Unit 1: Wave Optics

Q. State Huygens principle.

In 1679, Christian Huygens proposed the wave theory of light. According to Huygens wave theory, each point on the wave front is to be considered as a source of secondary wavelets. It explains reflection, refraction, dispersion, double refraction, diffraction, interference, and polarisation properties of light. It fails to explain, photoelectric effect, black body radiation etc.

Q. State principle of superposition of waves.

The principle of superposition says:

When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

Q. What is interference of waves? Explain constructive and destructive

interference.

Interference concept is explained on the basis of superposition of wave's concept.

When two light waves superimpose, then the resultant amplitude or intensity in the region of superposition is different from the amplitude of individual waves.

Definition:-

The modification in the distribution of intensity in the region of superposition is known as interference.

In case of interference pattern we observe two cases

- Constructive interference
- Destructive interference

Constructive interference

The waves are reaching at a point are in phase constructive interference occurs

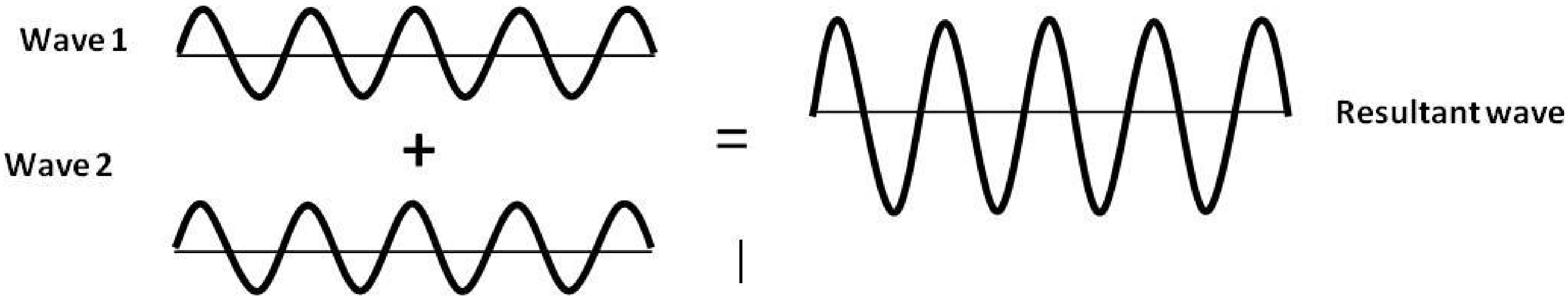
In constructive interference, the resultant amplitude is always equal to the **sum of the amplitudes of two individual waves.**

Condition

The path difference between the two waves is equal to the integral multiple of wavelength (λ) the constructive interference occurs.

Path difference $= n\lambda$

Where n = 0, 1, 2, 3, 4 …



Destructive interference

The waves are reaching at a point are in out of phase destructive interference occurs

In Destructive interference, the resultant amplitude is always equal to the

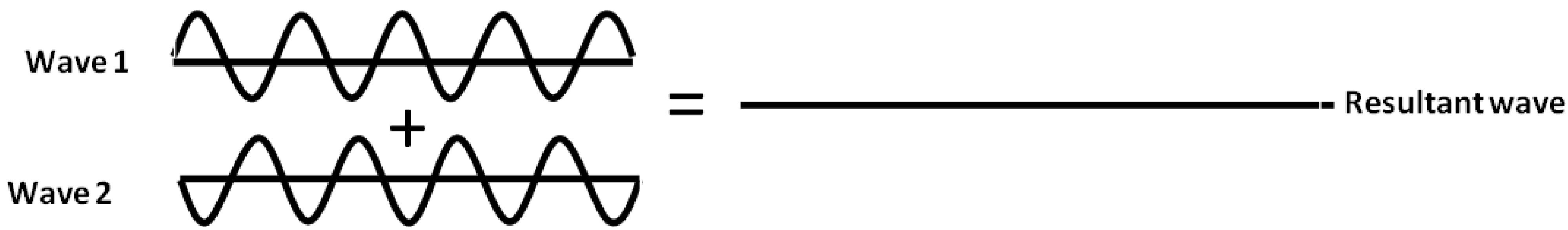
difference of the amplitudes of two individual waves.

Condition

The path difference between the two waves is equal to the odd integral multiple of $\lambda/2$ destructive interference occurs

Path difference $= \frac{(2n-1)\lambda}{2}$

where, n = 1, 2, 3, 4, …



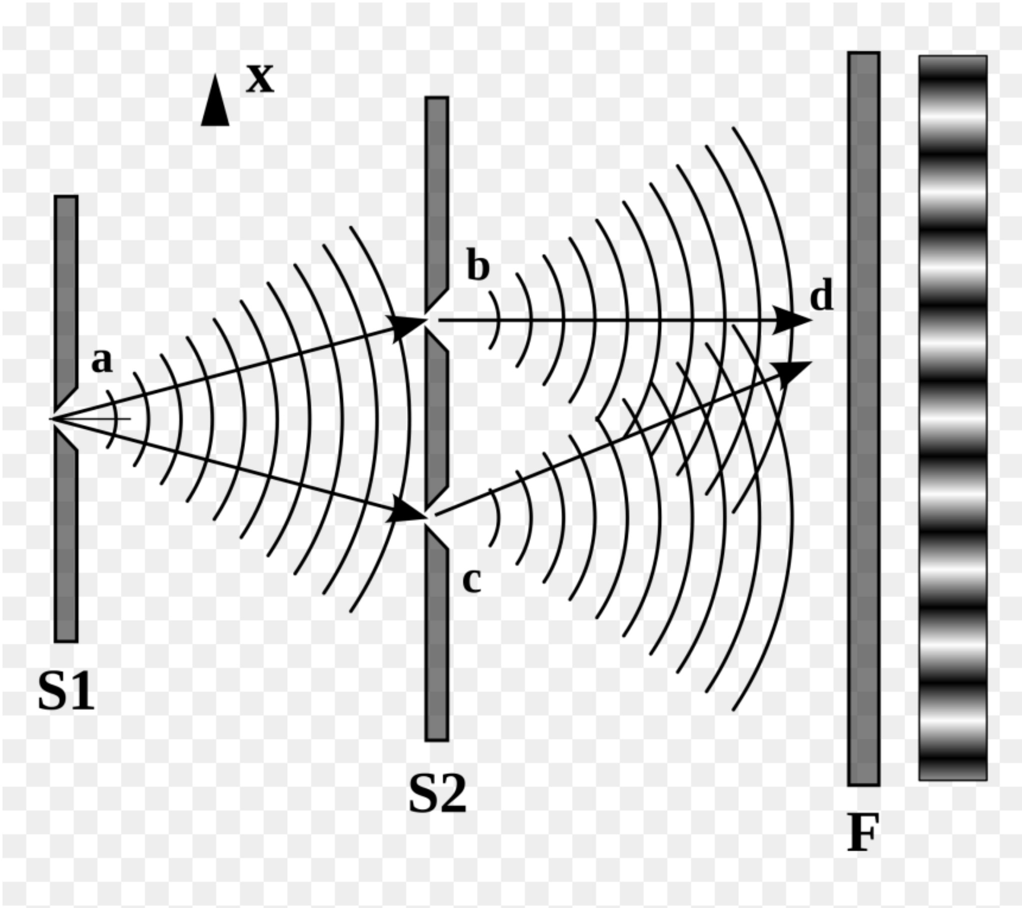
Types of interference:-

For the formation of interference patterns, two coherent light sources are required. To get two coherent sources from a single light source, two techniques are used. They are

1. Division of wave front
2. Division of amplitude

Division of wave front

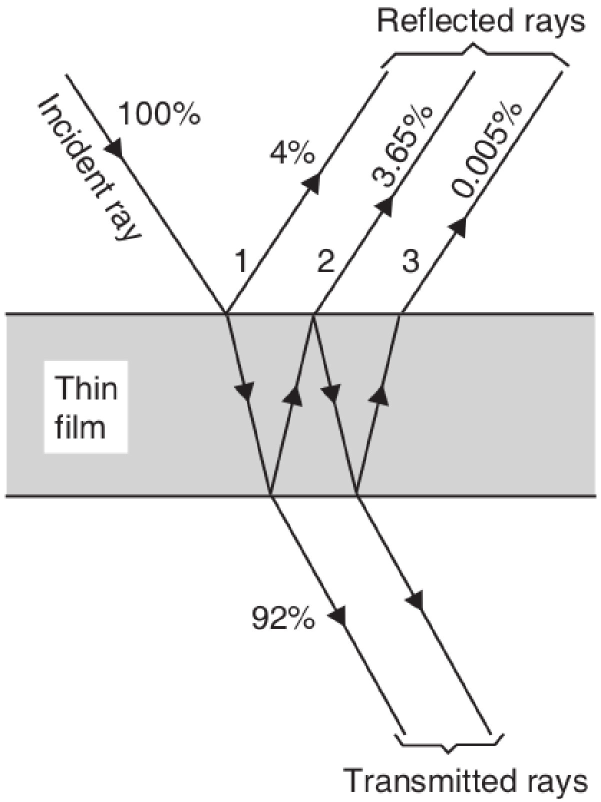
The wave front from a single light source is divided into two parts using the phenomenon of reflection, refraction, or diffraction. Young's double slit experiment belongs to this class of interference.



Division of amplitude

The amplitude of a single light beam is divided into two parts by parallel reflection or refraction.

Newton's ring experiment, Michelson's interferometer, belongs to this class of interference.



