B.E. All Branches First Semester (C.B.S.) / B.E. (Fire Engineering) First Semester

# Applied Mathematics - I

P. Pages : 3 **NIR/KW/18/3281/3936**

Time : Three Hours * 1 9 3 7 * Max. Marks : 80

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Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
9. Use of non programmable calculator is permitted.

**1.** a) If $y=\sin^{-1}x$, then show that $\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$. **6**

b) Evaluate **3**

i) $\lim_{x\to0}\dfrac{x\cos x-\sin x}{x^2\sin x}$

**3**

ii) $\lim_{x\to0}(\cot x)^{\sin x}$

**OR**

**2.** a) Using Taylor's theorem, find the valve of tan 46º correct to four decimal places. **6**

b) A curve is given by $x=a\sin\theta$, $y=b\cos2\theta$. Find the radius of curvature at $\theta=\pi/3$. **6**

**3.** a) If $u=\log(\tan x+\tan y+\tan z)$ show that. **6**

$\sin2x\dfrac{\partial u}{\partial x}+\sin2y\dfrac{\partial u}{\partial y}\sin2z\dfrac{\partial u}{\partial z}=2$.

b) If $u=\tan^{-1}\left[\dfrac{x^3+y^3}{x-y}\right]$ then prove that $x^2\dfrac{\partial^2u}{\partial x^2}+2xy\dfrac{\partial^2u}{\partial x\partial y}+y^2\dfrac{\partial^2u}{\partial y^2}=\sin4u-\sin2u$. **6**

c) If $u=f\left(\dfrac{x}{y},\dfrac{y}{z},\dfrac{z}{x}\right)$, then Show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}=0$. **6**

**OR**

**4.** a) If $u=3x+2y-z,\ v=x-2y+z,\ \&\ w=x+2y-z$ are u, v & w functionally related? If so, find the relationship? **6**

b) Expand $y^x$ in the neighborhood of $(1,1)$ up to the terms of second degree. **6**

c) Divide 24 into three parts such that the continued product of first, square of second and cub of the third is maximum. **6**

**5.** a) Determine the rank of the matrix **6**

$$A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$

b) Solve the system of equations by adjoint method. **6**

$$x + y + z = 6$$
$$2x - y + z = 3$$
$$3x + 2y + 2z = 13$$

**OR**

**6.** a) Find the inverse of the matrix by partitioning. **6**

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

b) Find for what value of $\lambda$ and $\mu$ the system of linear equations **6**

$x + y + z = 6,$

$x + 2y + 5z = 10,$ and

$2x + 3y + \lambda z = \mu$ will have

i) a unique solution.
ii) No solution.
iii) Infinite solution.

**7.** a) Solve **4**

$$(x+1)\frac{dy}{dx} - 2y = (x+1)^4$$

b) Solve $\sec^2 y \frac{dy}{dx} + x \tan y = x^3$. **4**

c) $(2x - y + 1)dx - (x - 2y + 1)dy = 0$. **4**

**OR**

**8.** a) Solve $p^3 - 4xyp + 8y^3 = 0$. **3**

b) Solve $y = 2px + p^4x^2$. **3**

c) The equation of electromotive force in terms of current i for an electrical circuit having resistance R and condenser of capacity C in series is: $E = Ri + \int \frac{i}{c} dt$. **6**

Find the current i at any time t when $E = E_m \sin wt$.

9. a) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{-2x} + \cos x$. **6**

b) Solve by the method of variation of parameter. **6**
$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$.

c) Solve $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2$ **6**

**OR**

10. a) Solve $\frac{d^2y}{dx^2} = \sec^2 y \tan y$, given that **6**
$y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$.

b) Solve, the following equations. **6**
$\frac{dx}{dt} + 5x - 2y = t$ and
$\frac{dy}{dt} + 2x + y = 0.$

c) The radial displacement u in a rotating disc at a distance r from the axis is given by **6**
$r^2 \frac{d^2u}{dr^2} + r\frac{du}{dr} - u + kr^3 = 0$
Where K is constant. Solve the equation under the condition
$u = 0$ when $r = 0$ and $u = 0$ when $r = a$.

11. a) Use De-Moiver's theorem to solve **4**
$x^5 + x^4 + x^3 + x^2 + x + 1 = 0$.

b) Find the values of $(1+i)^{2/3}$ **4**

**OR**

12. a) Prove that $\log\tan\left(\frac{\pi}{4} + i\frac{x}{2}\right) = i\tan^{-1}(\sinh x)$ **4**

b) Find the general value of log (-i) **4**

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