B.E. All Branches First Semester (C.B.S.) / B.E. (Fire Engineering) First Semester

Applied Mathematics - I

P. Pages : 3 **NRT/KS/19/3281/3936**

Time : Three Hours * 0 1 1 6 * Max. Marks : 80

Notes : 1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) If $y = a\cos(\log x) + b\sin(\log x)$ then show that $x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1)y_n = 0$ **6**

b) **3**

i) Evaluate $\lim_{x \to 0} \dfrac{e^x - e^{-x} + 2\sin x - 4x}{x^5}$

ii) Evaluate $\lim_{x \to \pi/2} (\sin x)^{\tan x}$ **3**

OR

2. a) Expand log (cos x) in ascending powers of x upto and including the term x^4 and calculate $\log_{10} \cos\left(\dfrac{\pi}{12}\right)$ up to three places of decimal. **6**

b) Find the radius of curvature at any θ of the cycloid $x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$ **6**

3. a) If $x^x\, y^y\, z^z = c$, show that at x = y = z $\dfrac{\partial^2 z}{\partial x\, \partial y} = -(x \log ex)^{-1}$ **6**

b) if $\phi = f(x, y, z)$, where $x = \sqrt{vw},\ y = \sqrt{uw},\ z = \sqrt{uv}$ then prove that **6**

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial v} + w\frac{\partial \phi}{\partial w} = x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} + z\frac{\partial \phi}{\partial z}$$

c) **6**

If $u = \text{cosec}^{-1}\left[\dfrac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}\right]^{1/2}$, show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 y}{\partial x\, \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left[\frac{13}{12} + \frac{\tan^2 u}{12}\right]$$

OR

4. a) If $u = 3x + 2y - z,\ v = x - 2y + z,\ w = x(x + 2y - z)$. Are u, v and w functionally related ? If so, find its relationship. **6**

b) Expand sin(xy) in powers of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$ as far as the term of 2nd degree. **6**

c) The temperature T at any point (x, y, z) in space is $T = 400\,xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. **6**

5. a) Solve the system of equation by adjoint method **6**
$2x + 3y + 4z = 15$, $3x - y + 2z = 9,\ x + y + z = 5$

b) Find the rank of matrix **6**

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

OR

6. a) Test the consistency and solve **6**
$x + y + z = 6,\ x - y + 2z = 5,\ 3x + y + z = 8,\ 2x - 2y + 3z = 7$

b) Find A^{-1} by partitioning method for the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$ **6**

7. a) Solve $(x+1)\frac{dy}{dx} - 2y = (x+1)^4$ **4**

b) Solve $\frac{dy}{dx} + y\tan x = y^3 \sec x$ **4**

c) Solve $xy^2dx + (2 + x^2y)dy = 0$ **4**

OR

8. a) Solve $y - 2px = \tan^{-1}(xp^2)$ where $p = \frac{dy}{dx}$ **4**

b) When a resistance R ohms is connected in series with an inductance L henries with constant emf of E volts, the current i amperes at time t is given by $L\frac{di}{dt} + Ri = E$, Find the current at any time t, if i=0 at t=0. **4**

9. a) Solve $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^x\sin x$ **6**

b) Solve $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=\frac{e^{2x}}{x}$ using method of separation of variable. **6**

c) Solve $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=2\left(x+\frac{1}{x}\right)$ **6**

OR

10. a) The radial displacement u in a rotating disc at a distance r from the axis is given by $r^2\frac{d^2u}{dr^2}+r\frac{du}{dr}-u+kr^3=0$, where k is a constant. Solve the equation under the condition u = 0 when r = 0, u = 0 when r = a. **6**

b) Solve $\frac{d^2y}{dx^2}=\sec^2 y\tan y$, given that y = 0 and $\frac{dy}{dx}=1$, when x = 0. **6**

c) Solve $\frac{dx}{dt}+y-\sin t=0,\ \frac{dy}{dt}+x-\cos t=0$ given that x = 2 and y = 0 when t = 0. **6**

11. a) Prove that $(a+ib)^{m/n}+(a-ib)^{m/n}=2(a^2+b^2)^{m/2n}\cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$ **4**

b) Find all the roots of $x^6-1=0$. **4**

OR

12. a) If $\tan(\theta+i\phi)=\tan\alpha+i\sec\alpha$, prove that $2\theta=n\pi+\frac{\pi}{2}+\alpha$. **4**

b) Find the general value of $\log_e(4+3i)$. **4**

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