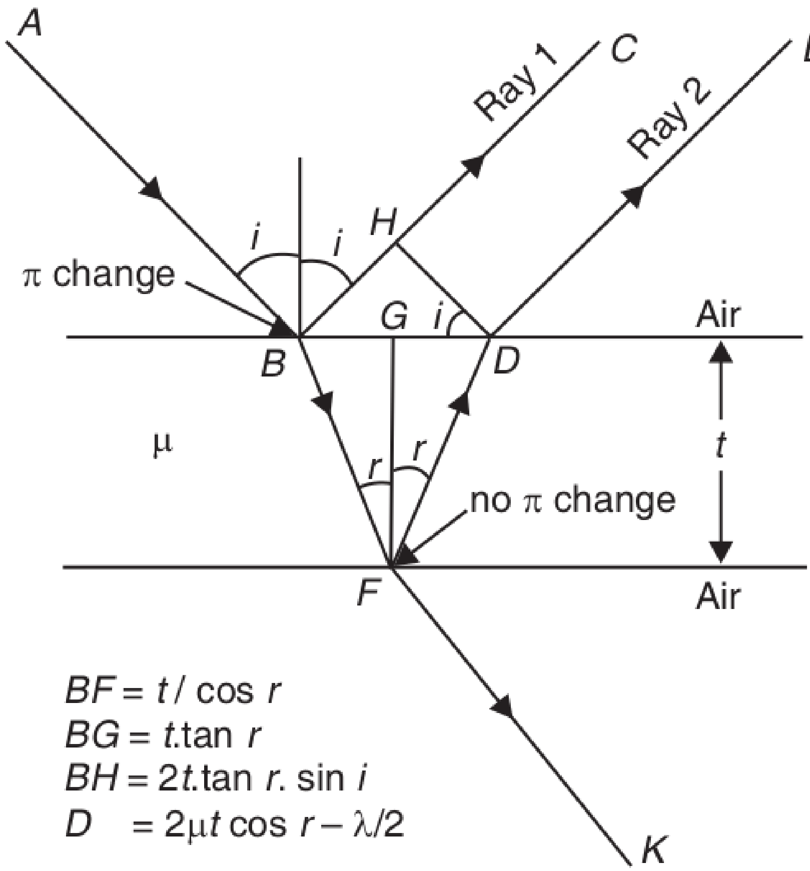
Conditions for sustained interference

1) Two light sources of emitting light waves should be coherent.
2) Two sources must emit continuous light waves of same wavelengths or frequency.
3) The separation between the two sources should be small.
4) The distance between the two sources and the screen should be large.

5) To view interference fringes, the background should be dark.
6) The amplitude of light waves should be equal or nearly equal.
7) The sources should be narrow.
8) The sources should be monochromatic.

Q. Derive an expression for path difference and conditions for constructive and destructive interference for phenomenon of interference in thin parallel film in reflected light.

Consider a transparent film of uniform thickness t, bounded by two parallel surfaces, as shown in figure. Let the refractive index of the material be μ. Let a monochromatic source illuminate the film. AB be one of the incident rays. The ray AB is partly reflected along BC and partly transmitted to the film along BF. BF is reflected partly as FD and transmitted partly as FK. Ray FD refracts at the boundary as DE. The waves travelling along BC and BFDE are coherent and produce interference. Condition of interference depends on the optical path difference between rays 1 and 2 as shown in figure.



The geometric path difference between the rays 1 and 2 are given as,

Path difference = BF + FD - BH

Optical path difference,

$$\Delta = \mu(BF + FD) - BH \qquad (i)$$

In ΔBFD,

∠BFG = ∠GFD = ∠r;

BF = FD and BG = GD

Further,

$$BF = \frac{FG}{\cos r} = \frac{t}{\cos r}$$

$$\therefore BF + FD = \frac{2t}{\cos r} \qquad (ii)$$

Also,

$$BG = FG.\tan r$$

$$BG = t.\tan r \qquad (iii)$$

In the ΔBHD,

$\angle HBD = (90 - i)$

And $\angle BHD = 90$

$$\therefore \angle BDH = i$$

Also by trigonometry,

$$BH = BD.\sin i$$

$$BH = 2BG\sin i$$

From (iii),

$$BH = 2t\tan r\sin i$$

From Snell's law,

$$\sin i = \mu\sin r$$

$$\therefore BH = 2t\tan r\,(\mu\sin r)$$

$$\therefore BH = \frac{2\mu t\sin^2 r}{\cos r} \qquad (iv)$$

From i, ii and iv,

$$\Delta = \mu\left(\frac{2\,t}{\cos r}\right) - \frac{2\mu t \sin^2 r}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$\Delta = \frac{2\mu t \cos^2 r}{\cos r}$$

$$\Delta = 2\,\mu\, t\, cos\, r$$

But at B, light is being reflected from a denser medium which introduces a path difference of $\lambda/2$ in ray 1 which does not happen with ray 2.

Therefore, true path difference,

$$\Delta = 2\mu t. \cos r - \frac{\lambda}{2}$$

Expression for maxima (Brightness):

If the path difference between the rays is integral multiple of wavelength (λ) of light then we get constructive interference or maxima i.e.

$$2\mu t.\cos r - \frac{\lambda}{2} = m\lambda$$

$$2\mu t.\cos r = m\lambda + \frac{\lambda}{2}$$

$$2\mu t.\cos r = (2m + 1)\frac{\lambda}{2}$$

This is the condition for maxima.

Expression for minima (Darkness):

If the path difference between the rays is odd multiple of $\lambda/2$ then we get destructive interference or minima i.e.

$$2\mu t.\cos r - \frac{\lambda}{2} = (2m + 1)\frac{\lambda}{2}$$

$$2\mu t.\cos r = (2m + 1 + 1)\frac{\lambda}{2} = (m + 1)\lambda$$

The phase relationship of the interfering waves does not change if one full wave is added to or subtracted from any of the interfering waves.

Hence,

$$2\mu t.\cos r = m\,\lambda$$

This is the condition for minima.

Q. What happens when thin film is exposed to sun (white) light?

OR

Q. Why do thin films exposed to sun (white) light exhibit colours?

White light consists of many wavelengths (colours) so when it is incident on thin film, all colours are reflected from the top and bottom surface. But all of them do not satisfy the condition of brightness (maxima).

Reflected light will have only those colours which satisfy the condition of maxima. The colours which satisfy the condition of darkness (minima) will remain absent. Hence thin film seems coloured under sun (white) light.

Q. What happens when film is very very thin?

When film is so thin i.e. made up of few layers of air molecules and the wavelength of light is such that path difference = $\lambda/2$, then the wave reflected from the upper surface and bottom surface of film will interfere destructively and the film appears dark.

Q. What is a wedge shaped thin film? Obtain an expression for fringe width in wedge shaped thin film.

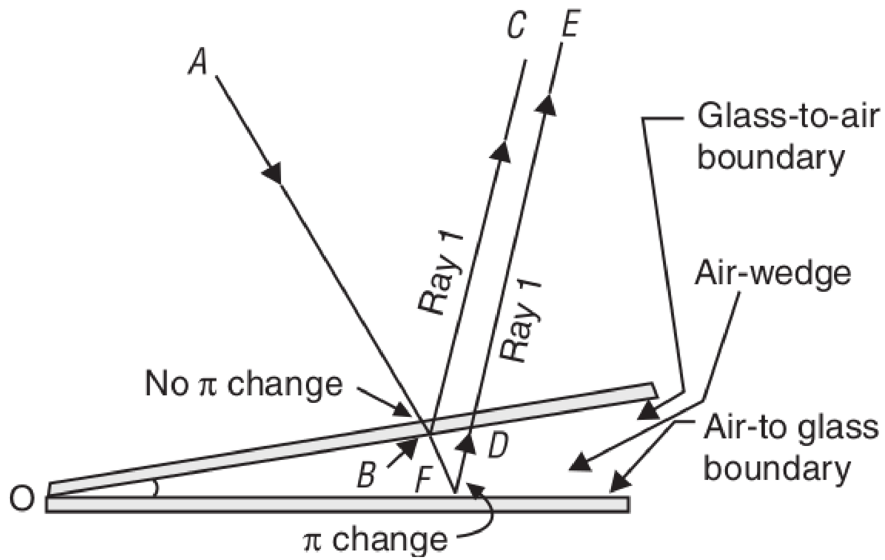
OR

Q. Obtain an expression for fringe width in the interference pattern of wedge shaped film. Explain why the fringe at the apex of the wedge is always dark.

Wedge shaped thin film is a thin film of varying thickness such that it has zero thickness on one end which uniformly increases towards the other end.

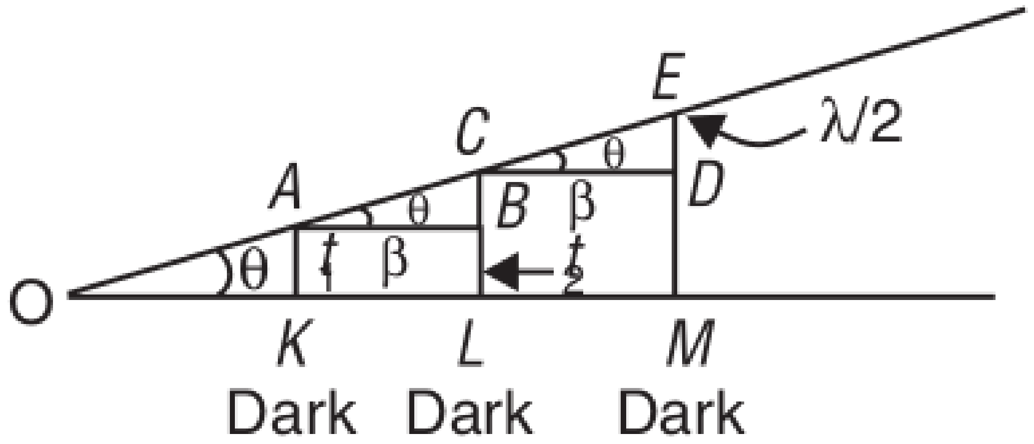
Wedge shaped thin film can be formed by placing two glass slides resting on each other at one edge and separated by a thin space at the opposite edge.

The interference pattern of wedge shaped thin film consists of alternate dark and bright bands of equal width called fringes.



Expression for fringe width (β):

Let us consider a wedge shaped film of refractive index μ and wedge angle θ. A monochromatic light of wavelength λ is incident normally on the wedge.



Let n^{th} dark band appear at A and $(n+1)^{th}$ dark band at C. Let film thickness at point A and C be t_1 and t_2 respectively.

The optical path difference is given as,

$$\Delta = 2\mu t \cos\theta - \frac{\lambda}{2}$$

where $\lambda/2$ takes in account the gain of half–wave due to the abrupt jump of π radians in the phase of the wave reflected from the bottom boundary of air to glass.

Applying condition of destructive interference we get,

$$2\mu t.\cos\theta - \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

$$2\mu t.\cos\theta = (2n + 1 + 1)\frac{\lambda}{2} = (n + 1)\lambda$$

Or

$$2\mu t.\cos\theta = n\lambda$$

At point A, the thickness of the air film is very very small.

