QUANTUM COMPUTING

UNIT 1

Fundamentals of Quantum Computing

Dr Anjaneyulu Pasala

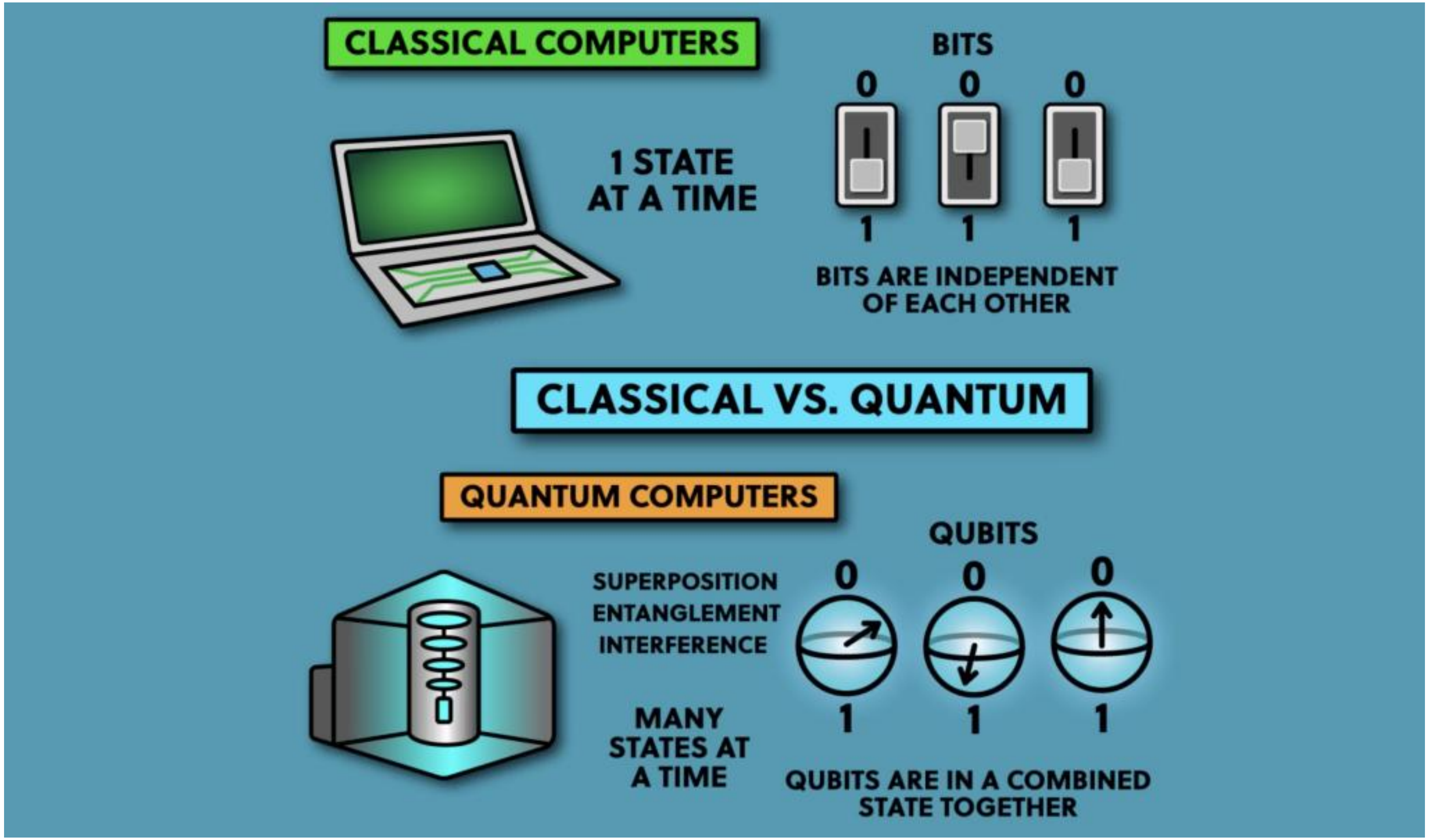
- Fundamentals of Quantum Computing: Overview of Quantum vs. Classical Computing, Basic Concepts in Quantum Mechanics: Superposition, Entanglement, Measurement, Qubits and the Bloch Sphere. Introduction to Quantum Gates: Pauli Gates, Hadamard Gate, Phase Shift, and CNOT, Quantum Circuit Design and Simulation.

Introduction

This unit introduces:

- **Quantum vs. Classical Computing** – comparing binary logic with quantum principles.
- **Quantum Mechanics Basics** – key ideas like **superposition** (qubits being in multiple states at once), **entanglement** (strong correlations between qubits), and **measurement** (how quantum states collapse when observed).
- **Qubits and the Bloch Sphere** – representing and visualizing quantum states.
- **Quantum Gates** – operations such as **Pauli gates**, **Hadamard**, **phase shift**, and **CNOT**, which manipulate qubits.
- **Quantum Circuits and Simulation** – designing simple circuits to understand computation at a quantum level.
- Together, these concepts form the **foundation** for exploring advanced quantum algorithms and applications.

Classical Computers vs Quantum Computer



Binary : Base-2

1 1 0 1 1 0 1

= 1 * 2 ^ 6 + 1 * 2 ^ 5 + 0 * 2 ^ 4 + 1 * 2^ 3 + 1 * 2 ^ 2 + 0 * 2 ^ 1 + 1 * 2 ^ 0

