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B.E.First Semester All Branches (C.B.S.) / B.E. First Semester (Fire Engineering)

Applied Mathematics - I

P. Pages : 4 **NKT/KS/17/7196**

Time : Three Hours * 0 0 4 7 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) If $y = \left(\sin^{-1} x\right)^2$, prove that **6**

$$\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$

b) Evaluate :

i) $\lim_{x\to 0} \frac{x - \sin x}{\tan^3 x}$ **3**

ii) $\lim_{x\to 0} (\cot x)^{\sin x}$ **3**

OR

2. a) Find the radius of curvature at any 'θ' of the cycloid. **6**

$x = a(\theta - \sin\theta),\ y = a(1-\cos\theta)$

b) Using Taylor's Theorem Find $f\left(\frac{11}{10}\right)$ where $f(x) = x^3 + 8x^2 + 15x - 24$. **6**

3. a) If $u = \log\left[\tan x + \tan y + \tan z\right]$ www.rtmnuonline.com **6**

Prove that,

$$(\sin 2x)\frac{\partial u}{\partial x} + (\sin 2y)\frac{\partial u}{\partial y} + (\sin 2z)\frac{\partial u}{\partial z} = 2$$

b) If $u = \sin^{-1}\left[\frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}\right]$, then find the value of **6**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

c) If $\phi = f(x,y,z)$ and $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$, then show that **6**

$$u\frac{\partial\phi}{\partial u} + v\frac{\partial\phi}{\partial v} + w\frac{\partial\phi}{\partial w} = x\frac{\partial\phi}{\partial x} + y\frac{\partial\phi}{\partial y} + z\frac{\partial\phi}{\partial z}$$

OR

4. a) Given $u = \dfrac{x+y}{1-xy}$ and **6**

$v = \tan^{-1}x + \tan^{-1}y$

find $\dfrac{\partial(u,v)}{\partial(x,y)}$, Are u and v functionally related? If so, find relation between them.

b) Expand $e^x \cos y$ in powers of x and $(y - \pi/2)$ up to the 3rd degree term. **6**

c) Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid. **6**

5. a) Find the inverse of the following matrix by using partitioning method. **6**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

b) Find for what value of λ and μ the system of linear equations : **6**

$x + y + z = 6$

$x + 2y + 5z = 10$

$2x + 3y + \lambda z = \mu$ will have

i) a unique solution

ii) no solution

iii) infinite solutions

OR

6. a) Determine the rank of the matrix. **6**

$$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$$

b) Solve the system of equations by adjoint method : **6**

$5x + 3y + 3z = 48$

$2x + 6y - 3z = 18$

$8x - 3y + 2z = 21$

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7. a) Solve **4**

$$\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1$$

b) Solve **4**

$$(1+x)\frac{dy}{dx} - \tan y = (1+x)^2 e^x \sec y$$

c) Solve **4**

$$\frac{dx}{dy} = \left[\frac{1+y^2+\cos^2 x}{y \sin 2x}\right]$$

OR

8. a) Solve **4**

$$y + px = x^4 p^2$$

b) Solve **4**

$$p(p+y) = x(x+y)$$

c) When a resistance R ohms is connected in series with an inductance L henries with constant emf of E volts, the current 'i' amperes at time 't' is given by $L\frac{di}{dt} + Ri = E$. Find the current at any time 't' if i = 0 at t = 0. **4**

9. a) Solve : **6**

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{2x} \sin x$$

b) Solve by method of variation of parameters **6**

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \frac{e^{2x}}{x}$$

c) Solve **6**

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = 2 + 5\log x$$

OR

10. a) Solve the simultaneous equations **6**

$$\frac{dx}{dt} + y = \sin T$$

$$\frac{dy}{dt} + x = \cos T$$

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b) A motion is governed by $\frac{d^2x}{dt^2} = 36x^{-2}$, given that at t = 0, x = 8 and $\frac{dx}{dt} = 0$, find the displacement at any time t. **6**

c) The differential equation for a circuit in which self - inductance and capacitance neutralize each other is $L\frac{d^2i}{dt^2} + \frac{i}{c} = 0$, find the current i. **6**

11. a) Find all the values of $(16)^{1/4}$ in $(a+ib)$ form. **4**

b) Use De - Moivre's theorem to solve **4**
$x^8 + x^5 + x^3 + 1 = 0$

OR

12. a) If $2\cos\theta = x + \frac{1}{x}$, **4**

$2\cos\phi = y + \frac{1}{y}$, prove that

$$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\theta + n\phi)$$

b) Prove that, **4**

$$(a+ib)^{m/n} + (a-ib)^{m/n} =$$

$$2\left(a^2+b^2\right)^{m/2n} \cos\left[\frac{m}{n}\tan^{-1}\frac{b}{a}\right]$$

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