$\therefore \theta \approx 0$ and $\cos\theta \approx 1$

Hence the path difference introduced will be

$$2\mu t_1 = n\lambda \qquad (i)$$

At point C,

$$2\mu t_2 = (n + 1)\lambda \qquad (ii)$$

Subtracting i from ii we get,

$$2\mu\left(t_2 - t_1\right) = \lambda$$

$$\left(t_2 - t_1\right) = \frac{\lambda}{2\mu}$$

$$\therefore BC = \left(t_2 - t_1\right) = \frac{\lambda}{2\mu} \qquad (iii)$$

And in ΔABC,

$$\tan\theta = \frac{BC}{AB}$$

where, AB = β is the fringe-width. Hence,

$$\tan\theta = \frac{\left(t_2 - t_1\right)}{\beta}$$

$$\left(t_2 - t_1\right) = \beta.\tan\theta$$

From (iii)

$$\frac{\lambda}{2\mu} = \beta\tan\theta$$

$$\therefore \beta = \frac{\lambda}{2\mu\tan\theta}$$

If θ is very small then tan θ ≈θ

$$\therefore \beta = \frac{\lambda}{2\mu\theta}$$

All the parameters on the right side are constant, hence fringes are equidistant.

Fringe at the apex is dark:

At the apex the two glass slides are in contact with each other. Therefore the thickness of the air film at the contact edge is negligible ($t \cong 0$). The optical path difference becomes,

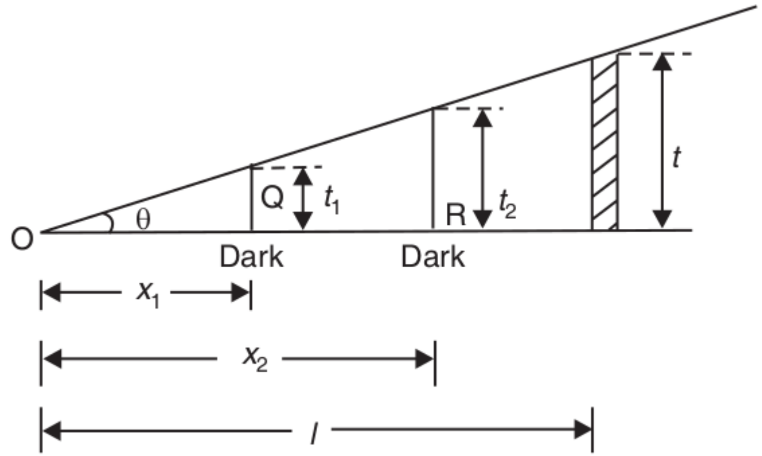
$$\Delta = 2\mu t \cos\theta - \frac{\lambda}{2}$$

$$\Delta = 0 - \frac{\lambda}{2}$$

$$\therefore \Delta = -\frac{\lambda}{2}$$

It implies that the two waves interfere destructively. Therefore, fringes at the apex are dark.

Q. Deduce expression for wedge angle in case of wedge shaped thin film.



Let us consider a wedge shaped film with very small wedge angle θ. Also assume that m[th] dark fringe appears at point Q and (m+N)[th] at R.

Hence at point Q dark fringe is given by

$$2\mu t_1 = m\lambda$$

But,

$$\tan\theta = \frac{t_1}{x_1}$$

$$t_1 = x_1 \tan\theta \cong x_1\theta$$

$$2\mu x_1\theta = m\lambda \qquad \text{(i)}$$

Similarly at B,

$$2\mu t_2 = (m + N)\lambda$$

Where N is the number of dark fringes between Q and R

But,

$$\tan\theta = \frac{t_2}{x_2}$$

$$t_2 = x_2 \tan\theta \cong x_2\theta$$

$$\therefore 2\mu x_2\theta = (m + N)\lambda \qquad (ii)$$

Subtracting (ii) –(i) we get,

$$2\mu\left(x_2 - x_1\right)\theta = N\lambda$$

$$\theta = \frac{N\lambda}{2\mu\left(x_2 - x_1\right)}$$

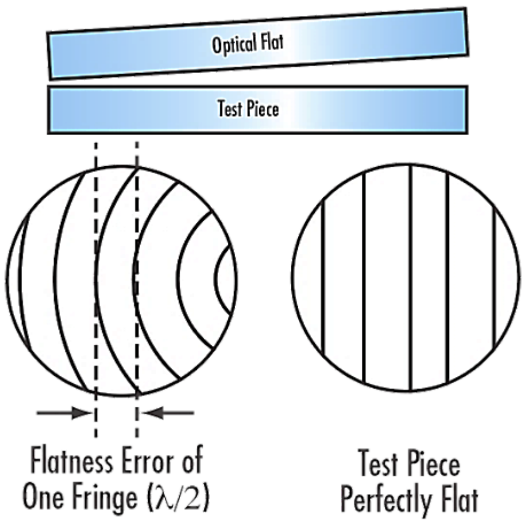
For μ = 1,

$$\theta = \frac{N\lambda}{2\left(x_2 - x_1\right)}$$

By knowing λ, x_1, x_2 and N we can calculate θ.

Q. How interference in wedge shaped thin film is used for testing the optically flat surface?

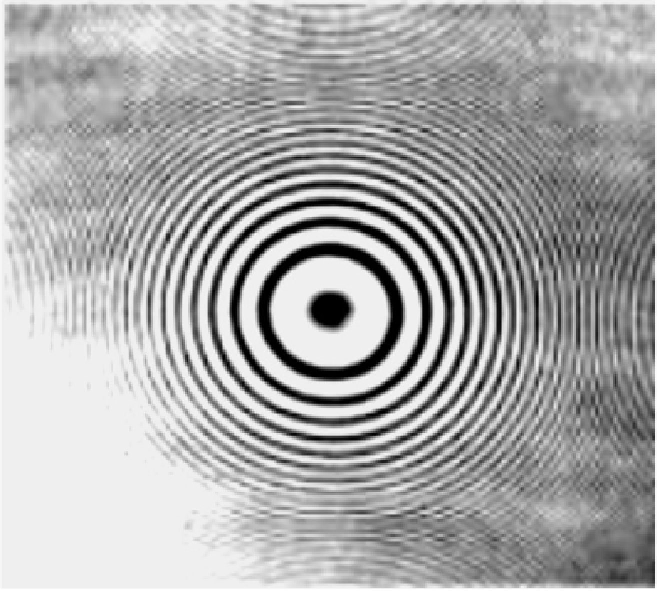
1. Phenomenon of interference in wedge shaped film can be used for checking the optical flatness of the surface.
2. The flatness of the surface can be tested by viewing the fringes formed in wedge shaped film.
3. A wedge shape is formed by placing an optically flat plate on the surface to be tested and is illuminated by light.
4. If the fringes are straight and parallel then the work piece is smooth. If instead curved fringes are seen, the surface is not smooth.



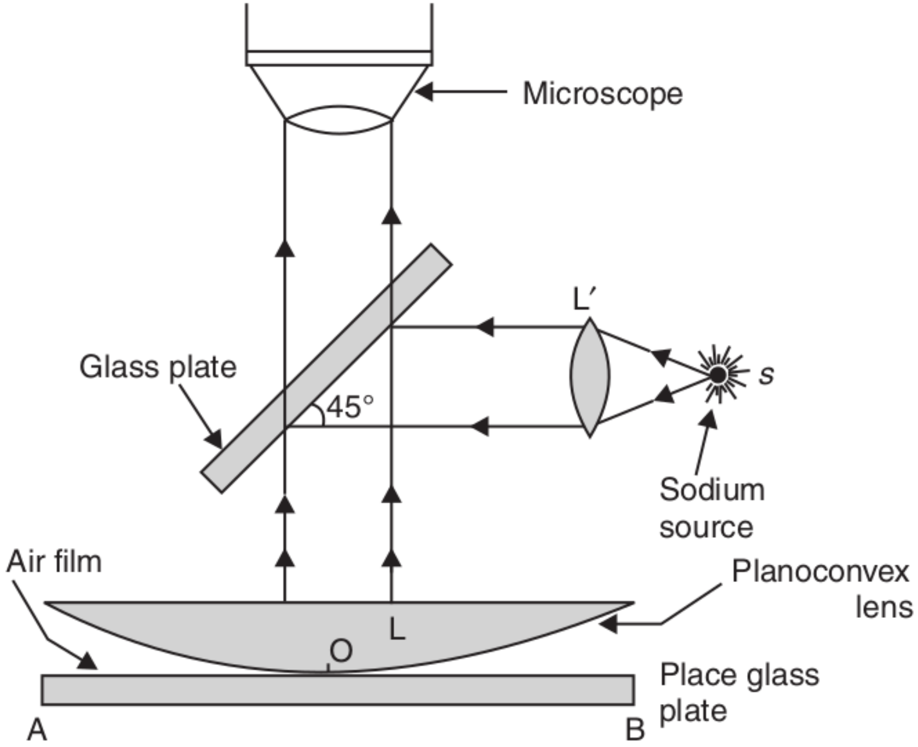
Q. How are Newton's rings formed? Derive expressions for radius of curvature of plano-convex lens.

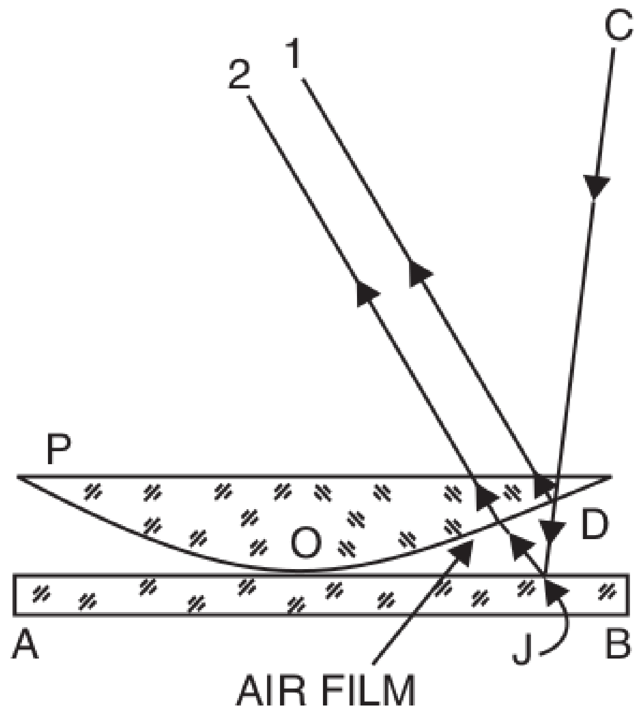
Q. Draw a neat diagram of the experimental set up for Newton's rings formation.

