B.E.(Computer Science & Engineering (New) / Computer Technology)
Semester Third (C.B.S.)

Applied Mathematics

Paper - I

P. Pages : 3 **KNT/KW/16/7232/7237**

Time : Three Hours Max. Marks : 80

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Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = F(S)$ then prove that **6**

$L\{f'(t)\} = s\,L\{f(t)\} - f(0)$

and hence find $L\left\{\frac{d}{dt}\left(\frac{\sin t}{t}\right)\right\}$.

b) Find $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$ by Convolution Theorem. **6**

OR

2. a) Express $f(t) = \begin{cases} t-1\,; & 1<t<2 \\ 3-t\,; & 2<t<3 \end{cases}$ **6**

in terms of unit step function and find its Laplace transform.

b) Solve $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$, given y(0) = 1 by using Laplace Transform. **6**

3. a) Obtain Fourier Series for $f(x) = 1 + \frac{2x}{\pi}\,;\ -\pi \le x \le 0$ **6**

$= 1 - \frac{2x}{\pi}\,;\ 0 \le x \le \pi$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.. = \frac{\pi^2}{8}$

b) Solve the integral equation **6**

$$\int_0^\infty f(t)\cos\lambda t\;dt = \begin{cases} 1\,, & 0 \le \lambda < 1 \\ 2\,, & 1 \le \lambda < 2 \\ 0\,, & \lambda \ge 2 \end{cases}$$

OR

4. a) Find Fourier sine transform of $\dfrac{e^{-ax}}{x}$. **6**

b) Draw the graph of the function **6**

$$f(x)=\begin{cases}-1, & -2\le x\le -1\\ x\ , & -1\le x\le 1\\ 1\ , & 1\le x\le 2\end{cases}$$

Discuss the symmetry and find the Fourier series for the function.

5. a) Prove that $Z\{n^p\}=-Z\dfrac{d}{dz}Z\{n^{p-1}\}$, p is a positive integer, hence find $Z\{n\}$. **6**

b) Prove that $\dfrac{1}{n!}*\dfrac{1}{n!}=\dfrac{2^n}{n!}$ where $*$ is a convolution operation. **6**

OR

6. a) Find inverse Z - transform of $\dfrac{Z^2+Z}{(Z-1)(Z^2+1)}$. **6**

b) By using Z - transform solve the difference equation **6**

$y_{n+2}+6y_{n+1}+9y_n=2^n$, given $y_0=y_1=0$.

7. a) If $u=y^3-3x^2y$, show that u is harmonic function. Find V and analytic function. **7**

b) Evaluate $\displaystyle\int_C \frac{\cos\pi Z^2}{(Z-1)(Z-2)}\,dz$, where C is circle $|Z|=3$. **7**

OR

8. a) Expand $f(Z)=\dfrac{Z^2-1}{(Z+2)(Z+3)}$ in the region **7**

i) $|Z|<2$

ii) $2<|Z|<3$ and

iii) $|Z|>3$

b) Evaluate $\displaystyle\int_0^{2\pi}\frac{\cos 2\theta}{5+4\cos\theta}\,d\theta$ by contour Integration. **7**

9. a) Find eigen vectors for the matrix **6**

$$A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$$

b) If $A=\begin{bmatrix}-1 & 3\\ 1 & 1\end{bmatrix}$, verify $2\sin A=(\sin 2)A$ by Sylvester's theorem. **6**

c) Determine the largest eigen value and corresponding eigen vector of the matrix : **6**

$$A=\begin{bmatrix}-4 & -5\\ 1 & 2\end{bmatrix}$$

OR

10. a) Verify Cayley Hamilton's Theorem for $A=\begin{bmatrix}1 & 2 & 4\\ 2 & 1 & 2\\ 4 & 2 & 1\end{bmatrix}$ and hence find A^{-1}. **6**

b) Are the following vectors are linearly dependent? If so, find the relation between them **6**

$X_1=[1,1,1,3]$, $X_2=[1,2,3,4]$, $X_3=[2,3,4,7]$

c) Solve by matrix method $\dfrac{d^2y}{dt^2}-3\dfrac{dy}{dt}-10y=0$ given $y(0)=3,\ y'(0)=15$. **6**

11. a) Each of three identical jewellary boxes has two drawers. In each drawer of the first box there is a gold watch. In each drawer of the second box there is a silver watch. In one drawer of the third box there is a gold watch while in other there is a silver watch. If we select a box at random, open one of the drawer and find it to contain a silver watch. What is the probability that the other drawers has gold watch. **6**

b) Let X be a random variable having density function **6**

$$f(x)=\begin{cases} cx & : 0\le x\le 2\\ 0 & : \text{otherwise}\end{cases}$$

find (i) the constant C, (ii) $P\left(\frac{1}{2}<x<\frac{3}{2}\right)$ and (iii) the distribution function.

OR

12. a) A random variable X has prob. density function **6**

$$f(x)=\begin{cases} \frac{1}{b-a} & , a\le x\le b\\ 0 & , \text{otherwise}\end{cases}$$

find (i) mean of X (ii) variance of X (iii) first two moments about origin.

b) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution. **6**

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