= 64 + 32 + 0 + 8 + 4 + 0 + 1

= 109

Hexadecimal : Base-16

0 1 2 3 4 5 6 7 8 9	A	B	C	D	E	F
	10	11	12	13	14	15

A 4 F 6

=10 * 16 ^ 3 + 4 * 16 ^ 2 + 15 * 16 ^ 1 + 6 * 16 ^ 0

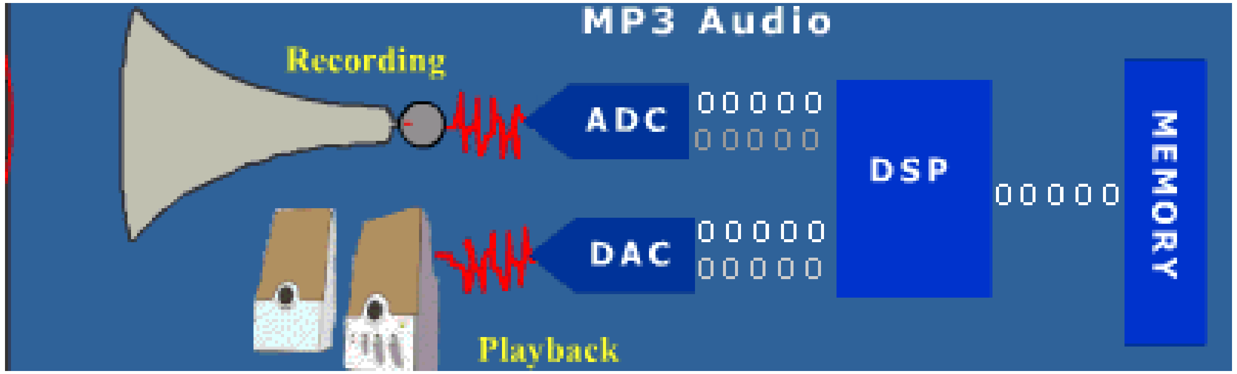
=40960 + 1024 + 240 + 6

=42230

ASCII conversion Chart

Decimal	Binary	Octal	Hex	ASCII	Decimal	Binary	Octal	Hex	ASCII	Decimal	Binary	Octal	Hex	ASCII	Decimal	Binary	Octal	Hex	ASCII
0	00000000	000	00	NUL	32	00100000	040	20	SP	64	01000000	100	40	@	96	01100000	140	60	`
1	00000001	001	01	SOH	33	00100001	041	21	!	65	01000001	101	41	A	97	01100001	141	61	a
2	00000010	002	02	STX	34	00100010	042	22	"	66	01000010	102	42	B	98	01100010	142	62	b
3	00000011	003	03	ETX	35	00100011	043	23	#	67	01000011	103	43	C	99	01100011	143	63	c
4	00000100	004	04	EOT	36	00100100	044	24	$	68	01000100	104	44	D	100	01100100	144	64	d
5	00000101	005	05	ENQ	37	00100101	045	25	%	69	01000101	105	45	E	101	01100101	145	65	e
6	00000110	006	06	ACK	38	00100110	046	26	&	70	01000110	106	46	F	102	01100110	146	66	f
7	00000111	007	07	BEL	39	00100111	047	27	'	71	01000111	107	47	G	103	01100111	147	67	g
8	00001000	010	08	BS	40	00101000	050	28	(	72	01001000	110	48	H	104	01101000	150	68	h
9	00001001	011	09	HT	41	00101001	051	29	)	73	01001001	111	49	I	105	01101001	151	69	i
10	00001010	012	0A	LF	42	00101010	052	2A	*	74	01001010	112	4A	J	106	01101010	152	6A	j
11	00001011	013	0B	VT	43	00101011	053	2B	+	75	01001011	113	4B	K	107	01101011	153	6B	k
12	00001100	014	0C	FF	44	00101100	054	2C	,	76	01001100	114	4C	L	108	01101100	154	6C	l
13	00001101	015	0D	CR	45	00101101	055	2D	-	77	01001101	115	4D	M	109	01101101	155	6D	m
14	00001110	016	0E	SO	46	00101110	056	2E	.	78	01001110	116	4E	N	110	01101110	156	6E	n
15	00001111	017	0F	SI	47	00101111	057	2F	/	79	01001111	117	4F	O	111	01101111	157	6F	o
16	00010000	020	10	DLE	48	00110000	060	30	0	80	01010000	120	50	P	112	01110000	160	70	p
17	00010001	021	11	DC1	49	00110001	061	31	1	81	01010001	121	51	Q	113	01110001	161	71	q
18	00010010	022	12	DC2	50	00110010	062	32	2	82	01010010	122	52	R	114	01110010	162	72	r
19	00010011	023	13	DC3	51	00110011	063	33	3	83	01010011	123	53	S	115	01110011	163	73	s
20	00010100	024	14	DC4	52	00110100	064	34	4	84	01010100	124	54	T	116	01110100	164	74	t
21	00010101	025	15	NAK	53	00110101	065	35	5	85	01010101	125	55	U	117	01110101	165	75	u
22	00010110	026	16	SYN	54	00110110	066	36	6	86	01010110	126	56	V	118	01110110	166	76	v
23	00010111	027	17	ETB	55	00110111	067	37	7	87	01010111	127	57	W	119	01110111	167	77	w
24	00011000	030	18	CAN	56	00111000	070	38	8	88	01011000	130	58	X	120	01111000	170	78	x
25	00011001	031	19	EM	57	00111001	071	39	9	89	01011001	131	59	Y	121	01111001	171	79	y
26	00011010	032	1A	SUB	58	00111010	072	3A	:	90	01011010	132	5A	Z	122	01111010	172	7A	z
27	00011011	033	1B	ESC	59	00111011	073	3B	;	91	01011011	133	5B	[	123	01111011	173	7B	{
28	00011100	034	1C	FS	60	00111100	074	3C	<	92	01011100	134	5C	\	124	01111100	174	7C	|
29	00011101	035	1D	GS	61	00111101	075	3D	=	93	01011101	135	5D	]	125	01111101	175	7D	}
30	00011110	036	1E	RS	62	00111110	076	3E	>	94	01011110	136	5E	^	126	01111110	176	7E	~
31	00011111	037	1F	US	63	00111111	077	3F	?	95	01011111	137	5F	_	127	01111111	177	7F	DEL

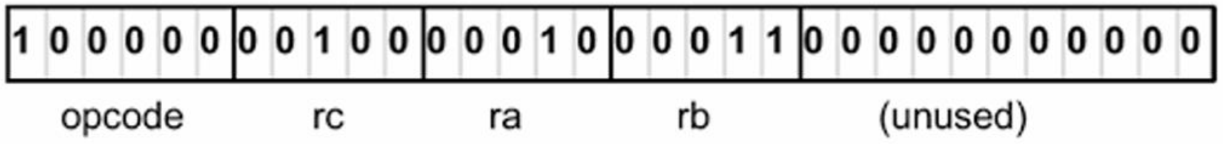
Classical System



Machine Learning

Assembly

High level language

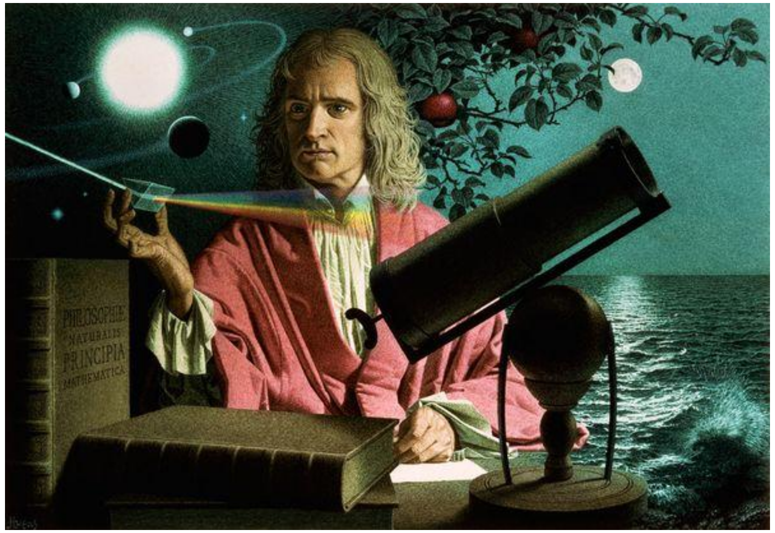


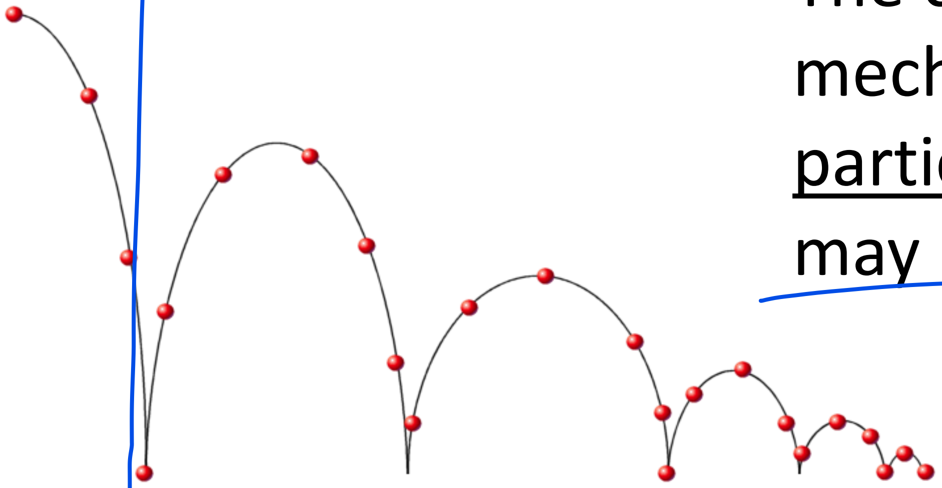
ADD (R2, R3, R4)

A = B + C;

Classical computing (Newtonian Mechanics)

- Classical mechanics provide extreme accurate results while dealing with movements of objects in normal scale.
- The states are deterministic.