Experimental Set up of Newton's Rings:

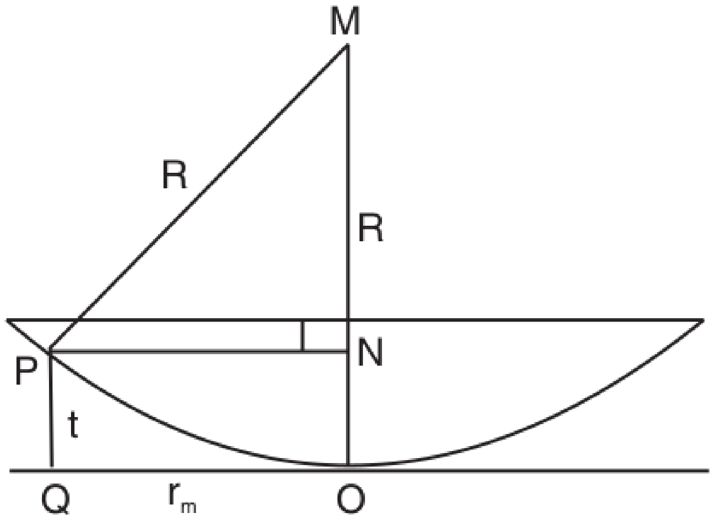


When a plano-convex lens of a very large radius is placed on the plane glass plate then a thin air film of varying thickness is formed in between these two. The nature of interference is decided by the film's thickness. Locus of points having the same thickness is circular in nature, hence the interference pattern is circular in nature i.e. ring shaped.





Expression for the radius of ring:



For Dark ring:

Let R be the radius of curvature of the lens. Let a dark fringe be located at Q. Let the thickness of the air film at Q be PQ = t. Let the radius of the circular fringe at Q be $OQ = r_m$. By the Pythagorus theorem,

$$PM^2 = PN^2 + MN^2$$

$$R^2 = r_m^2 + (R - t)^2$$

$$R^2 = r_m^2 + R^2 - 2Rt + t^2$$

$$r_m^2 = 2Rt - t^2$$

Since, R >> t, $2Rt >> t^2$.

Hence,

$$r_m^2 \cong 2Rt$$

Or,

$$r_m \cong \sqrt{2Rt}$$

The condition for darkness at Q is that,

$$2t = m\lambda$$

Therefore,

$$r_m^2 = m\lambda R$$

Or,

$$r_m = \sqrt{m\lambda R}$$

The radii of dark fringes can be found by inserting values 1,2,3, ... for m.

Expression for radius of curvature of plano convex lens:

Radius of n^{th} dark ring is given as,

$$r_n = \sqrt{n\lambda R}$$

$$\therefore D_n = 2r_n = 2\sqrt{n\lambda R} = \sqrt{4n\lambda R}$$

For n^{th} dark ring ,

$$D_n^2 = 4n\lambda R$$

Similarly for $(n+p)^{th}$ dark ring,

$$D_{n+p}^2 = 4(n + p)\lambda R$$

$$\therefore D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\therefore \lambda = \frac{D_{n+p}^{2} - D_{n}^{2}}{4pR}$$

Or,

$$R = \frac{D_{n+p}^{2} - D_{n}^{2}}{4p\lambda}$$

Where p is an integer, λ is the wavelength of light and R is the radius of curvature of a plano convex lens.

Q. In Newton's rings experiment, why:

i. **A Plano convex lens should have a larger radius.**
ii. **Rings get closer away from the centre.**
iii. **Central fringe is dark in reflected light.**
iv. **Fringes are circular.**

OR

Q. Why in Newton's ring experiment the central spot is dark?

Plano convex lens has large radius of curvature:

The radius of n^th ring is given by,

$$r_n = \sqrt{2Rt}$$

i.e.

$$r_n \propto \sqrt{R}$$

Where R is the radius of curvature of the lens.

Thus if R is larger, radii of rings are also larger and the pattern is more clear and distinct. Also if R is larger, thickness t is small. i.e. film will be more thin as required. Hence the plano-convex lens should have a larger radius.

Rings get closer away from the centre i.e. they are not evenly spaced:
The diameter of n^{th} dark ring is given as,

$$D_n = \sqrt{4n\lambda R}$$

$$\therefore D_n \propto \sqrt{n}$$

From the above relation it is clear that the increase in diameter is proportional to square root of increase in n. Hence the diameters do not increase in the same proportion as n increases. Hence the rings get closer and closer as they move away from the centre. Hence they are not evenly spaced.

Central spot (fringe) is dark:

At centre i.e. at point of contact of plano-convex lens and glass plate,

$$t \ll \lambda$$

Hence the path difference between the rays,

$$\Delta = 2\mu t - \frac{\lambda}{2} \cong \frac{\lambda}{2}$$

This is the condition of darkness and hence the central fringe is dark.

Fringes are circular:

The fringes are formed due to the rays reflected from the layers of film of equal thickness. The locus of points of equal thickness is a circle around the point of contact. Hence, the fringes are circular.

Q. How can Newton's rings experiment be used to determine the refractive index of a liquid?

To find the refractive index of liquid:

In Newton’s ring experiment with air in the gap, the radius of nth dark ring is given by,

$$\left[r_n\right]_{air} = \sqrt{n\lambda R}$$

Diameter of nth dark ring is,

$$\left[D_n\right]_{air} = \sqrt{4n\lambda R} \qquad (i)$$

And diameter of (n+p)th dark ring is,

$$\left[D_{n+p}\right]_{air} = \sqrt{4(n + p)\lambda R} \qquad (ii)$$

From equation i and ii,

$$\left[D_{n+p}\right]_{air}^2 - \left[D_n\right]_{air}^2 = 4p\lambda R \qquad (iii)$$

Similarly, in Newton’s ring experiment with liquids in the gap,
radius of nth dark ring is,

$$\left[r_n\right]_{liquid} = \sqrt{\frac{n\lambda R}{\mu}}$$

Diameter of n^{th} dark ring is

$$\left[D_n\right]_{liquid} = \sqrt{\frac{4n\lambda R}{\mu}} \qquad (iv)$$

Diameter of $(n+p)^{th}$ ring is,

$$\left[D_{n+p}\right]_{liquid} = \sqrt{\frac{4(n+p)\lambda R}{\mu}} \qquad (v)$$

From equation iv and v,

$$\left[D_{n+p}\right]^2_{liquid} - \left[D_n\right]^2_{liquid} = \frac{4p\lambda R}{\mu} \qquad (vi)$$

Dividing equation (iii) by (vi),

$$\frac{\left[D_{n+p}\right]^2_{air} - \left[D_n\right]^2_{air}}{\left[D_{n+p}\right]^2_{liquid} - \left[D_n\right]^2_{liquid}} = \frac{4p\lambda R}{\frac{4p\lambda R}{\mu}}$$

Or,

$$\mu = \frac{\left[D_{n+p}\right]^2_{air} - \left[D_n\right]^2_{air}}{\left[D_{n+p}\right]^2_{liquid} - \left[D_n\right]^2_{liquid}}$$

Q. What is antireflection coating? State the principle behind it.

Antireflection coating:

A thin transparent film coated on a surface in order to suppress the reflection is called antireflection film (AR coating).

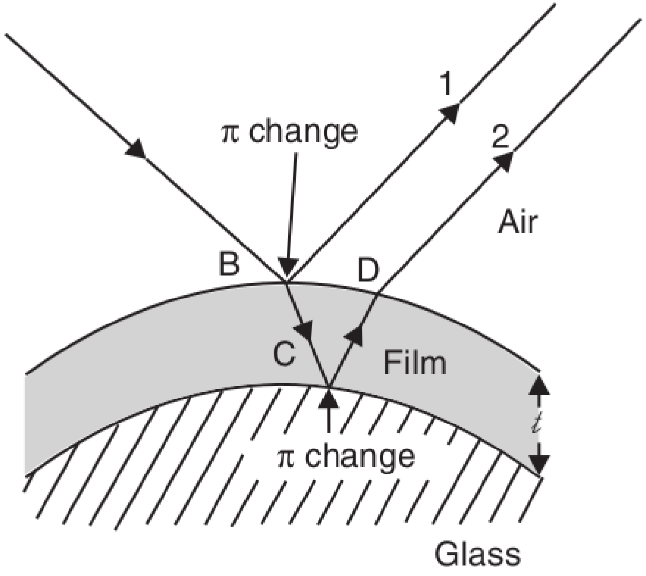
1. When light is incident on a glass surface the light is reflected back from the surface and very less amount of light is transmitted through it.
2. Due to this the quality of image of an object will be poor.
3. To increase the transmission i.e. to suppress the reflection of light, the lens surface is coated with antireflection coating.

Principle: The coating will suppress reflection and increase transmission if the rays reflected from the upper and lower surfaces of this coating have equal amplitudes and are 180^0 out of phase with each other.

This causes total destructive interference of reflected rays and enhances transmission of light.

To ensure that the rays reflected from the film interfere destructively, following conditions must be satisfied.

1. **Amplitude condition:** Amplitudes of the two rays (waves) must be the same.
2. **Phase condition:** The waves must be exactly 180^0 out of phase.



$$\mu_a < \mu_f < \mu_g$$

Q. Obtain the condition of refractive index of coating material for zero reflectivity. (Amplitude condition)

From the amplitude condition, we can say that amplitude of the waves reflected from the upper and lower surface of the coating should be equal.

i.e.

$$A_1 = A_2 \qquad (i)$$

Where A_1 is the amplitude of wave reflecting from the upper surface of coating and A_2 is the amplitude of wave reflecting from lower surface of coating.

