Hours]

[Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

- (1) Attempt
- SIX**
- questions as follows :

Que. No. 1 **OR** Que. No. 2Que. No. 3 **OR** Que. No. 4Que. No. 5 **OR** Que. No. 6Que. No. 7 **OR** Que. No. 8 rtmnuonline.comQue. No. 9 **OR** Que. No. 10Que. No. 11 **OR** Que. No. 12

- (2) Figures to the right indicate full marks.

- (3) Use of non-programmable calculator is permitted.

1. (a) If
- $B(n, 3) = \frac{1}{3}$
- and
- n
- is Positive Integer, find
- n
- . 5

- (b) Using differentiation under Integral sign, evaluate

$$\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)} dx. \quad \text{rtmnuonline.com} \quad 7$$

OR

2. (a) A rod of length 'a' is divided at random into two parts. Find the mean value of the sum of the squares of these two segments. 6

- (b) Evaluate
- $\int_0^1 (x \log x)^3 dx$
- . 6

3. (a) Trace the curve
- $9ay^2 = (x-2a)(x-5a)^2$
- . 6

- (b) Find the area of the loop of the curve
-
- $ay^2 = x^2(a-x)$
- . 6

OR

4. (a) If
- S
- is the arc of the curve
- $y^2 = x\left(1 - \frac{x}{3}\right)^2$
- measured from the origin to the point
- (x, y)
- . Show that

$$S^2 = y^2 + \frac{4}{3}x^2 \quad 6$$

- (b) Find the volume of the solid generated by revolution of the curve
- $y = \frac{a^3}{a^2 + x^2}$
- about its asymptote. 6

5. (a) Evaluate
- $\iint e^{2x+3y} dx dy$
- , over the triangle bounded by
- $x = 0$
- ,
- $y = 0$
- and
- $x + y = 1$
- . 6

- (b) Evaluate
- $\int_0^a \int_0^x \frac{\cos y}{\sqrt{(a-x)(a-y)}} dy dx$
- by changing the order of integration. 6

- (c) Evaluate
- $\iint (a^2 - x^2 - y^2) dx dy$
- , over the semicircle
- $x^2 + y^2 = ax$
- in the Positive quadrant by changing into Polar co-ordinates. 6

OR

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6. (a) Evaluate $\iint r \sin \theta \, dr d\theta$ over the cardioid $r = a(1 + \cos \theta)$ above the initial line. 6

(b) Find the mass of the area between $y = x^3$ and $x = y^2$ if $\rho = k(x^2 + y^2)$. 6
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(c) Evaluate $\int_{-1}^1 \int_0^2 \int_{x-2}^{x+2} (x + y + z) \, dy dx dz$. 6

7. (a) Show that a vector $\vec{d} = x\vec{a} + y\vec{b} + z\vec{c}$, where x, y, z are Scalars can be expressed in the form

$$\vec{d} = \frac{[\vec{d}\vec{b}\vec{c}]\vec{a} + [\vec{d}\vec{c}\vec{a}]\vec{b} + [\vec{d}\vec{a}\vec{b}]\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$$

where $[\vec{a}\vec{b}\vec{c}] \neq 0$. 5

(b) The position vector of a point at time t is given by $\vec{r} = e^t(\cos t \, \vec{i} + \sin t \, \vec{j})$. Show that $\vec{a} = 2(\vec{v} - \vec{r})$, where \vec{a} , \vec{v} are acceleration and velocity of a Particle. 7
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(c) Find the angle between the tangents to the curve $y = x^2$, $z = x^3$ at $(1, 1, 1)$ and $(-1, 1, -1)$. 6

OR

8. (a) Prove that :

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\vec{c}\vec{d}]\vec{b} - [\vec{b}\vec{c}\vec{d}]\vec{a}. \quad 4$$

(b) Find the directional derivative of $\phi = 4e^{2x-y+z}$ at the point $(1, 1, -1)$ in the direction towards the point $(-3, 5, 6)$. 7
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(c) Show that $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational. Also find its Scalar Potential. 7

9. Using the line integral, compute the work done by the force $\vec{F} = (2y + 3)\vec{i} + xz\vec{j} + (yz - x)\vec{k}$ when it moves a particle from the point $(0, 0, 0)$ to the point $(2, 1, 1)$ along the curve $x = 2t^2$, $y = t$, $z = t^3$. 7

OR

10. Use Divergence theorem to evaluate $\iiint_S \vec{F} \cdot \hat{n} \, ds$ where

$\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$, $z = 3$. 7

11. (a) Fit a curve of the form $y = ae^{bx}$ to the following data : 7
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x	1	2	3	4	5	6
y	14	27	40	55	68	300

(b) Obtain the function whose first difference is $x^3 + 3x^2 + 5x + 12$. 6

OR

12. (a) Obtain the rank correlation coefficient for the following data :

x	25	28	39	25	24	30	48	45	38	25
y	44	34	28	32	34	29	35	42	24	30

(b) Solve $(E^2 - 5E + 6)y_n = 4^n(n^2 - n + 5)$. 6
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