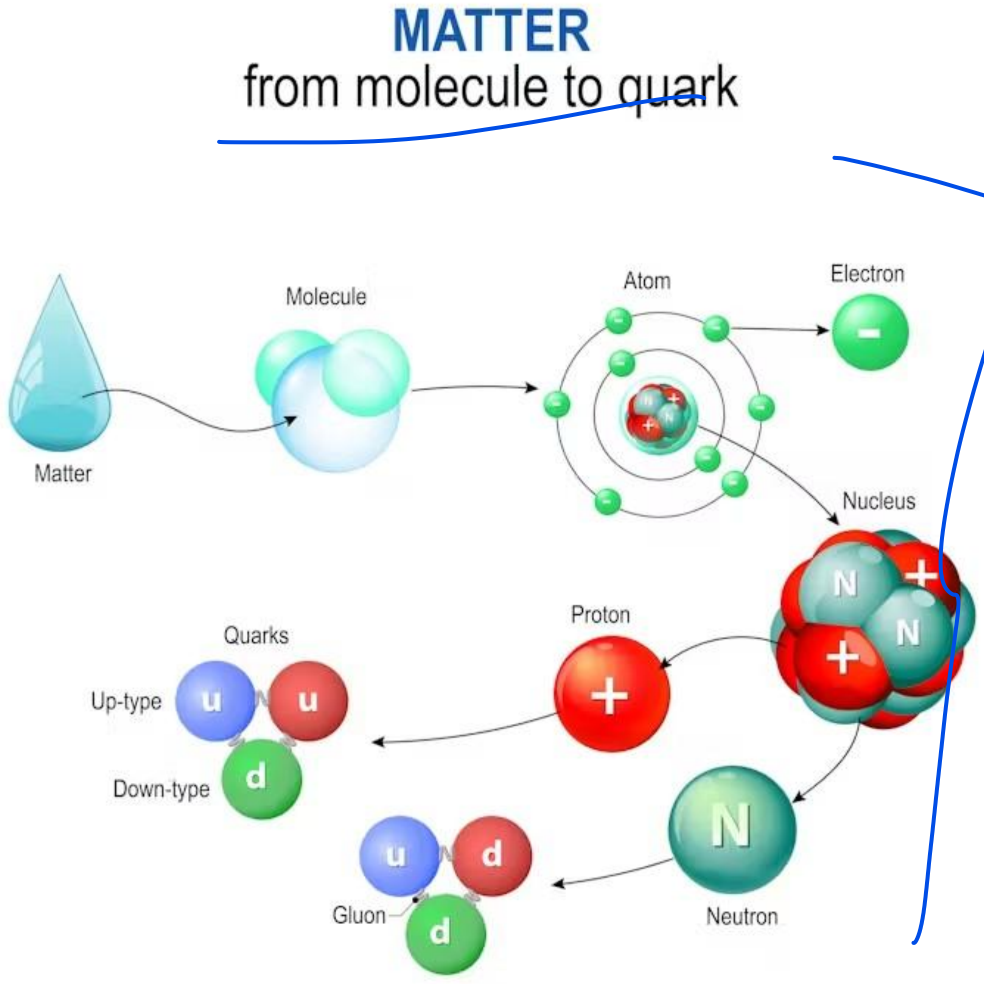


The classical theories are simple, but this branch of mechanics cannot be applied to extremely small particles moving at very high speed, as the results may be inaccurate

Quantum Mechanics

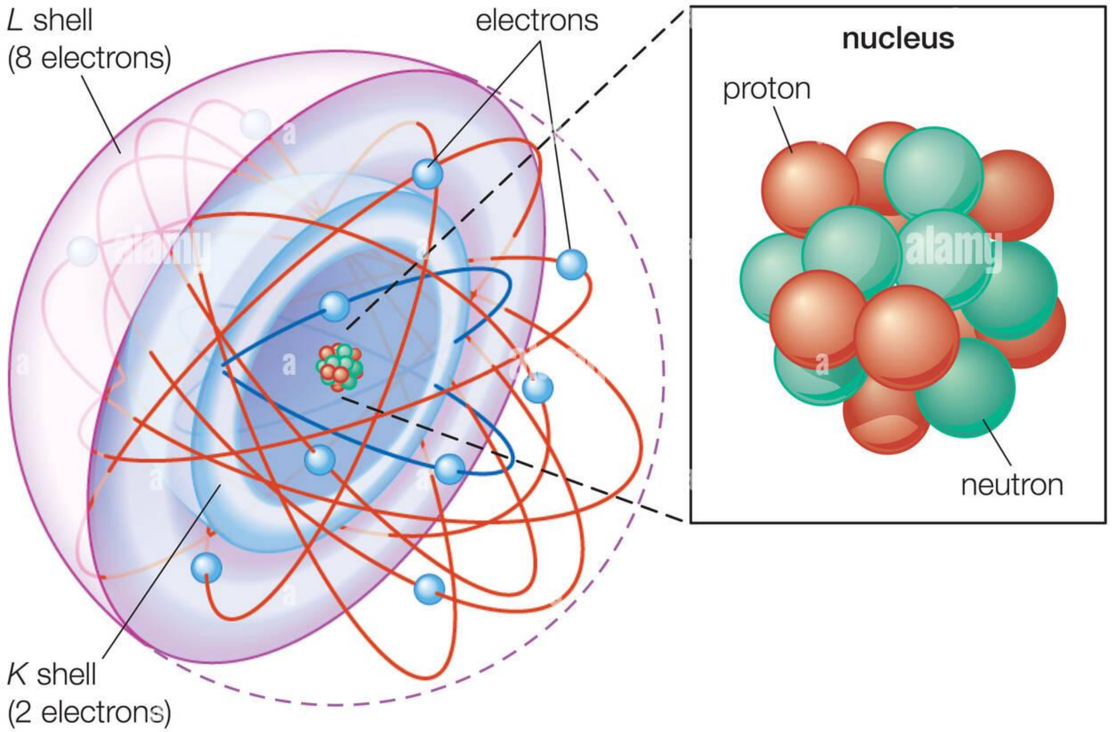
It is a branch of physics that explains the behavior of matter and energy at atomic and subatomic scales, where particles can exist in multiple states at once and are influenced by probability rather than certainty