It can be shown that,

$$A_1 \propto \left[\frac{\mu_f - \mu_a}{\mu_f + \mu_a}\right]^2$$

And

$$A_2 \propto \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2$$

Where, μ_a = R.I of air = 1

μ_f = R.I. of film

μ_g = R.I. of glass

Then, from (i) we can write,

$$\left[\frac{\mu_f - 1}{\mu_f + 1}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2$$

Expanding the above equation,

$$\frac{\mu_f^2-2\mu_f+1}{\mu_f^2+2\mu_f+1} = \frac{\mu_g^2-2\mu_g\mu_f+\mu_f^2}{\mu_g^2+2\mu_g\mu_f+\mu_f^2}$$

$$\mu_f^2\mu_g^2 + 2\mu_f^3\mu_g + \mu_f^4 - 2\mu_f\mu_g^2 - 4\mu_f^2\mu_g - 2\mu_f^3 + \mu_g^2 + 2\mu_g\mu_f + \mu_f^2$$
$$= \mu_f^2\mu_g^2 - 2\mu_f^3\mu_g + \mu_f^4 + 2\mu_f\mu_g^2 - 4\mu_f^2\mu_g + 2\mu_f^3 + \mu_g^2 - 2\mu_g\mu_f + \mu_f^2$$

$$4\mu_f^3\mu_g - 4\mu_f\mu_g^2 - 4\mu_f^3 + 4\mu_g\mu_f = 0$$

Dividing by $4\mu_f$ and rearranging the terms,

$$\mu_f^2\mu_g - \mu_g^2 - \mu_f^2 + \mu_g = 0$$

$$\mu_f^2(\mu_g - 1) - \mu_g(\mu_g - 1) = 0$$

$$(\mu_g - 1)(\mu_f^2 - \mu_g) = 0$$

Since, $\mu_g \neq 1$ therefore dividing by $(\mu_g - 1)$ we get,

$$\mu_f^2 - \mu_g = 0$$

$$\mu_f^2 = \mu_g$$

$$\therefore \mu_f = \sqrt{\mu_g}$$

It implies that the refractive index of thin film should be less than that of the glass and possibly nearer to its square root.

Q. Obtain condition for minimum thickness of antireflection coating. (Phase condition).

The phase condition states that the waves reflected from top and bottom of the film must be exactly 180^0 out of phase.

It means that their optical path difference must be equal to $\lambda/2$.

Optical path difference is given as

$$\Delta = 2\,\mu_f\, t \cos r - \frac{\lambda}{2} - \frac{\lambda}{2}$$

$$\Delta = 2\,\mu_f\, t \cos r - \lambda$$

$$\Delta = 2\,\mu_f\, t\ \cos\ r$$

For normal incidence, $\cos r = \cos 0^0 = 1$

$$\therefore \Delta = 2\,\mu_f\, t$$

But for destructive interference,

$$\Delta = (2n + 1)\frac{\lambda}{2}$$

$$\therefore 2\,\mu_f\, t = (2n + 1)\frac{\lambda}{2}$$

For minimum thickness of the film, $t = t_{min}$, $n = 0$,

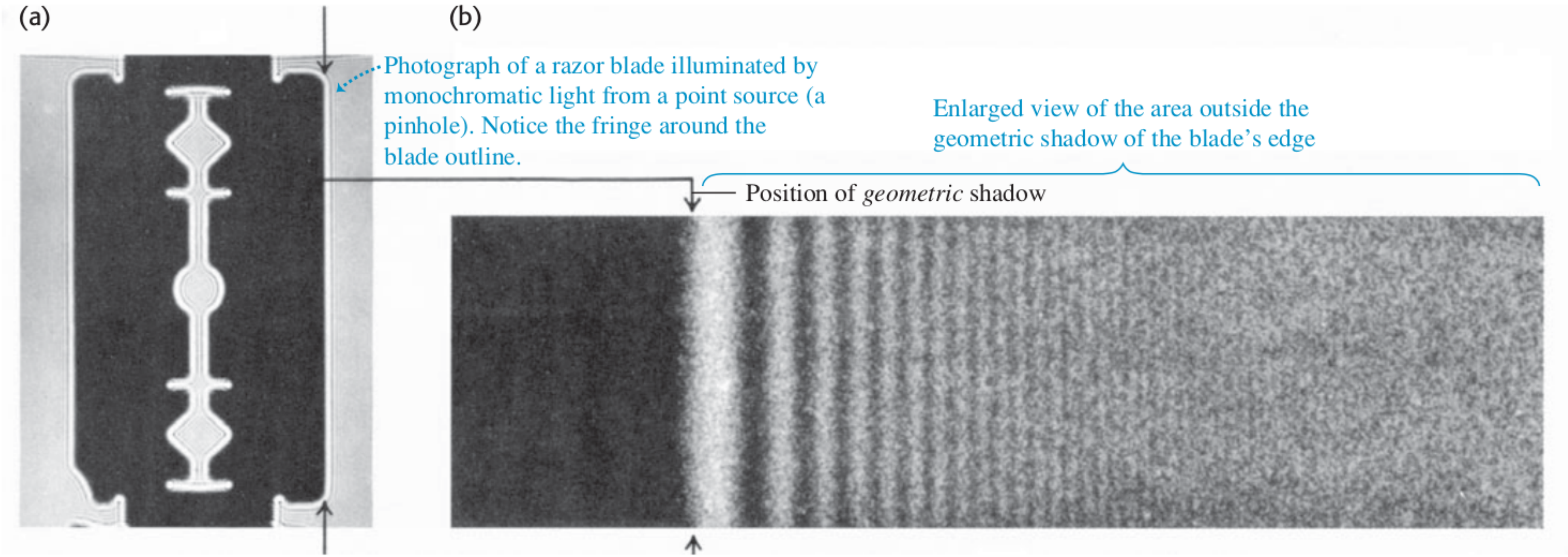
$$\therefore 2\,\mu_f\, t_{min} = \frac{\lambda}{2}$$

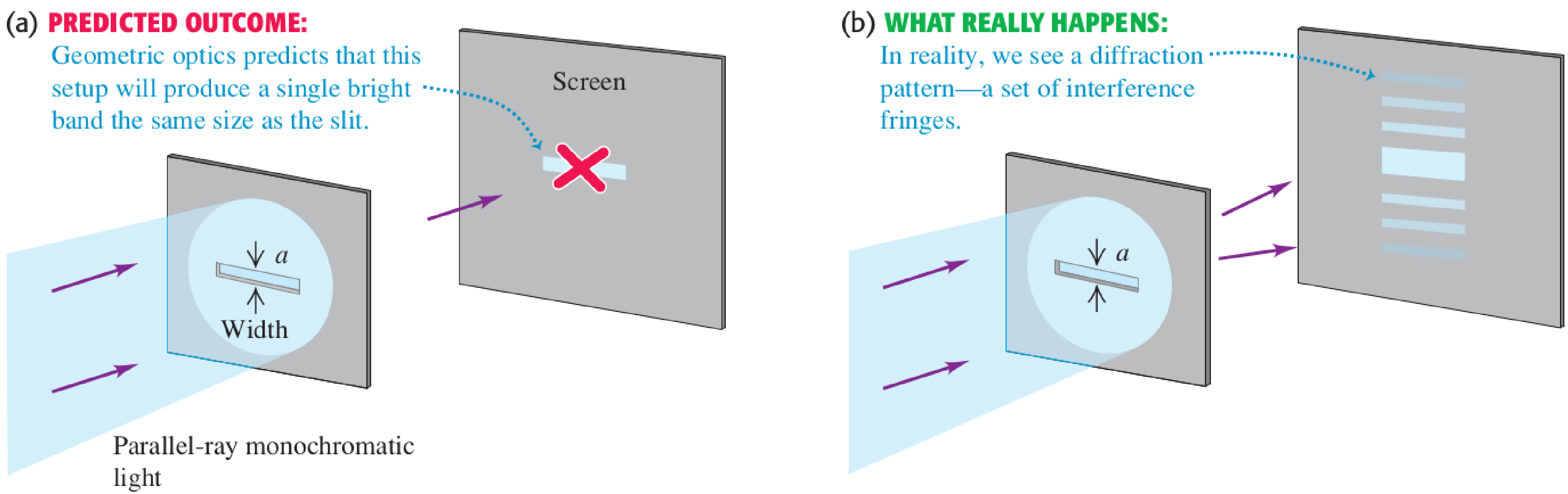
$$\therefore t_{min} = \frac{\lambda}{4\mu_f}$$

This means, for proper cancellation the thickness of the film should be equal to quarter wavelength.

Q. Explain the concept of diffraction.

According to geometric optics, when an opaque object is placed between a point light source and a screen, the shadow of the object forms a perfectly sharp line. No light at all strikes the screen at points within the shadow, and the area outside the shadow is illuminated nearly uniformly. But the wave nature of light causes effects that can't be understood with geometric optics. An important class of such effects occurs when light strikes a barrier that has an aperture or an edge. The interference patterns formed in such a situation are grouped under the heading diffraction.


