

B.Tech. Fourth Semester (Computer Science & Engineering / Computer Technology / Computer Engineering / Information Technology) (C.B.C.S.) Summer 2023
Discrete Mathematics & Graph Theory

P. Pages : 4

Time : Three Hours



MSP/KS/23/2582/2588/2594/2600

Max. Marks : 70

- Notes :
1. All questions carry marks as indicated.
 2. Solve Question 1 OR Questions No. 2.
 3. Solve Question 3 OR Questions No. 4.
 4. Solve Question 5 OR Questions No. 6.
 5. Solve Question 7 OR Questions No. 8.
 6. Solve Question 9 OR Questions No. 10.
 7. Assume suitable data whenever necessary.
 8. Use of non programmable calculator is permitted.

1. a) Prove that $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$. 7

b) Let f be the set of all one-one and onto mappings from X to X , where $X = \{1, 2, 3\}$. Find all elements of f and find inverse of each element. 7

OR

2. a) If $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$, then find $R, S, R \circ S, S \circ R, \tilde{R}, \tilde{S}$ 7

b) Using the properties of characteristics function, show that 7

i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii) $(A^c)^c = A$

3. a) Define the following 7

i) Equality of two fuzzy sets

ii) Union of two fuzzy sets

iii) Difference of two fuzzy sets

iv) Algebraic sum of two fuzzy sets

v) Algebraic product of two fuzzy sets

b) If $A = \left\{ \frac{0.3}{30}, \frac{0.7}{60}, \frac{1}{100}, \frac{0.2}{120} \right\}$ and $B = \left\{ \frac{0.2}{20}, \frac{0.4}{40}, \frac{0.6}{60}, \frac{0.8}{80} \right\}$ 7

Are fuzzy sets, then find the relation $R = A \times B$. Also find R^2 .

OR

4. a) Consider the fuzzy sets A, B, C defined on the interval $X = [0, 10]$ of integers by the membership grade function $\mu_A(x) = \frac{x}{x+2}$, $\mu_B(x) = 2^{-x}$, $\mu_C(x) = \frac{1}{1+10(x-2)^2}$. 7

Determine the mathematical formula for

i) \bar{A}

ii) $A \cup B \cup C$

- b) If R and S are fuzzy relations given by 7

$$R = \begin{matrix} & y_1 & y_2 \\ x_1 & 0.5 & 0.1 \\ x_2 & 0.2 & 0.9 \\ x_3 & 0.8 & 0.6 \end{matrix}, \quad S = \begin{matrix} & y_1 & y_2 \\ x_1 & 0.6 & 0.5 \\ x_2 & 0.4 & 0.8 \\ x_3 & 0.7 & 0.9 \end{matrix}$$

Then find $R \cup S$, $R \cap S$, \bar{R} , \bar{S}

5. a) Prove that the set $\{1, 2, 3, 4, 5, 6\}$ of order 6 is a finite abelian group under multiplication modulo 7 as composition. 7
- b) Show that the set of numbers of the form $a + b\sqrt{2}$, where a and b are rational numbers, is a field. 7

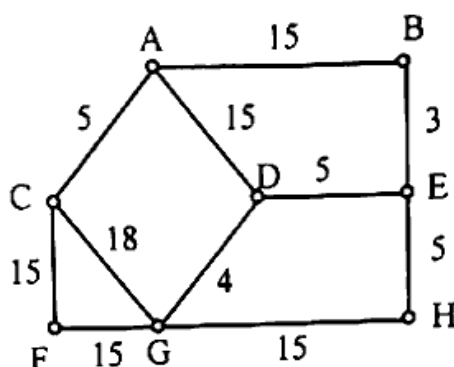
OR

6. a) Define : (i) Subgroup (ii) Normal subgroup. Show that the intersection of any two normal subgroups of a group G, is a normal subgroup of G. 7
- b) If R is a ring such that $a^2 = a$, $\forall a \in R$, then show that 7
- i) $a + a = 0$, $\forall a \in R$
- ii) $a + b = 0 \Rightarrow a = b$, $\forall a, b \in R$
- iii) R is a commutative ring
7. a) Draw a digraph corresponding to the following adjacency matrix and interpret the result AA^T , $A^T A$, A^2 , A^3 , where 6

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

- b) Determine a railway network of minimal cost for the following cities using Prim's algorithm.

6



- c) Represent the following algebraic expressions using binary tree. Also draw the Venn diagrams.

6

- $((a+b)/c) + (x+y)$
- $(x + (y+z)) * (a \times (b+c))$
- $(3 - 2(-(11 - (9 - 4)))) \div (2 + (3 + (4 + 7)))$

OR

8. a) Draw the digraphs corresponding to the adjacency matrices A, B, A^T , B^T , where

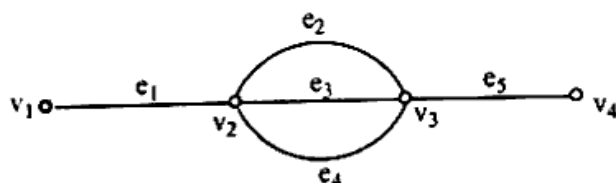
6

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

show that the digraphs corresponding to A^T and B^T are isomorphic.

- b) Consider the following graph.

6



- How many simple paths are there from v_1 to v_4
- How many trails are there from v_1 to v_4
- How many paths are there from v_1 to v_4

- c) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
 $T = \{(2, 3), (2, 1), (4, 5), (4, 6), (5, 8), (6, 7), (4, 2), (7, 9), (7, 10)\}$
 Identify the root and show that T is a rooted tree. Also give the corresponding binary tree. 6

9. a) A box contains 5 red and 6 white marbles. In how many ways can 6 marbles be selected so that there are at least two balls of each colour. 5

- b) Find the closed form of generating function for 5

i) $a_n = 2 + 3n$

ii) $a_n = 3^n$

OR

10. a) Find the minimum number of students in a class to be sure that four out of them are born in the same month? 5

- b) Solve the following recurrence relation using generating function. 5
 $a_n - 9a_{n-1} + 20a_{n-2} = 0, a_0 = -3, a_1 = -10$

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