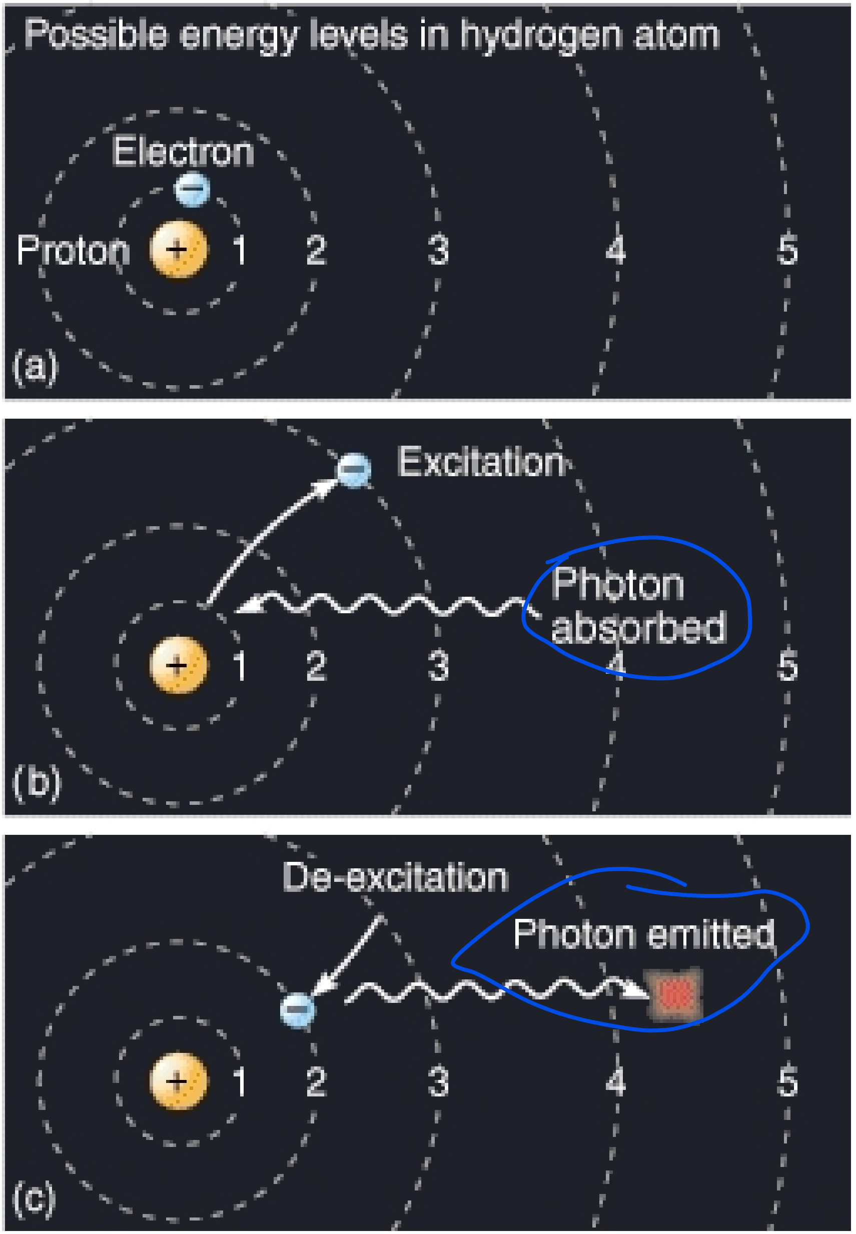
- Particles can be in more than one state at the same time : Superposition.
- Measuring a particle changes its state : Observer effect.
- Particles can be "entangled", the state of one affects another, even at a distance.
- Outcomes are probabilistic, not deterministic : we can only predict the likelihood of an event, not its certainty.



Single Quantum of Energy

- Quantum is a discrete packet of energy.
- One photon carries exactly one quantum of energy.

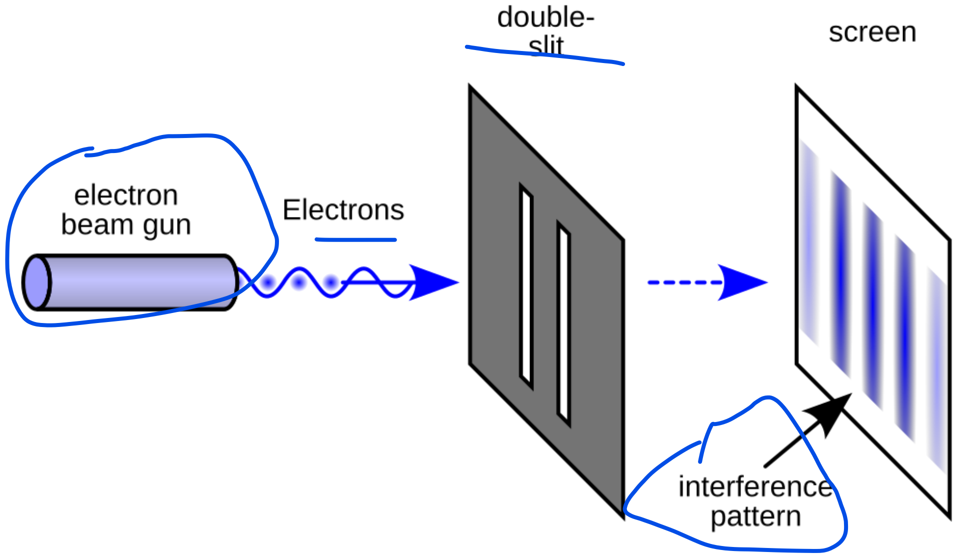


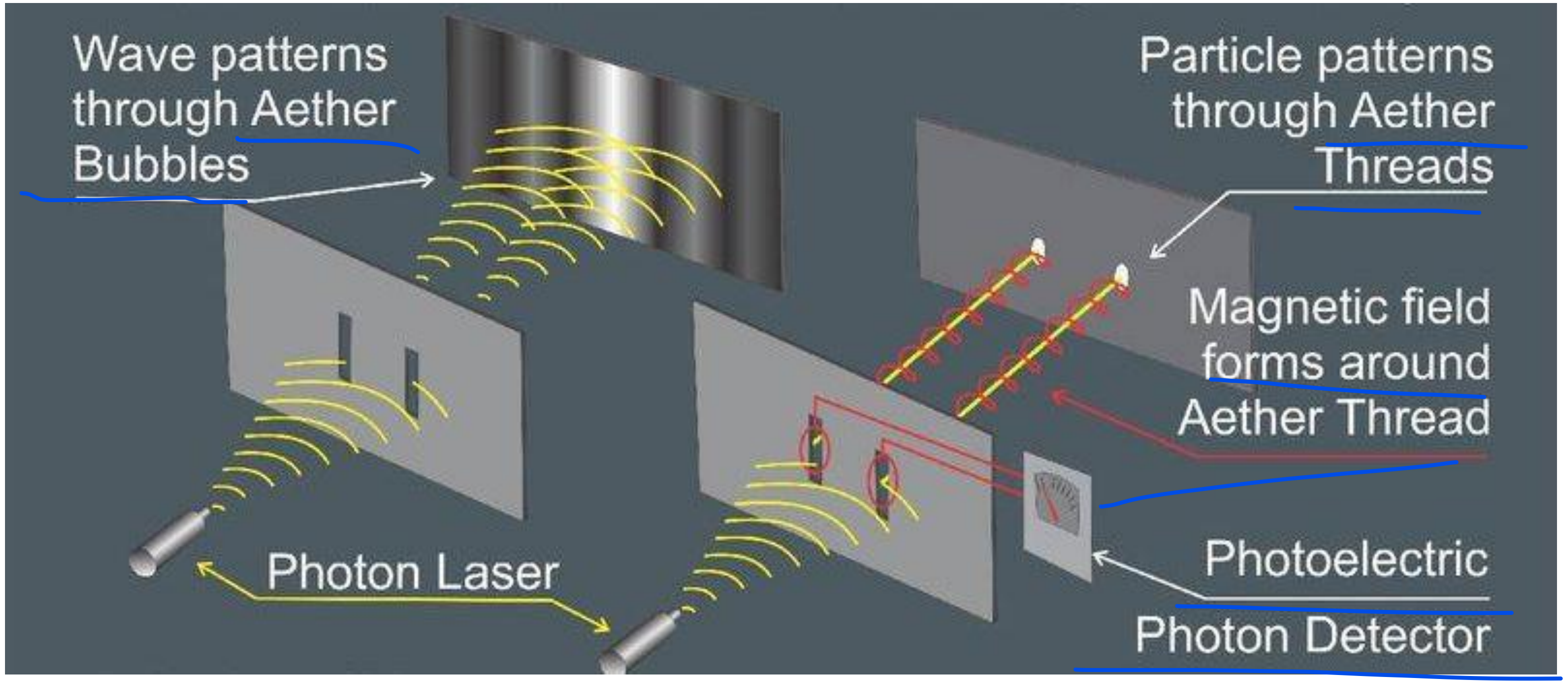


Double Slit

A single particle like an electron or photon can interfere with itself, producing a wave like interference pattern even when fired one at a time until observed.