(a)
(b)
Photograph of a razor blade illuminated by monochromatic light from a point source (a pinhole). Notice the fringe around the blade outline.
Enlarged view of the area outside the geometric shadow of the blade's edge
Position of *geometric* shadow



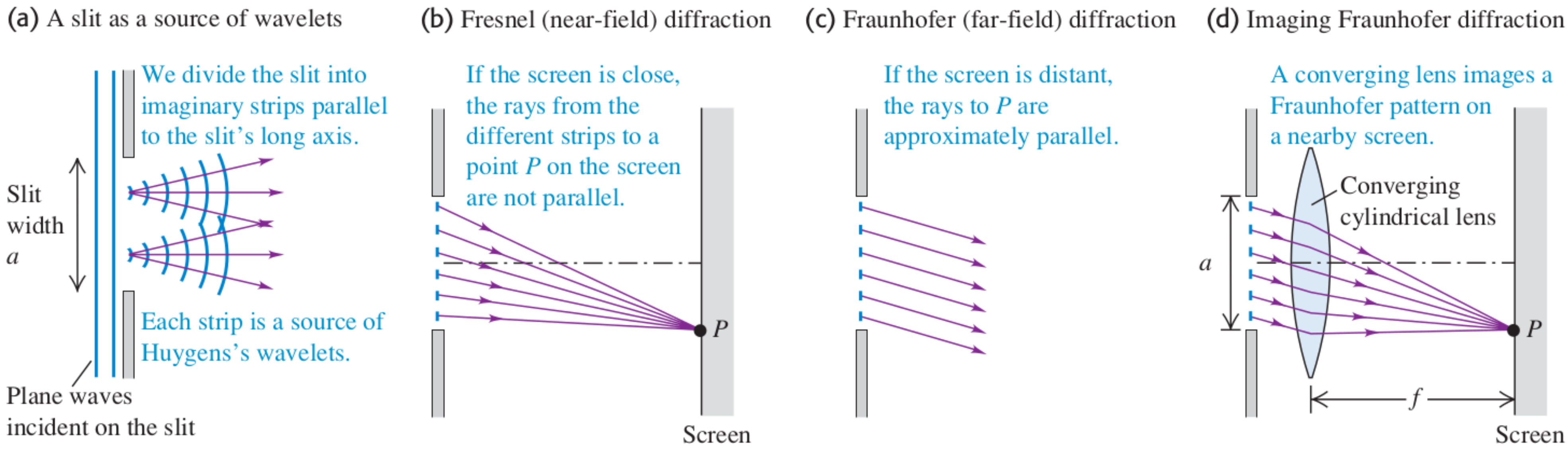
Diffraction is sometimes described as "the bending of light around an obstacle." But the process that causes diffraction is present in the propagation of every wave. When part of the wave is cut off by some obstacle, we observe diffraction effects that result from interference of the remaining parts of the wave fronts. Optical instruments typically use only a limited portion of a wave; for example, a telescope uses only the part of a wave that is admitted by its objective lens or mirror.

There is no fundamental distinction between interference and diffraction. Interference involves waves from a small number of sources, usually two. Diffraction usually involves a continuous distribution of Huygens's wavelets across the area of an aperture, or a very large number of sources or apertures. But both interference and diffraction are consequences of superposition and Huygens's principle.

Types of Diffraction:

Fresnel Diffraction: When both the point source and the screen are relatively close to the obstacle forming the diffraction pattern, it is described as near-field diffraction or Fresnel diffraction.

Fraunhofer diffraction: The situation in which the source, obstacle, and screen are far enough apart that we can consider all lines from the source to the obstacle to be parallel, and can likewise consider all lines from the obstacle to a given point on the screen to be parallel is called Fraunhofer diffraction.



Q. Explain Fraunhofer Diffraction from a single slit and circular aperture.

According to geometric optics, the transmitted beam should have the same cross section as the slit. But the beam spreads out vertically after passing through the slit. The diffraction pattern consists of a central bright band, which may be much broader than the width of the slit, bordered by alternating dark and bright bands with rapidly decreasing intensity.

About 85% of the power in the transmitted beam is in the central bright band, whose width is inversely proportional to the width of the slit. In general, the smaller the width of the slit, the broader the entire diffraction pattern. You can observe a similar diffraction pattern by looking at a point source, such as a distant street light, through a narrow slit formed between your two thumbs held in front of your eye; the retina of your eye corresponds to the screen.

Central Maxima:

We can justify the central bright fringe by noting that the Huygens wavelets from all points in the slit travel about the same distance to reach the centre of the pattern and thus are in phase there.

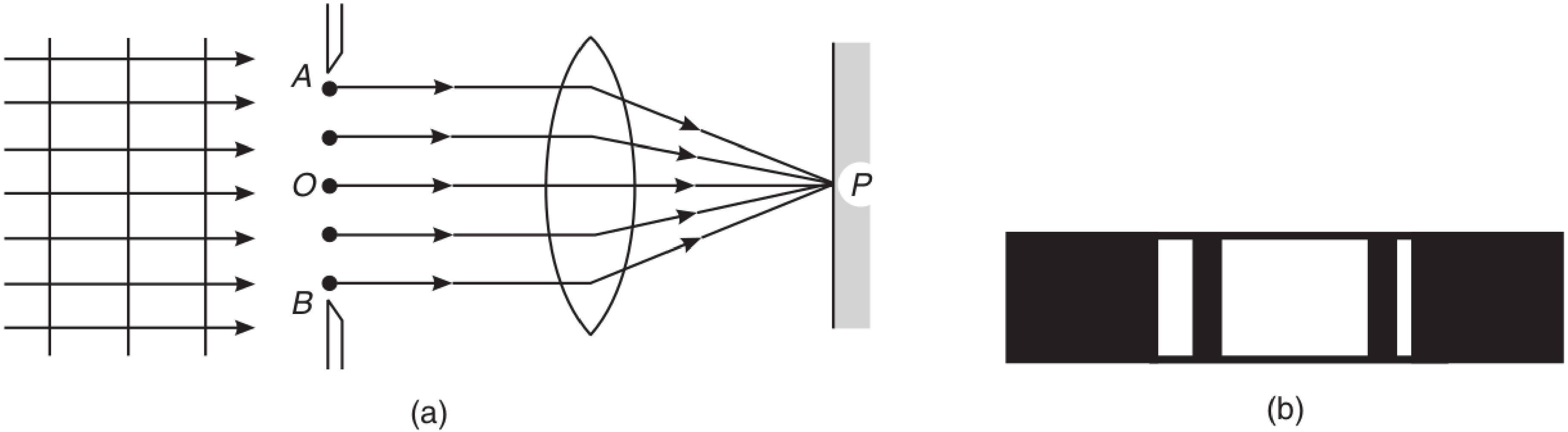


Fig. 7.6: Fraunhofer diffraction at a single slit (a) Conditions at the central maximum. The rays parallel to axis come to focus at P giving a bright band. (b) Typical diffraction pattern consisting of a central bright band flanked by weaker bright bands.

Minima:

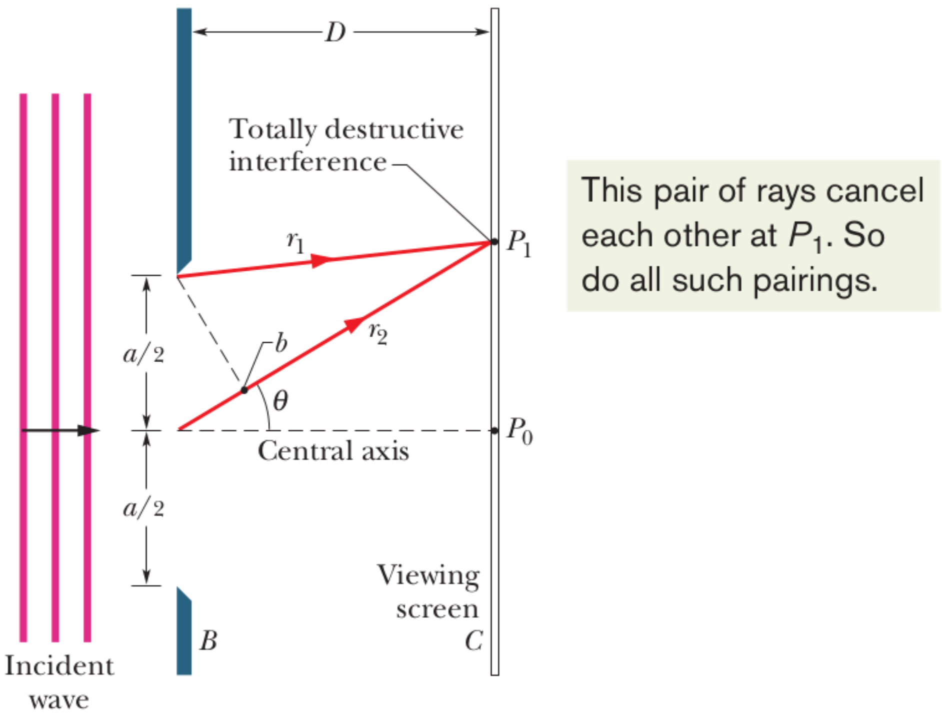


Fig. 36-4 Waves from the top points of two zones of width $a/2$ undergo fully destructive interference at point P_1 on viewing screen C.

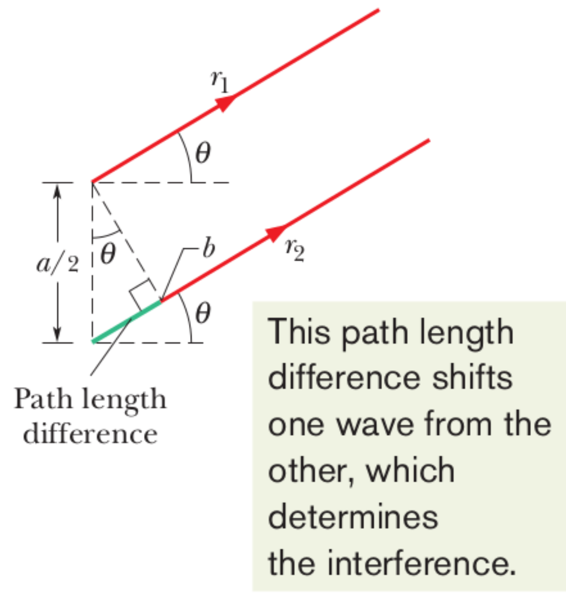


Fig. 36-5 For $D \gg a$, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis.

To find the dark fringes, we shall use a strategy that involves pairing up all the rays coming through the slit and then finding what conditions cause the wavelets of the rays in each pair to cancel each other. We apply this strategy to locate the first dark fringe P_1. First, we mentally divide the slit into two zones of equal widths a/2. Then we extend to P_1 a light ray r_1 from the top point of the top zone and a light ray r_2 from the top point of the bottom zone. We want the wavelets along these two rays to cancel each other when they arrive at P_1 . Then any similar pairing of rays from the two zones will give cancellation. A central axis is drawn from the centre of the slit to screen C, and P_1 is located at an angle θ to that axis.

The wavelets of the pair of rays r_1 and r_2 are in phase within the slit because they originate from the same wavefront passing through the slit, along the width of the slit. However, when they reach P_1 ; there is a phase difference of $\lambda/2$ between them. This is due to the extra path followed by r_2 from slit to screen. There is a point b on ray r_2 such that the path length from b to P_1 matches the path length of ray r_1 . Then the path length difference between the two rays is the distance from the centre of the slit to b.

