$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

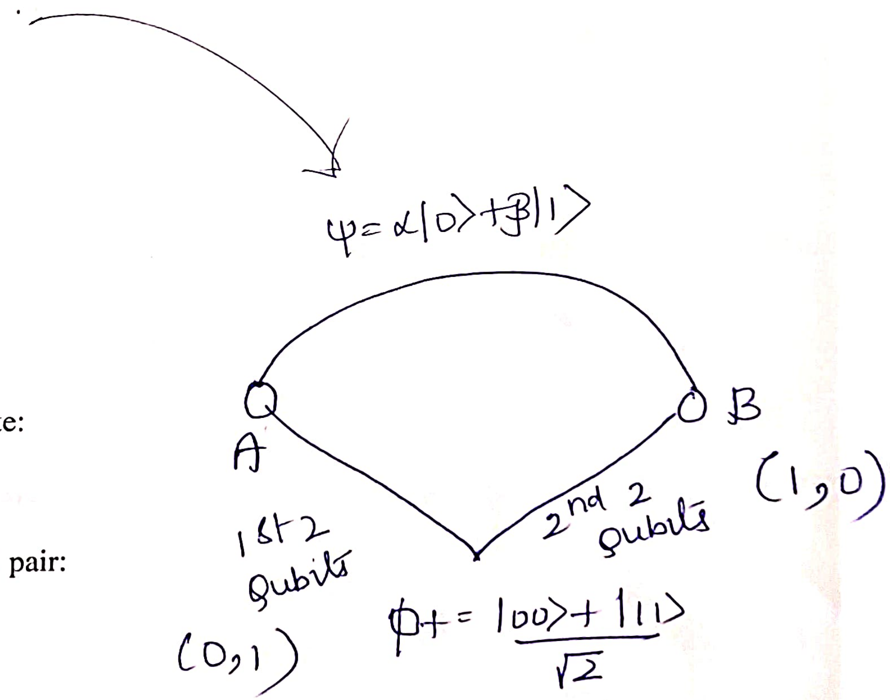
$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

: key properties

- Maximal entanglement
- Measurement of one determines the other
- Cannot be prepared by local operations

Simple Diagram



Characters

- **Alice** (sender)
- **Bob** (receiver)

Initial Resources

- Alice has unknown state:

  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

- Alice & Bob share Bell pair:

System label

- Qubit 1: Unknown state (Alice)
- Qubit 2: Alice's half of Bell pair
- Qubit 3: Bob's half of Bell pair

STEP 1 — Combine unknown qubit with Bell pair

Full state:

|ψ⟩⊗|Φ+⟩

Expand: $\psi = \beta_{00} = (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{1}{\sqrt{2}}[|00\rangle + |11\rangle]$

$= \frac{1}{\sqrt{2}}[\alpha\underbrace{|00\rangle}_{A} + \alpha\underbrace{|011\rangle}_{A} + \beta\underbrace{|100\rangle}_{A} + \beta\underbrace{|111\rangle}_{A}]$

$= \frac{1}{\sqrt{2}}[\alpha\underbrace{|000\rangle}_{A} + \alpha\underbrace{|011\rangle}_{A} + \beta\underbrace{|100\rangle}_{A} + \beta\underbrace{|111\rangle}_{A}]$

Explain tensor-product expansion clearly.
This is the **raw starting point.**

STEP 2 — Alice applies CNOT on (1→2)

x	y	CNOT
0	0	00
0	1	01
1	0	11
1	1	10

- Control: qubit 1
- Target: qubit 2

Show effect:

|00⟩→|00⟩, |01⟩→|01⟩, |10⟩→|11⟩, |11⟩→|10>

$= \frac{1}{\sqrt{2}}[\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle]$

Resulting 3-qubit state:

STEP 3 — Alice applies Hadamard on qubit 1

Hadamard

$H|0\rangle = (|0\rangle + |1\rangle)/\sqrt{2} = H|+\rangle$

$H|1\rangle = (|0\rangle - |1\rangle)/\sqrt{2} = H|-\rangle$

Hadamard acts as:

$= \frac{1}{\sqrt{2}}[\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle$

$= \frac{1}{\sqrt{2}}[\alpha|+00\rangle + \alpha|+11\rangle + \beta|-10\rangle + \beta|-01\rangle$

$= \frac{1}{\sqrt{2}}[\alpha[\underbrace{|000\rangle}_{A} + \underbrace{|100\rangle}_{A}] + \alpha[\underbrace{|011\rangle}_{A} + \underbrace{|111\rangle}_{A}] + \beta[\underbrace{|010\rangle}_{A} - \underbrace{|110\rangle}_{A}]$

$+ \beta[\underbrace{|001\rangle}_{A} - \underbrace{|101\rangle}_{A}]]$

Interpretation

- Measurement results (00, 01, 10, 11) determine which version of **Bob's qubit** he will receive.

