Applied Mathematics-III

SPM/KW/22/2529

1. a) Find by Newton Raphson method the real root of $3x - \cos x - 1 = 0$ 7

b) Solve $\frac{dy}{dx} = x^2y - 1$, given $y(0) = 1$. Find $y(0.1)$ by Taylor's series method 7

OR

2. a) Using Regula - Falsi method find the root of the equation $x \log_{10} x - 1.2 = 0$ 7

b) Solve the differential equation $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 2, y(0.2) = 2.0933, y(0.4) = 2.1755$, $y(0.6) = 2.2493$. Find $y(0.8)$ by Milne's predictor – corrector method 7

3. a) Investigate the linear dependence and independence of vectors 7
$X_1 = (1,2,4), X_2 = (2,-1,3), X_3 = (0,1,2), X_4 = (-3,7,2)$

b) Use Sylvester's theorem to show that 7
$2 \sin A = (\sin 2)A$, where $A = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$

OR

4. a) Diagonalize the matrix, where 7
$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

b) Find the largest eigen value and corresponding eigen vector for the matrix 7
$$A = \begin{bmatrix} -4 & -5 \\ 1 & 2 \end{bmatrix}$$

5. a) A random variable X is defined by 7
$$X = \begin{cases} -2, & \text{prob } 1/3 \\ 3, & \text{prob } 1/2 \\ 1, & \text{prob } 1/6 \end{cases}$$
Find
i) $E(X)$ iii) $E(2X+3)$
ii) $E(X^2)$ iv) $Var(X)$
v) $S.D(X)$

b) Out of 800 families with 5 children each, how many would you expect to have 7
i) 3 boys
ii) 5 girls
iii) either 2 or 3 boys?
Assume equal probabilities for boys and girls.

OR

6. a) Find the moment generating function of the random variable 7
$$X = \begin{cases} 1/2, & \text{prob } 1/2 \\ -1/2, & \text{prob } 1/2 \end{cases}$$
Also find first four moments about the origin.

b) An aptitude test for selecting engineers in an industry is conducted on 100 candidates, the average score is 42 & standard deviation of score is 24. Assuming Normal distribution for this distribution, find 7
i) The number of candidates whose score is more than 60
ii) The number of candidates whose score lies between 30 and 60.

7. a) Find the mean deviation of the following frequency distribution 7

Class	0 - 6	6 - 12	12 - 18	18 - 24	24 - 30
Frequency	8	10	12	9	5

b) Let x_1, x_2, x_3 are three variables measured from their mean with $n = 10$, $\sum x_1^2 = 90$, $\sum x_2^2 = 160, \sum x_3^2 = 40, \sum x_1x_2 = 60, \sum x_1x_3 = 40$ and $\sum x_2x_3 = 60$. Calculate the multiple correlation coefficient $R_{1.23}$ 7

OR

8. a) Calculate coefficient of Skewness of the following distribution. 7

x	0	1	2	3	4	5	6	7	8
f	7	12	32	56	70	56	28	8	1

b) Find the multiple linear regression equation of X_1 on X_2 and X_3 from the data relating to three variables given below 7

X_1	4	6	7	9	13	15
X_2	15	12	8	6	4	3
X_3	30	24	20	14	10	4

9. a) The process $\{X(t)\}$ whose probability distribution under certain condition is given by 7
$$P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,3,\ldots \\ \frac{at}{1+at}, & n = 0 \end{cases}$$
Show that it is not stationary.

b) In an investigation of health and nutrition of two groups of children of different social status, the following results are obtained 7

Social status / Health	Poor	Rich	Total
Below Normal	130	20	150
Normal	102	108	210
Above Normal	24	96	120
Total	256	224	480

Discuss the relation between health and social status

Critical value of χ^2 at 5% significance level is 5.99

OR

10. a) A transition matrix P is given by 7

$$P = \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

Assume $X_0 = 1$ and let R be the first time that the Markov chain returns to state 1 i. e

$R = \min\{n \geq 1 | X_n = 1\}$

Find $E[R | X_0 = 1]$

b) Pizza delivery times of two cities are given below 7

City 1: Number of delivery times observed = 28, variance = 38

City 2: Number of delivery times observed = 25, variance = 83

Check if the delivery times of city 1 are lesser than city 2 at 0.05 alpha level

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