When viewing screen C is near screen B, as in Fig. 36-4, the diffraction pattern on C is difficult to describe mathematically. However, we can simplify the mathematics considerably if we arrange for the screen separation D to be much larger than the slit width a. Then we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis (Fig. 36-5). We can also approximate the triangle formed by point b, the top point of the slit, and the centre point of the slit as being a right triangle, and one of the angles inside that triangle as being θ. The path length difference between rays r_1 and r_2 (which is still the distance from the centre of the slit to point b) is then equal to $\frac{a}{2}\sin\theta$.

We can repeat this analysis for any other pair of rays originating at corresponding points in the two zones (say, at the midpoints of the zones) and extending to point P_1. Each such pair of rays has the same path length difference $\frac{a}{2}\sin\theta$. This path length difference will produce a dark fringe if it is equal to $\lambda/2$. So that,

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

which gives us,

$$a\sin\theta = \lambda$$

for the first minima.

Similarly we can find the successive minima by dividing the slit into further partitions. We would find that the dark fringes above and below the central axis can be located with the general equation,

$a \sin\theta = m\lambda$

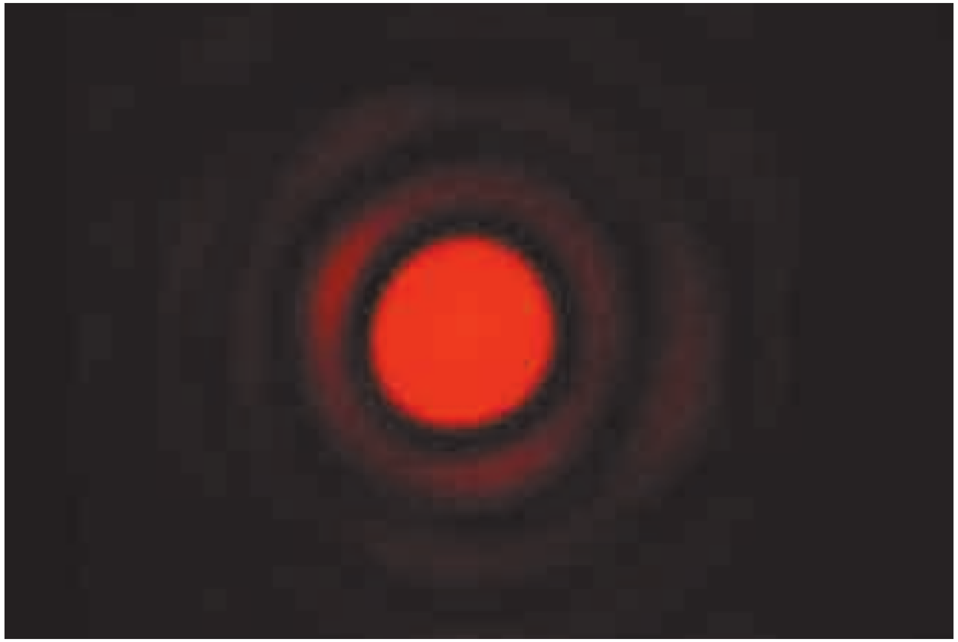
for $m = 1, 2, 3,....$

Hence, in a single-slit diffraction experiment, dark fringes are produced where the path length differences (a sin θ) between the top and bottom rays are equal to λ, 2λ, 3λ and so on.

Secondary Maxima

The secondary maxima are approximately halfway between adjacent dark fringes.

Q. Explain the Fraunhofer diffraction by a circular aperture.



Here we consider diffraction by a circular aperture — that is, a circular opening, such as a circular lens, through which light can pass. Figure shows the image formed by light from a laser that was directed onto a circular aperture with a very small diameter. This image is not a point, as geometrical optics would suggest, but a circular disk surrounded by several progressively fainter secondary rings. Here, however, the aperture is a circle of diameter d rather than a rectangular slit. The (complex) analysis of such patterns shows that the first minimum for the diffraction pattern of a circular aperture of diameter d is located by,

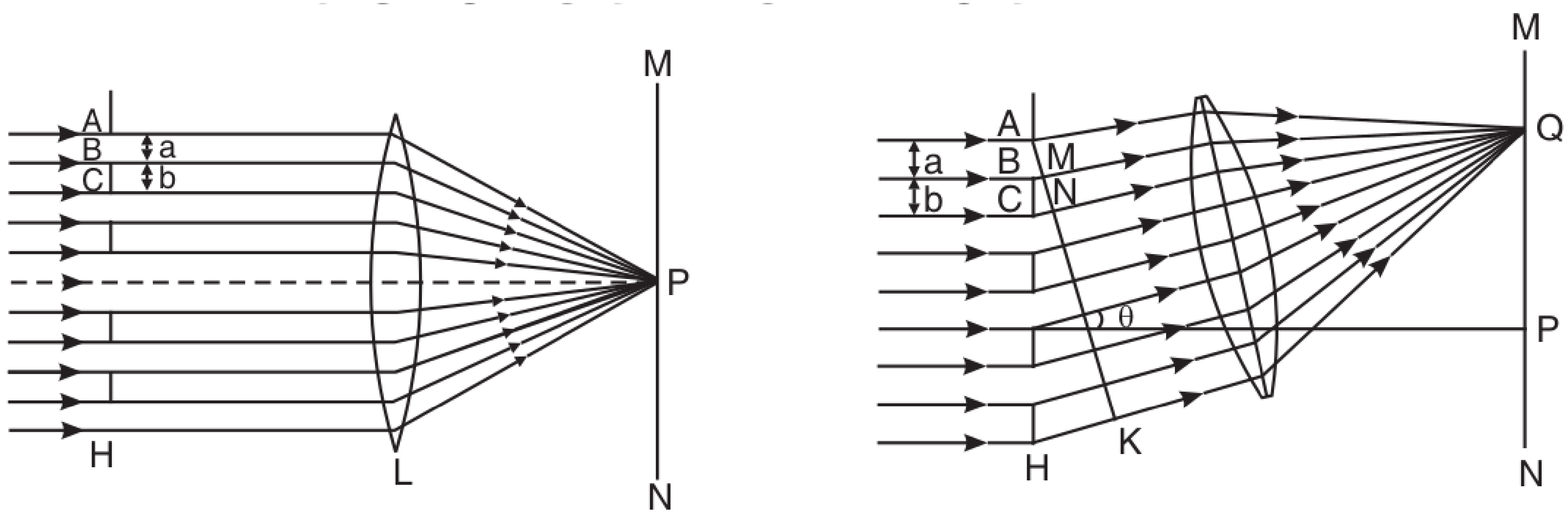
$$\sin\theta = 1.22\frac{\lambda}{d}$$

The angle θ Here is the angle from the central axis to any point on that (circular) minimum.

Q. Explain the diffraction due to diffraction grating (normal incidence)

Let us now consider the diffraction pattern produced by N-slits, each of width a. The separation between consecutive slits is d = a + b, where a is the width of the open portion and b is the width of the opaque portion. Such a device consisting of a large number of parallel slits of equal width and separated from one another by equal opaque spaces is called a **diffraction grating**. The distance d between the centres of the adjacent slits is known as the grating period. Rowland (1848-1901) produced transmission gratings by ruling extremely close, equidistant and parallel lines on optically plain glass plates with a diamond point. The rulings (diamond scratch) scatter light and are effectively opaque while the parts without ruling transmit light and act as slits. Because of the expenses and difficulty involved in fabrication, commonly used gratings are reproduced from the original ruled gratings. The replica gratings are made by pouring a thin layer of collodion solution over the surface of a ruled grating and the solution is allowed to harden. The collodion film is peeled carefully afterwards from the grating. The film retains the impression of the rulings of the original grating in the form of ridges. The ruled lines, which scatter light, act as opaque spaces whereas the spaces between them which transmit incident light act as parallel slits. The film is mounted between glass plates and it acts as a plane transmission grating. The number of lines on a plane transmission grating is of the order of 6000 lines per cm.

PLANE DIFFRACTION GRATING - THEORY



Let us consider the plane transmission grating held normal to the plane of the page and represented by the section ABC…H. Let the width of the transparent portion AB be equal to a and the opaque portion BC be equal to b. The distance (a + b) = d and is called the grating constant or grating period. Let a parallel beam of monochromatic light of wavelength λ be incident normally on the grating surface. Then all the secondary waves travelling in the same direction as that of the incident light will come to focus at the point P on the screen. The screen is placed at the focal plane of the collecting lens, L. The point P where all the secondary waves reinforce one another corresponds to the position of the central bright maximum.

Now let us consider the secondary waves travelling in a direction inclined at an angle θ with the direction of the incident light. The waves travel different distances and it is obvious that there is a path difference between the waves coming out from each slit and bending at an angle θ. These secondary waves come to focus at the point Q on the screen. The intensity at Q will depend on the path difference between the secondary waves originating from the corresponding points A and C of two neighbouring slits. In the Fig, AB = a and BC = b. The path difference between the secondary waves starting from A and C is equal to $AC \sin\theta$.

But, $AC = AB + BC = a + b$

Path difference = $AC \sin\theta = (a + b)\sin\theta$

The point Q will be of maximum intensity if this path difference is equal to integral multiples of λ. It means that all the secondary waves originating from the corresponding points of the neighbouring slits reinforce one another and the angle θ gives the direction of maximum intensity. In general,

$(a + b)\sin\theta_m = m\lambda$

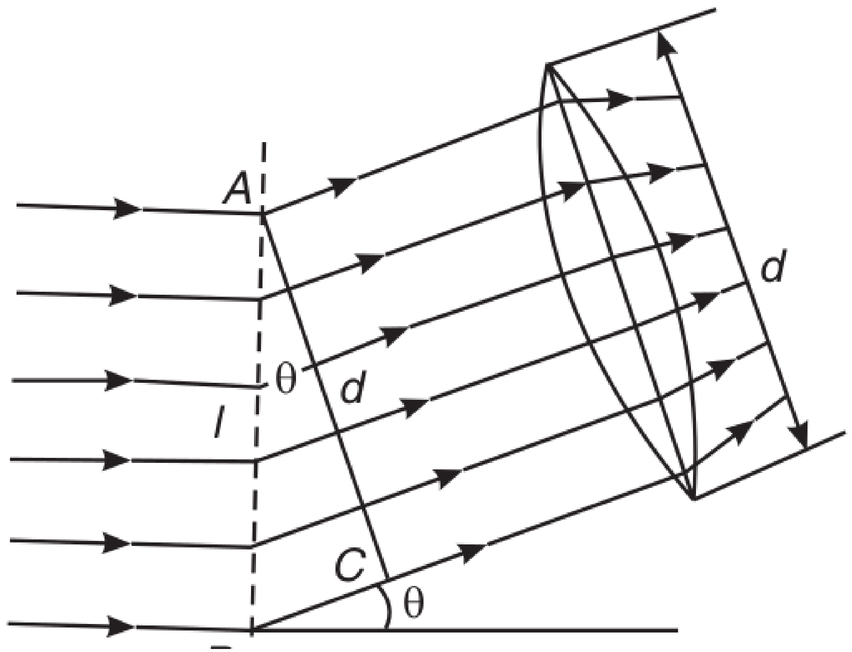
where θ_m is the direction of the m^{th} principal maximum. If $(a + b)\sin\theta = \lambda$, we obtain maximum intensity at Q. When $(a + b)\sin\theta = 2\lambda$, there will be again a maximum and so on. Between the central maximum P and the first maximum at Q there will be minimum intensity and so on. Similar maxima and minima are obtained on the other side of the central maximum. Thus, on each side of the central maximum at P, principal maxima and minimum intensity are observed due to diffracted light.