Measure.

$\Rightarrow \frac{1}{2}\left[\alpha\left[|000\rangle+|100\rangle\right] + \alpha\left[|011\rangle+|111\rangle\right] + \beta\left[|010\rangle - |110\rangle\right] + \beta\left[|001\rangle - |101\rangle\right]\right]$

$|00\rangle_A = \alpha|0\rangle+\beta|1\rangle$ $\quad |01\rangle = \alpha|1\rangle+\beta|0\rangle$

$|10\rangle_A = \alpha|0\rangle-\beta|1\rangle$ $\quad |11\rangle = \alpha|1\rangle-\beta|1\rangle.$

Diagram:

Alice: [Q1: ψ] — CNOT — H — Measure ——— classical bits ——▶ Bob

Bob: apply correction

STEP 4 — Alice measures qubits 1 and 2 (Bell basis)

Possible outputs:

- 00
- 01
- 10
- 11

Each occurs with probability 1/4.

Bob's qubit collapses accordingly:

Corresponding collapsed states

Alice outcome	**Bob receives**	
00	(\alpha	$\alpha|0\rangle+\beta|1\rangle$
01	(\alpha	$\alpha|1\rangle+\beta|0\rangle$
10	(\alpha	$\alpha|0\rangle-\beta|1\rangle$
11	(\alpha	$\alpha|1\rangle-\beta|1\rangle$

STEP 5 — Alice sends 2 classical bits to Bob

Explain:

- Shows quantum teleportation is **not faster than light.**
- Classical communication ensures causality.

STEP 6 — Bob applies correction operations

Outcome	**Correction**	**Meaning**
00	I	Do nothing $\rightarrow \alpha|0\rangle+\beta|1\rangle$

Outcome	Correction	Meaning
01	X	Bit flip
10	Z	Phase flip
11	ZX	Bit + Phase flip

$$\left.\begin{aligned} \alpha|1\rangle+\beta|0\rangle &\xrightarrow{X} \alpha|0\rangle+\beta|1\rangle \\ \alpha|0\rangle-\beta|1\rangle &\xrightarrow{Z} \alpha|0\rangle+\beta|1\rangle \\ \alpha|1\rangle-\beta|0\rangle &\xrightarrow{XZ} \alpha|0\rangle+\beta|1\rangle \end{aligned}\right\} \psi$$

Explain Pauli gates:

- **X gate:** $|0\rangle \leftrightarrow |1\rangle$
- **Z gate:** $|1\rangle \rightarrow -|1\rangle$

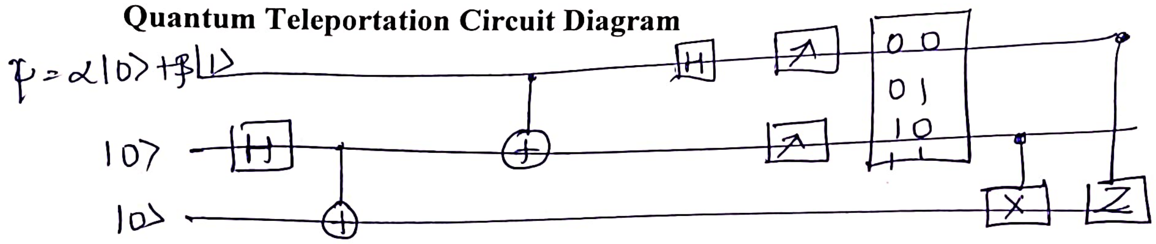
STEP 7 — Bob reconstructs the unknown state

After applying the correction:

$|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$

Teleportation complete.

Quantum Teleportation Circuit Diagram



- b1, b2 are measurement results.

Applications

- Quantum repeaters
- Quantum internet nodes
- Distributed quantum computing
- Secure communication (with QKD + teleportation)

Extensions to teach

- Teleportation of multi-qubit states
- Continuous-variable teleportation
- Entanglement swapping
- GHZ state teleportation