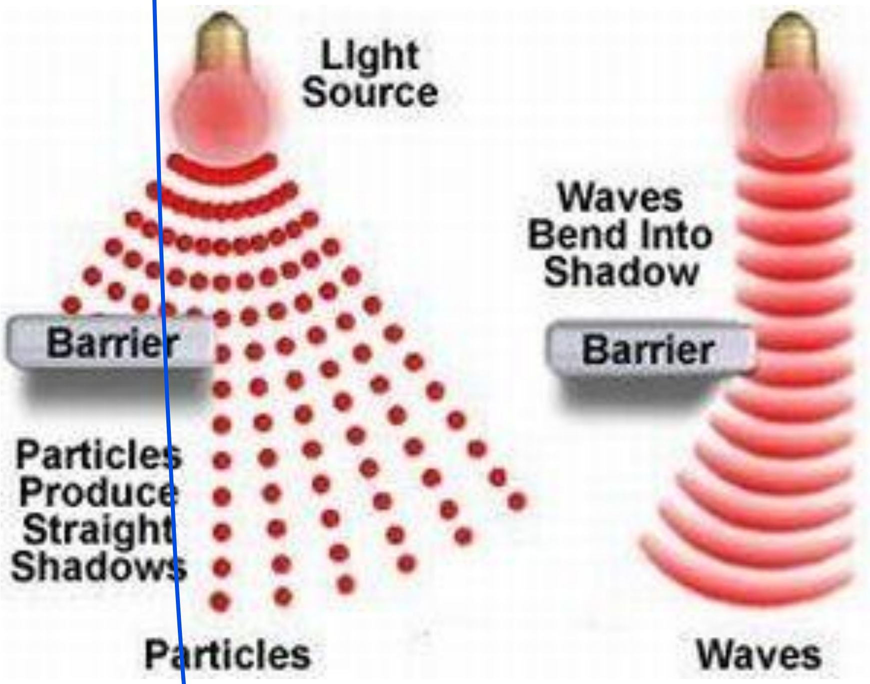
- Each particle went through both slits at once and interfered with itself.
- The particle exists in a state where it takes both paths simultaneously.
- If you try to measure which slit the particle goes through, the interference pattern disappears.
- Particle behaved like a classical particle, not a wave.
- Measurement collapses the quantum state.

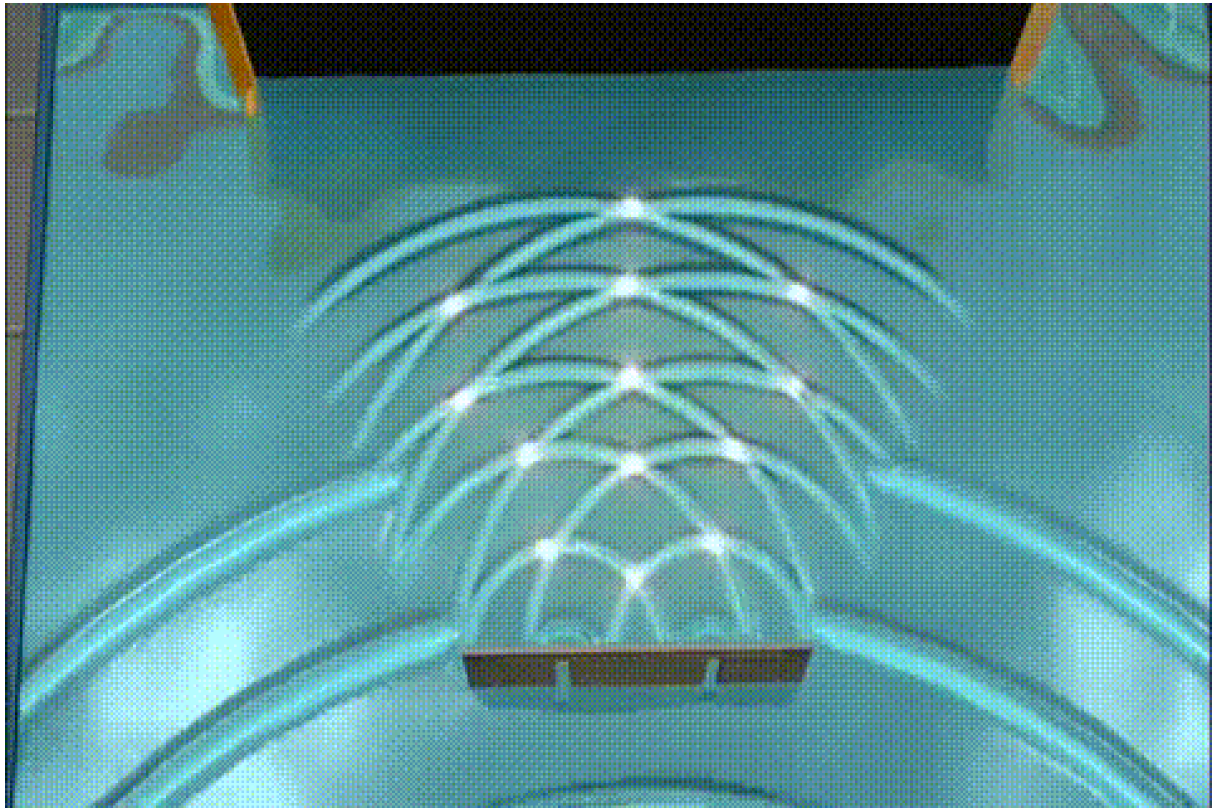




Wave Duality

The dual nature of light means that light can exist as both a particle and a wave.





Qubit

- It is a basic unit of quantum information.
- It can be in 0,1 or superposition of 0 and 1
- When they are measured they can be either 0 or 1
- There are 2^n possible states.

Quantum Bits (Qubits)	Equivelant Classical Bits
3	8
10	1024
20	1,048,576
...	...
300	2.037035976...E90

Notes

- Newtonian mechanics is deterministic; quantum computing relies on probabilistic quantum states.
- Newton's laws cannot describe superposition and entanglement, essential for qubits.

- Classical physics lacks tools for managing quantum decoherence and errors.
- Newtonian mechanics applies to macroscopic objects, not quantum-scale particles.

