

Faculty of Engineering & Technology
First Semester B.E. (C.B.S.) Examination
APPLIED MATHEMATICS—I
Paper—I

Time—Three Hours]

[Maximum Marks—80

N.B. :— (1) All questions carry marks as indicated.

(2) Use of Non-programmable calculator is permitted.

(3) Solve :

Que. No. 1 OR Que. No. 2

Que. No. 3 OR Que. No. 4

Que. No. 5 OR Que. No. 6

Que. No. 7 OR Que. No. 8

Que. No. 9 OR Que. No. 10

Que. No. 11 OR Que. No. 12

1. (a) If $Y = (\sin^{-1} x)^2$; then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.

6

(b) Evaluate $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2\cos x}{x \sin x}$.

3

(c) Evaluate $\lim_{x \rightarrow \infty} \left[\frac{1}{x} - \cot x \right]$.

3

OR

2. (a) If $x = a \cos^4 \theta$; $y = a \sin^4 \theta$, find the curvature at $\theta = \pi/6$. 6

- (b) Use Taylor's Series Expansion to evaluate $\sin 60^\circ 30'$ correct up to three decimal places. 6

3. (a) If $v = At^{\frac{1}{2}} e^{\frac{1}{4t}}$

then prove that

$$\frac{\partial v}{\partial t} = a^{-1} \frac{\partial^2 v}{\partial x^2}$$

6

- (b) If $u = \sin^{-1} \left(\frac{x^2 - y^2}{\sqrt{x} + \sqrt{y}} \right)$

find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

6

- (c) If $\phi = f(x, y, z)$

$$\text{and } x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$$

then show that

$$u \frac{\partial \phi}{\partial u} + v \frac{\partial \phi}{\partial v} + w \frac{\partial \phi}{\partial w} = x \frac{\partial \phi}{\partial x} + y \frac{\partial \phi}{\partial y} + z \frac{\partial \phi}{\partial z}$$

6

OR

4. (a) If $y_1 = \frac{x_2 x_1}{x_1}, y_2 = \frac{x_1 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_1}$

$$\text{find the value of } \frac{\partial(x_1, x_2, x_3)}{\partial(y_1, y_2, y_3)}$$

6

- (b) Expand $e^x \cos y$ in the power of x and y as far as the term of third degree. 6

- (c) Given $f(x, y, z) = \frac{5xyz}{x + 2y + 4z}$ Find the values of x, y, z for which $f(x, y, z)$ is maximum Subject to the condition $x + y + z = 8$ (by Lagrange method).

5. (a) Find the inverse of matrix by partitioning method: 6

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

6

(b) Find the rank of matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

6

OR

6. (a) Test the consistency and solve :

$$x + y + z = 6$$

$$x - y + 2z = 5$$

$$3x + y + z = 8.$$

6

(b) Solve the system of equations by adjoint method :

$$2x + 3y + 4z = 15$$

$$3x - y + 2z = 9$$

$$x + y + z = 5.$$

6

7. (a) Solve $\frac{dy}{dx} - y \tan x = 3e^{\sin x}$.

4

(b) Solve $3y^2 \frac{dy}{dx} + 2xy^3 = 4xe^{x^2}$

4

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4

Cond.

(c) Solve $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$.

4

OR

8. (a) Solve $p^2 + 2px + py + 2xy = 0$.

4

(b) Solve $y = x + 2 \tan^{-1} p$.

4

(c) A resistance $R = 50$ ohms and an inductance $L = 10$ henries are connected in series with a constant voltage of $E = 100$ volts. If the current is zero at $t = 0$.

Find :

(i) The equation of i (ii) The current at $t = 0.5$ seconds.

4

9. (a) Solve $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 13y = e^{2x} \sin 3x$.

6

(b) Solve by method of variation of parameter

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = e^x \tan x.$$

6

(c) Solve $x^3 \frac{d^3y}{dx^3} - 2x^2 \frac{d^2y}{dx^2} + 2x = 10 \left(x + \frac{1}{x} \right)$.

OR

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Contd.

10. (a) In an L.C.R. circuit the charge q on a plate of a condenser is given by.

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E \sin pt$$

The circuit is tuned to

resonance so that $P^2 = \frac{1}{LC}$. If initially current i and

the charge q be zero show that for small value of R , the current in the circuit at time t is given by $\frac{Et}{2L} \sin pt$. 6

(b) Solve :

$$\frac{dx}{dt} + y - \sin t = 0$$

$$\frac{dy}{dt} + x - \cos t = 0.$$

- (c) Solve $\frac{d^2 y}{dx^2} = 2(y^3 + y)$ under the condition $\dot{y} = 0$;

$$\frac{dy}{dx} = 1 \text{ when } x = 0.$$

11. (a) Prove that $\log \tan \left(\frac{\pi}{4} + i \frac{x}{2} \right) = i \tan^{-1} (\sinh x)$.

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6

Contd.

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- (b) Find all the values of $\left[\frac{1}{2} + \frac{\sqrt{3}}{2} i \right]^{1/4}$ and show that the continued product of all the values is 1. 4

OR

12. (a) Solve using De-Moivre's theorem $x^7 - x^4 + x^3 - 1 = 0$. 4

- (b) Separate $\tan^{-1}(x + iy)$ into real and imaginary parts. 4

