Faculty of Engineering & Technology

First Semester B.E. (C.B.S.) Examination

APPLIED MATHEMATICS—I

Paper—I

Time : Three Hours] [Maximum Marks : 80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Use of non-programmable calculator is permitted.

(3) Solve :

Question No. **1** **OR** Question No. **2**

Question No. **3** **OR** Question No. **4**

Question No. **5** **OR** Question No. **6**

Question No. **7** **OR** Question No. **8**

Question No. **9** **OR** Question No. **10**

Question No. **11** **OR** Question No. **12**.

1. (a) If y = a cos (log x) + b sin (log x) show that : $x^2 y_{n+2} + (2n+1) x y_{n+1} + (n^2+1) y_n = 0$. 6

(b) Evaluate : $\lim_{x \to 0} \dfrac{x \cos x - \sin x}{x^2 \sin x}$. 3

(c) Evaluate : $\lim_{x \to 0} \left(\dfrac{a^x + b^x + c^x}{3} \right)^{1/x}$. 3

OR

2. (a) If $x = a \cos^4 \theta$, $y = a \sin^4 \theta$; find the curvature at $\theta = \pi/6$. 6

(b) Using Taylor's series find the value of cos 64° correct to four decimal places. 6

3. (a) If $u(x+y) = x^2 + y^2$, then prove that :

$$\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right).$$ 6

(b) If $u = \tan^{-1} \left[\dfrac{x^3 + y^3}{x - y} \right]$ prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u.$$ 6

(c) If u = f(x/y, y/z, z/x) find the value of :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}.$$ 6

OR

4. (a) Given $u = \dfrac{x - y}{x + y}$, $v = \dfrac{x + y}{x}$,

find $\dfrac{\partial(u, v)}{\partial(x, y)}$. Are u and v functionally related ? If so, find the relation between them. 6

(b) Expand y^x in the neighbourhood of (1, 1) upto the term of second degree. 6

(c) Find the points on the surface $z^2 = xy + 1$ nearest to origin. 6

5. (a) Find the inverse of matrix by partitioning method :

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}.$$ 6

(b) Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}.$$ 6

OR

6. (a) Test the consistency and solve :

$$x + y + z = 3$$

$$x + 2y + 3z = 4$$

$$x + 4y + 9z = 6.$$ 6

(b) Solve the system of equations by adjoint method :

$$x - 2y + 3z = 2$$

$$2x - 3z = 3$$

$$x + y + z = 0.$$ 6

7. (a) Solve : $(x + 1)\dfrac{dy}{dx} - 2y = (x + 1)^4$. 4

(b) Solve : $(1 + x)\dfrac{dy}{dx} - \tan y = (1 + x)^2 e^x \sec y$. 4

(c) Solve : $\dfrac{dy}{dx} + \dfrac{x + y\cos x}{1 + \sin x} = 0$. 4

OR

8. (a) Solve : $P(P + y) = x(x + y)$. 4

(b) Solve : $y = 2px + p^n_o$ where $P = \dfrac{dy}{dx}$. 4

(c) Solve : $P^3 - 4xyP + 8y^2 = 0$. 4

9. (a) Solve : $\dfrac{d^2y}{dx^2} + 4y = \cos 2x + e^{3x}$. 6

(b) Solve by method of variation of parameter :

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}.$$ 6

(c) Solve :

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x \log x.$$ 6

OR

10. (a) Solve :

$$\frac{dx}{dt} + 2x - 3y = t$$

$$\frac{dy}{dt} - 3x + 2y = e^{2t}.$$ 6

(b) Solve :

$$\frac{d^2y}{dx^2} = 3\sqrt{y}, \text{ given that}$$

$y = 1$, $\frac{dy}{dx} = 2$, when $x = 0$. 6

(c) In an L-C-R circuit, the charge q on a plate of a condenser is given by $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E \sin pt$. The circuit is tuned to resonance so that $P^2 = \frac{1}{LC}$. If initially current i and the charge q be zero, show that for small values of R/L, the current in the circuit at time t is given by $\left(\frac{Et}{2L}\right)\sin pt$. 6

11. (a) If $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that :

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4} \text{ and } \phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right).$$ 4

(b) Find all the values of $\left(\frac{1}{2} + \frac{\sqrt{3}\,i}{2}\right)^{3/4}$ and show that their continual product is 1. 4

OR

12. (a) Using De-Moivre's theorem, solve :

$$x^5 + x^4 + x^3 + x^2 + x + 1 = 0.$$ 4

(b) If $2\cos\theta = x + \frac{1}{x}$, $2\cos\phi = y + \frac{1}{y}$, show that :

$$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\theta + n\phi).$$ 4