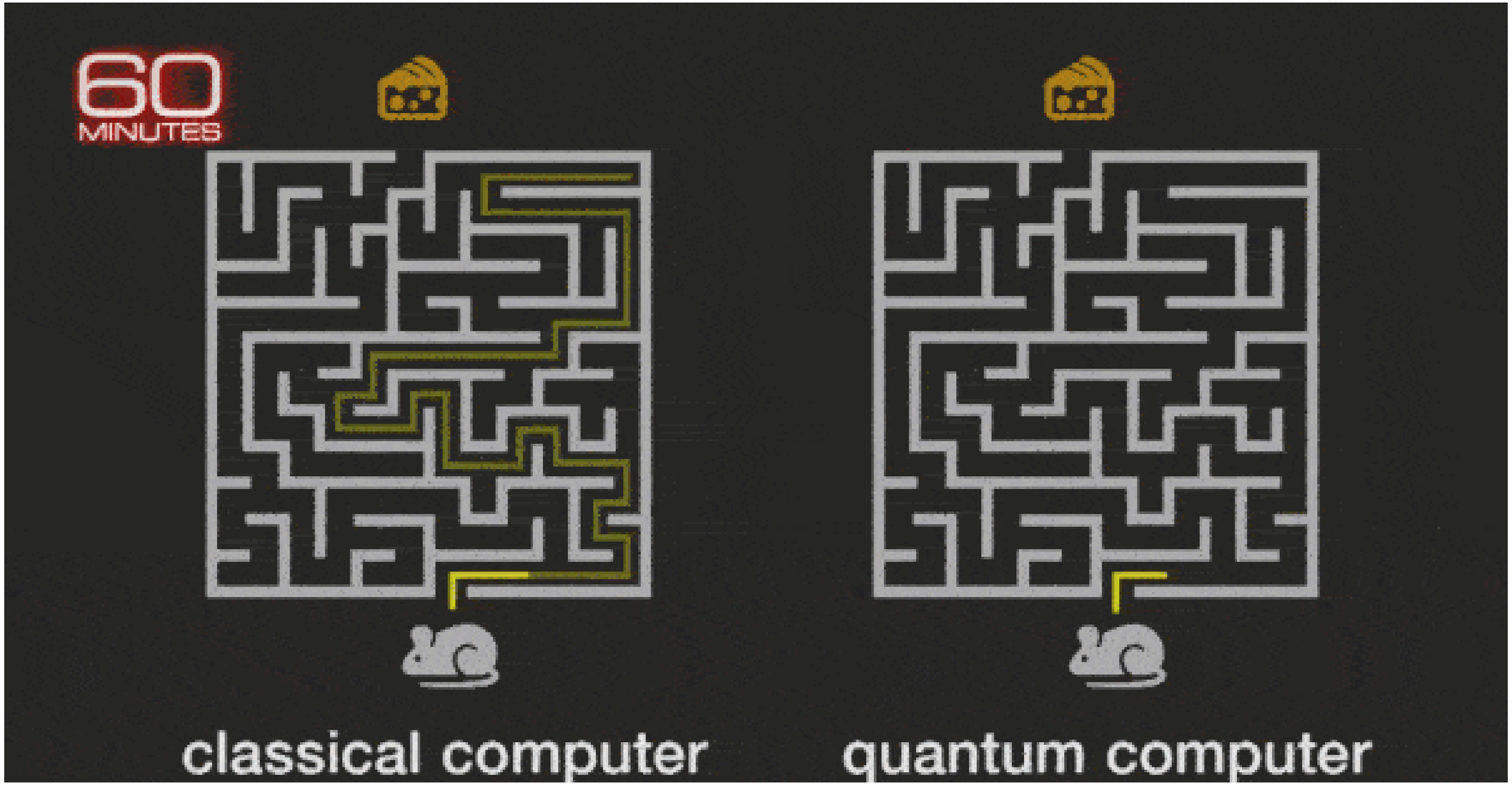
- Quantum computing requires quantum mechanics formalism, not classical laws.
- Quantum computing works with quantum particles (such as electrons, photons, or ions) as qubits, which harness quantum properties like superposition and entanglement.

- These quantum properties arise from the fundamental nature of particles at the atomic and subatomic level (atoms, electrons, quarks).
- Quantum processors manipulate the states of these particles to perform computations that are impossible for classical computers, taking advantage of the same behaviors that govern the building blocks of matter depicted in the image.

- Quantum computers operate at this scale, utilizing the quantum behaviors of particles such as electrons and ions—components represented in this diagram—to encode, process, and transfer information far beyond what's achievable with classical computers

Qubit vs Classical Bit

- Classical bit can do only single calculation at a time.
- Qubit can exist simultaneously in multiple states.
- Adv of qubit- parallel processing.



Classical vs Quantum Computing

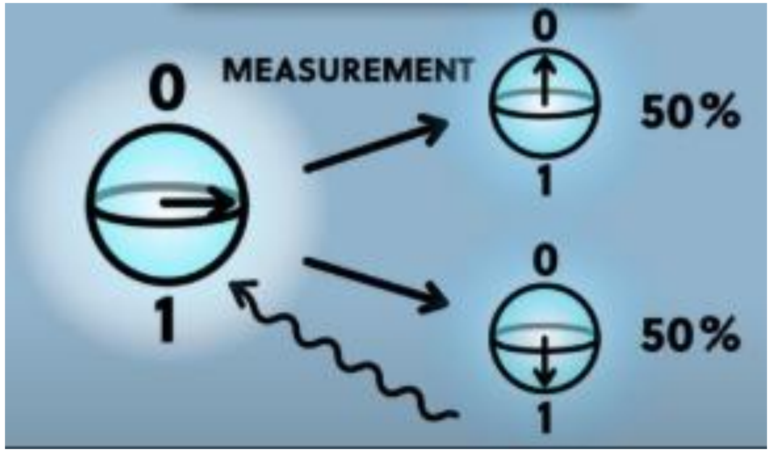
Feature	Classical Computing	Quantum Computing
Basic Unit	Bit → takes value 0 or 1	Qubit → can be in $\|0\rangle$, $\|1\rangle$, or superposition of both
Data Representation	Binary states stored in transistors	Quantum states represented as vectors in a complex Hilbert space
Logic / Operations	Boolean logic gates: AND, OR, NOT, NAND, etc.	Quantum gates : Pauli (X, Y, Z), Hadamard (H), Phase (S, T), CNOT, etc.
Parallelism	Limited → achieved by multi-core processors	True quantum parallelism via superposition and entanglement
Copying Information	Possible → data can be duplicated	Not possible due to the No-Cloning Theorem
Error Handling	Mature, robust error-correction codes	High error rates, requires quantum error-correction schemes
Measurement	Directly reads bit value (0 or 1) - Deterministic	Measurement collapses the quantum state to one of the basic states ($\|0\rangle$ or $\|1\rangle$) with probabilities determined by the squared amplitudes.
Performance Scaling	Scales linearly/polynomially with number of cores/resources	Potential exponential speed-up for certain problems (e.g., factoring, unstructured search)
Current State	Mature, widely used, low-cost	Emerging, experimental, requires cryogenic and error-tolerant systems

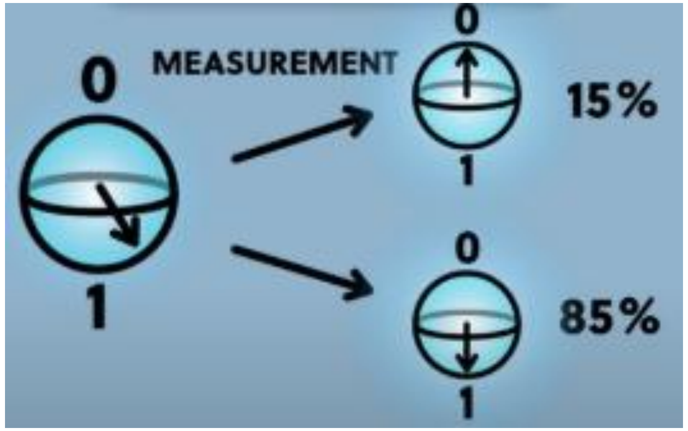
No cores in a processor – current state

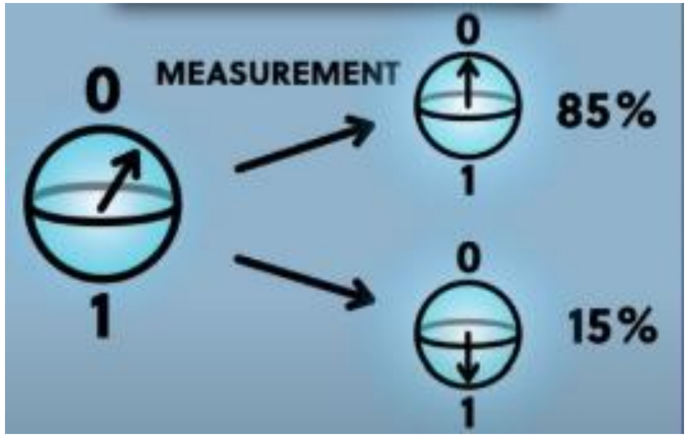
Processor / Family	Number of Cores (P-cores + E-cores) / Type
Intel "Arrow Lake" desktop CPUs (Core Ultra, Series 2)	**24 cores** = 8 Performance (P) + 16 Efficiency (E) cores
Intel "Lunar Lake" mobile CPUs (Core Ultra 200V series)	**8 cores** = 4 P-cores + 4 E-cores
Intel "Nova Lake" (rumored flagship)	Up to **52 cores** (16 P-cores + 32 E-cores + 4 Low-Power E-cores) in leaks
Intel Xeon / server line (Granite Rapids etc.)	Some models up to **288 cores** in E-core only configs (for server use)