The position of m^{th} minimum is given by,

$(a + b)\sin\theta_m = (2m + 1)\lambda/2$

Q. Derive expression for resolving power of a diffraction grating.

One of the important properties of a diffraction grating is its ability to resolve spectral lines, which have nearly the same wavelength. The spectral resolving power of a grating is defined in terms of the smallest wavelength interval ($d\lambda$) that can be detected by it. It is given by $\lambda/d\lambda$ where λ is the average of the two wavelengths and $d\lambda$ is their difference.



$$Resolving\ power, R = \frac{\lambda}{d\lambda} = \frac{\lambda}{d\theta}\frac{d\theta}{d\lambda}$$

Let us now find the values of $\frac{\lambda}{d\theta}$ and $\frac{d\theta}{d\lambda}$.

The diffraction grating equation is

$(a + b)\sin\theta = m\lambda$.

Differentiating the above equation both sides, we get,

$(a + b)\cos\theta\, d\theta = m d\lambda$

$$\frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta}$$

where $d\theta$ is the angle between the two diffracted beams whose difference in wavelength is $d\lambda$.

The light diffracted from the grating enters the objective of a telescope in a grating spectrometer. If the diffracted beam completely fills the objective then width of the beam equals the diameter d of the objective lens. The angular limit of resolution of a telescope objective is given by,

$$d\theta = \frac{\lambda}{d}$$

Now $d = AB\cos\theta = l\cos\theta$ where l is the grating.

$$d\theta = \frac{\lambda}{l\cos\theta}$$

$$\frac{\lambda}{d\theta} = l\cos\theta$$

Resolving power,

$$R = \frac{\lambda}{d\lambda} = \frac{\lambda}{d\theta}\frac{d\theta}{d\lambda}$$

$$R = l\cos\theta \times \frac{m}{(a+b)\cos\theta}$$

$$R = \frac{ml}{a+b}$$

$$R = mN$$

Where, $N = l/(a + b)$ = number of rulings on the grating surface and m is the order of the spectrum. Hence, the resolving power of a grating is given by the simple expression,

$$R = mN$$

List of Formulae:

1. Diameter of n^{th} dark ring, $D_n = \sqrt{4n\lambda R}$
2. Radius of curvature of plano convex lens, $R = \frac{D_{n+p}^2 - D_n^2}{4p\lambda}$
3. Fringe-width, $\beta = \frac{\lambda}{2\mu\theta}$ $(for\ \tan\theta \approx \theta)$
4. Fringe-width, $\beta = \frac{\lambda}{2\mu\tan\theta}$
5. Minimum film thickness, $t_{min} = \frac{\lambda}{4\mu_f}$

1. The fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 0.1mm and wavelength of the light is 5893 A°. The angle of the wedge is ...

Given Data:

Refractive index of glass, $\mu = 1.52$

Fringe width, $\beta = 0.1\, mm = 0.1 \times 10^{-3} m$

Wavelength, $\lambda = 5893 A^{\circ} = 5893 \times 10^{-10} m$

To Find:

Wedge angle, $\theta = ?$

Solution:

Fringe-width, $\beta = \frac{\lambda}{2\mu\theta}$

$$\theta = \frac{\lambda}{2\mu\beta}$$

$$\theta = \frac{5893\times10^{-10}}{2\times1.52\times0.1\times10^{-3}}$$

$$\theta = 1.94 \times 10^{-3} rad$$

$\theta = 1.94 \times 10^{-3} \times \frac{180}{\pi}$degree

$\theta = 0.11^{0}$

2. In Newton's ring experiment we get the diameter of the 10^{th} ring as 0.5 cm, the wavelength of the light in air is 6000 A°, how much is the radius of curvature of the lens.

Given Data:

Diameter of 10^{th} ring, $D_{10} = 0.5\ cm = 0.5 \times 10^{-2} m$

Wavelength of light in air, $\lambda = 6000$ A° $= 6000 \times 10^{-10} m$

To Find:

Radius of curvature, $R = ?$

Solution:

Diameter of n^{th} dark ring, $D_n = \sqrt{4n\lambda R}$

$$D_{10} = \sqrt{4 \times 10 \times 6000 \times 10^{-10} \times R}$$

$$0.5 \times 10^{-2} = \sqrt{4 \times 10 \times 6000 \times 10^{-10} \times R}$$

Squaring both sides,

$$(0.5 \times 10^{-2})^2 = 4 \times 10 \times 6000 \times 10^{-10} \times R$$

$$R = \frac{(0.5\times10^{-2})^2}{4\times10\times6000\times10^{-10}}$$

$$R = 1.041\, m$$

$$R = 104.1\, cm$$

3. Find the thickness of water film with a refractive index of 1.33 formed on a glass windowpane to act as non-reflecting film. Given λ= 5500 Å

Given Data:

Wavelength of light, $\lambda = 5500Å = 5500 \times 10^{-10} m$

Refractive index of water, $\mu = 1.33$

To Find:

Thickness of water film, t_{min} =?

Solution:

Minimum film thickness, $t_{min} = \frac{\lambda}{4\mu_f}$

$t_{min} = \frac{5500\times10^{-10}}{4\times1.33}$

$t_{min} = 1.033 \times 10^{-7} m$

$t_{min} = 1033Å$

4. A glass microscope lens is coated with magnesium fluoride (μ=1.38) film to increase the transmission of normally incident light of wavelength 6800 Å. What is the minimum film thickness needed for optimum result?

Given Data:

Wavelength of light, $\lambda = 6800Å = 6800 \times 10^{-10} m$

Refractive index of film, $\mu = 1.38$

To Find:

Thickness of film, t_{min} =?

Solution:

Minimum film thickness, $t_{min} = \frac{\lambda}{4\mu_f}$

$$t_{min} = \frac{6800\times10^{-10}}{4\times1.38}$$

$$t_{min} = 1.2318 \times 10^{-7} m$$

$$t_{min} = 1231.8Å$$

5. In a Newton's ring experiment, the diameter of the 15th ring was found to be 0.59 cm and that of the 5th ring was 0.336 cm. If the radius of the plano-convex lens is 100 cm. Calculate the wavelength of light used.

Given Data:

Diameter of 15th ring, $D_{n+p} = D_{15} = 0.59\, cm = 0.59 \times 10^{-2} m$

Diameter of 5th ring, $D_n = D_5 = 0.336\, cm = 0.336 \times 10^{-2} m$

Here, $n + p = 15$ and $n = 5$

Hence, $p = 10$

Radius of plano-convex lens, $R = 100\ cm = 1m$

To Find:

Wavelength of light, $\lambda =?$

Solution:

Radius of curvature of plano convex lens, $R = \frac{D_{n+p}^2 - D_n^2}{4p\lambda}$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

$$\lambda = \frac{(0.59\times10^{-2})^2 - (0.336\times10^{-2})^2}{4\times10\times1}$$

$$\lambda = 5.8801 \times 10^{-7} m$$

1. In Newton’s rings experiment, the diameter of n^{th} ring and $(n + 14)^{th}$ rings are 4.2 mm and 7 mm respectively. Radius of plano convex lens is 1 m. Calculate the wavelength of light used. [3m]
2. In a Newton’s ring experiment the diameter of the 15^{th} ring was found to be 0.59 cm and that of 5^{th} ring was 0.336 cm. If the radius of the plano convex lens is 100 cm. Calculate the wavelength of light used. [3m]
3. In Newton’s ring experiment, diameter of 10^{th} dark ring due to wavelength 6000 A° in air is 0.5 cm. Find the radius of curvature of lens. [3m]
4. In Newton’s ring experiment, the diameter of 5^{th} ring is 0.336 cm and the diameter of 15^{th} ring is 0.590 cm. Find the radius of curvature of plano-convex lens if the wavelength of light used is 5890 A°. [3m]
5. Newton’s rings are observed in reflected light of wavelength 590 nm. The diameter of the 10^{th} ring is 0.6 cm. Find the radius of the curvature of the lens. [2m]
6. When a wedge shaped air film is viewed by a monochromatic source of light incident normally, the interference fringes 0.4 mm apart are observed. If the air

space is filled with water ($\mu = 1.33$). How far apart will the fringes be observed? [3m]

7. Fringes of equal thickness are observed in a thin glass wedge of RI 1.52. The fringe spacing is 0.1 mm, wavelength of light being 5893 $\mathring{A}$. Calculate wedge angle. [3m]
8. A glass microscope lens is coated with magnesium fluoride ($\mu = 1.38$) film to increase the transmission of normally incident light of wavelength 6800 $\mathring{A}$. What is the minimum film thickness needed for optimum result? [2m]
9. A glass microscope lense ($\mu = 1.5$) is coated with magnesium fluoride ($\mu = 1.30$) film to increase the transmission of normally incident light ($\lambda = 5800$ $\mathring{A}$). What minimum film thickness would be deposited on the lense? [3m]
10. Find the thickness of the water film with a refractive index of 1.33 formed on a glass window pane to act as a non-reflecting film. Given $\lambda = 5500$ $\mathring{A}$. [2m]