B.E. All Branches Semester First (C.B.S.) / B.E. Semester First (Fire Engineering)

Applied Mathematics - I

Paper - I

P. Pages : 4

Time : Three Hours

|| *0 6 3 0* ||

KNT/KW/16/7196

Max. Marks : 80

_____

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) If $y = \cos(m\sin^{-1} x)$
prove that

$$(1 - x^2)\, y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$$

6

b) Evaluate

3

i) $\displaystyle\lim_{x \to 0}\left[\frac{1}{\sin^2 x} - \frac{1}{x^2} \right]$

ii) $\displaystyle\lim_{x \to \frac{\pi}{2}} (\cos x)^{\cos x}$

3

OR

2. a) A curve is given by $x = a\sin\theta$; $y = b\cos 2\theta$. Find the radius of curvature at $\theta = \frac{\pi}{3}$.

6

b) Expand $2x^3 + 7x^2 + x - 7$ in powers of $(x - 2)$.

6

3. a) If $u(x + y) = x^2 + y^2$ Then prove that

$$\left[\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right]^2 = 4\left[1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right]$$

6

b) If $u = \tan^{-1}\left[\dfrac{x^{1/2} + y^{1/2}}{x^{1/3} - y^{1/3}} \right]$

find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x\, \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

6

1

c) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ **6**
Then prove that

$$\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$$

OR

4. a) If $u = \dfrac{x+y}{x-y}$ & $v = \dfrac{xy}{(x-y)^2}$ **6**

Then show that u & v are functionally related? If so find the relation between them.

b) Find the points on the surface $z^2 = xy + 1$ nearest to the origin. **6**

c) Obtain Taylor's expansion of $\tan^{-1}\left(\dfrac{y}{x}\right)$ about (1, 1) upto the third degree terms. **6**

5. a) Find the inverse of matrix by partitioning **6**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 4 & 12 \end{bmatrix}$$

b) Test the consistency and solve **6**

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

OR

6. a) Determine the rank of the matrix : **6**

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

b) Using adjoint method, solve **6**

$$x + 2y + z = 7$$
$$x + 3z = 11$$
$$2x - 3y = 1$$

7. a) $\dfrac{dy}{dx} - y\tan x = 3e^{-\sin x}$ **4**

b) $\dfrac{dx}{dy} = \dfrac{1 + y^2 + \cos^2 x}{y\sin 2x}$ **4**

c) $\dfrac{dy}{dx} + y \tan x = y^3 \sec x$

<div align="center">OR</div>

8. a) $x^2 \left(\dfrac{dy}{dx}\right)^2 + xy \dfrac{dy}{dx} - 6y^2 = 0$ **3**

b)   Solve : $y + px = x^4 p^2$ **3**

c)   The equation of electromotive force in terms of current i for an electrical circuit having resistance R and condenser of capacity C in series is :   **6**

$E = Ri + \int \dfrac{i}{c} dt$ Find the current i at any time t when $E = E_m \sin wt$.

9. a)   Solve $(D^2 - 5D + 6) y = e^{2x} + \cos x$.   **6**

b)   Solve by using variation of parameters method   **6**

$$\dfrac{d^2 y}{dx^2} - 4 \dfrac{dy}{dx} + 4y = \dfrac{e^{2x}}{x}$$

c)   Solve $x^2 \dfrac{d^2 y}{dx^2} - 2x \dfrac{dy}{dx} - 4y = x^2 + 2 \log x$ **6**

<div align="center">OR</div>

10. a)   Solve $\dfrac{d^2 y}{dx^2} = e^{-2y}$ **6**

Given that $y = 0, \dfrac{dy}{dx} = 0$ when $x = 0$.

b)   Solve $\dfrac{d^2 x}{dt^2} = b \dfrac{dy}{dt}$ and   **6**

$\dfrac{d^2 y}{dt^2} = a - b \dfrac{dx}{dt}$

c)   A body executes damped forced vibrations given by the equation   **6**

$\dfrac{d^2 x}{dt^2} + 2k \dfrac{dx}{dt} + b^2 x = e^{-kt} \sin wt$

solve the equation when $w^2 = b^2 - k^2$.

11. a)　If $2\cos\theta = x + \dfrac{1}{x}$ and **4**

$2\cos\phi = y + \dfrac{1}{y}$ then prove that

$x^m y^n + \dfrac{1}{x^m y^n} = 2\cos(m\theta + n\phi)$

4

b)　Find all the values of $\left[\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right]^{3/4}$ and show that the continued product of all the values

is 1.

OR

12. a)　Solve using De - Moivre's theorem **4**
$x^7 - x^4 + x^3 - 1 = 0$

b)　Separate $\tan^{-1}(x + iy)$ into real and imaginary parts. **4**

****