GPU / Model	CUDA Cores / Shader Cores
NVIDIA RTX 5090	**21,760 CUDA cores** (NVIDIA)
NVIDIA RTX 4090	**16,384 CUDA cores** (TechRadar)
NVIDIA RTX 5080	**10,752 CUDA cores** (NVIDIA)
NVIDIA RTX 5070	**6,144 CUDA cores** (NVIDIA)

Superposition

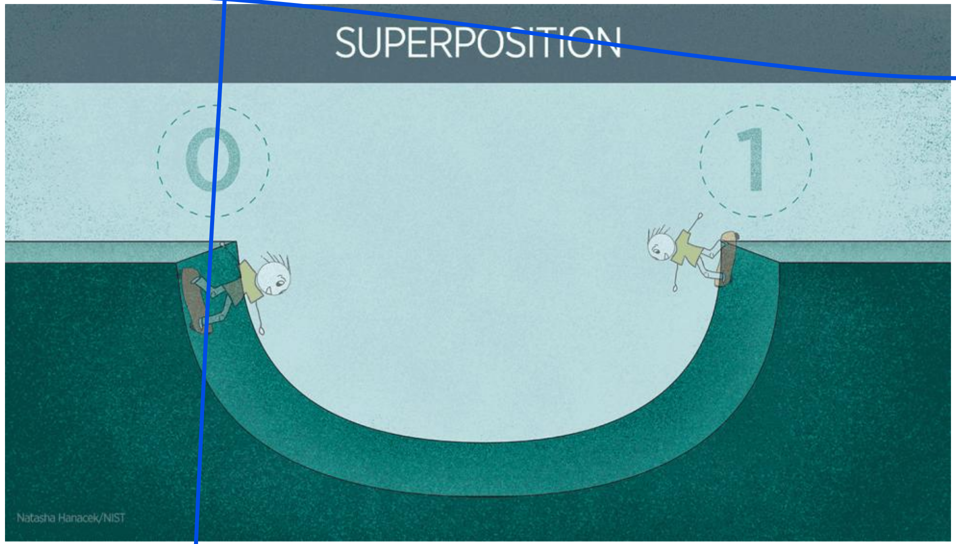






Superposition is a property of **individual qubits**, allowing them to be in multiple states at once.
A Qubit can exist as a 0, a 1. or simultaneously as both 0 and 1, with numerical coefficient representing probability for each state.

Superposition- Schrödinger's cat



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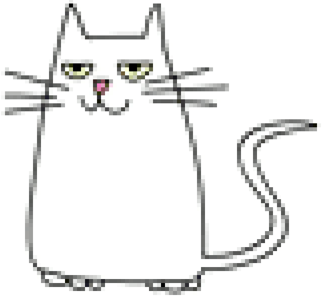
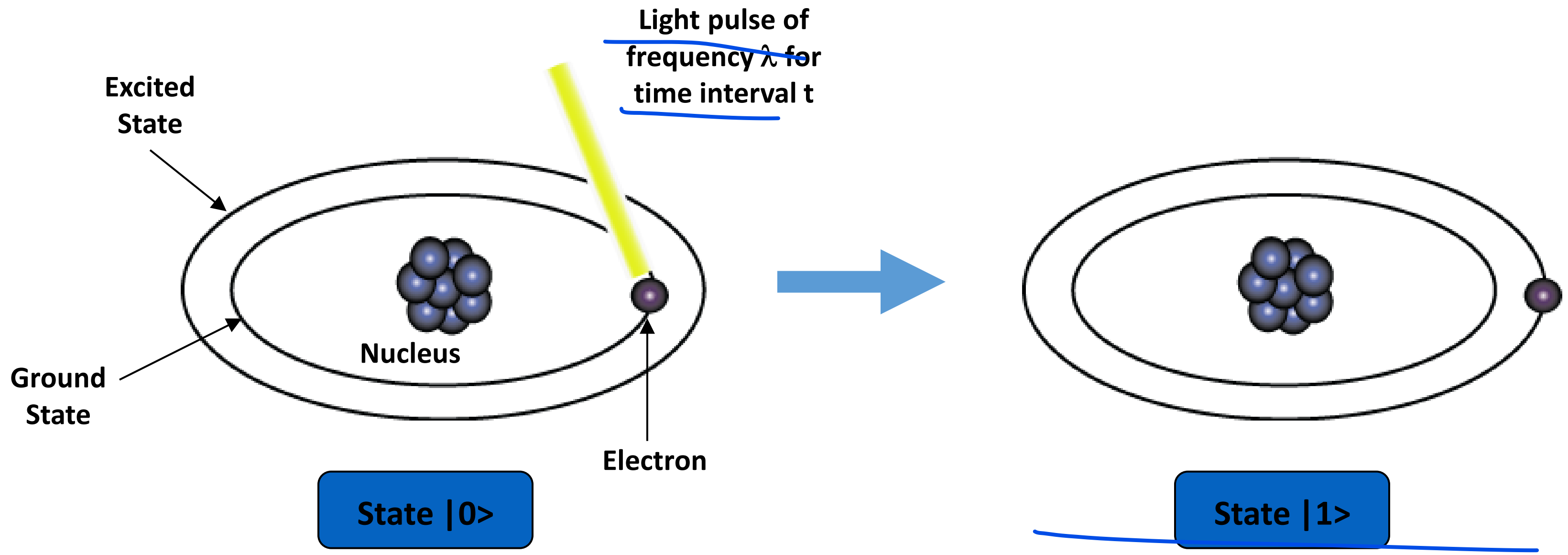


Representation of Data - Qubits

A bit of data is represented by a single atom that is in one of two states denoted by |0> and |1>. A single bit of this form is known as a ***qubit***

A physical implementation of a qubit could use the two energy levels of an atom. An excited state representing |1> and a ground state representing |0>.



Representation of Data - Superposition

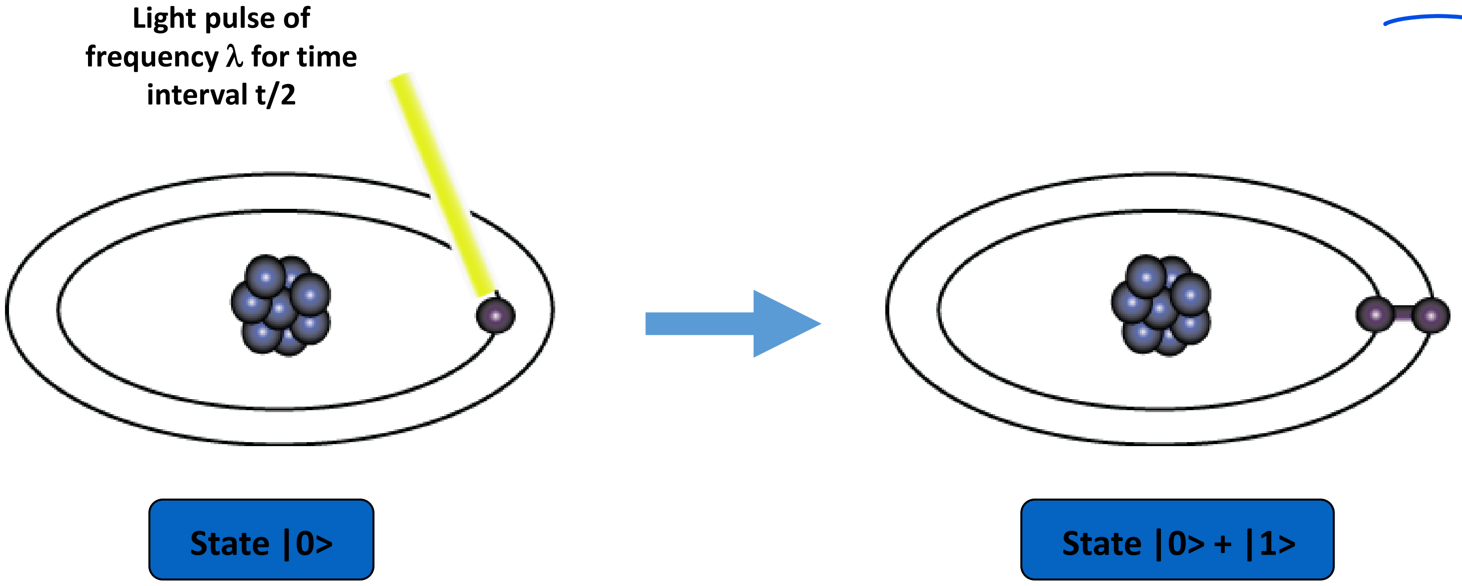
A single qubit can be forced into a ***superposition*** of the two states denoted by the addition of the state vectors:

$$|\psi\rangle = \alpha_1 |0\rangle + \alpha_2 |1\rangle$$

Where α_1 **and** α_2 are complex numbers and $|\alpha_1|^2 + |\alpha_2|^2 = 1$

A qubit in superposition is in both of the states |1> and |0 at the same time

Representation of Data - Superposition



- Consider a 3 bit qubit register. An equally weighted superposition of all possible states would be denoted by:

$$|\psi\rangle = \frac{1}{\sqrt{8}}|000\rangle + \frac{1}{\sqrt{8}}|001\rangle + \ldots + \frac{1}{\sqrt{8}}|111\rangle$$

Data Retrieval

- In general, an n qubit register can represent the numbers 0 through 2^n-1 ~~simultaneously.~~

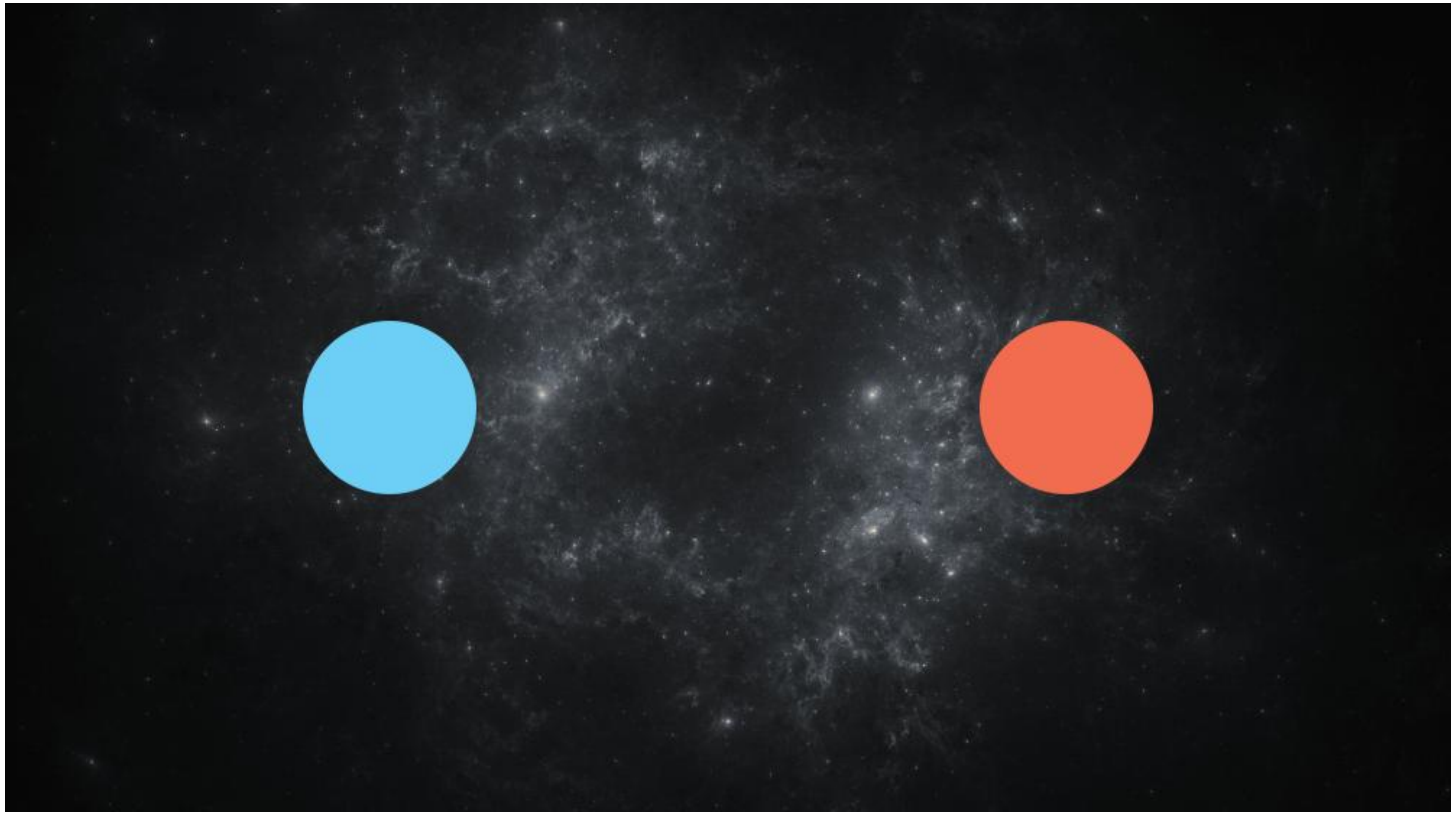
Sound too good to be true?…It is!

- If we attempt to retrieve the values represented within a superposition, the **superposition randomly collapses** to represent just one of the original values.

In our equation: $|\psi\rangle = \alpha_1 |0\rangle + \alpha_2 |1\rangle$, α_1 represents the probability of the superposition collapsing to $|0\rangle$. The α's are called probability amplitudes. In a balanced superposition, $\alpha = 1/\sqrt{2}$ where n is the number of qubits.

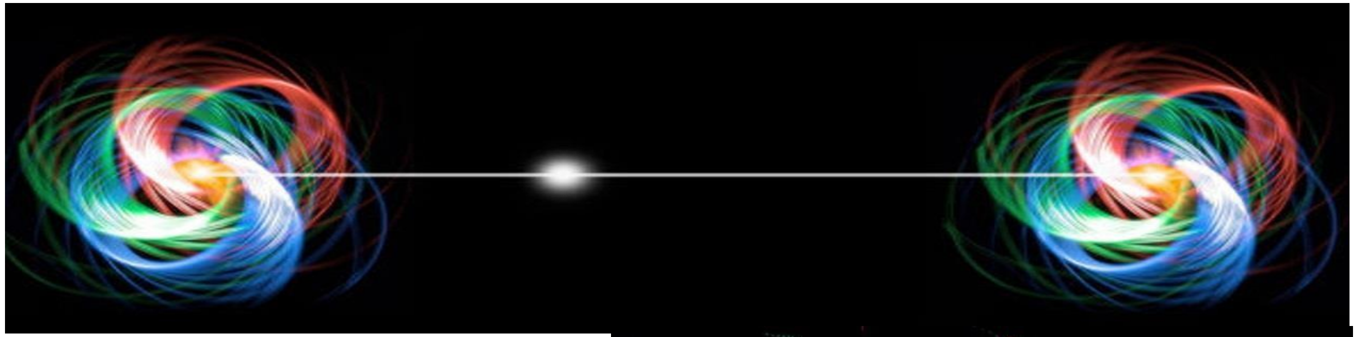
n

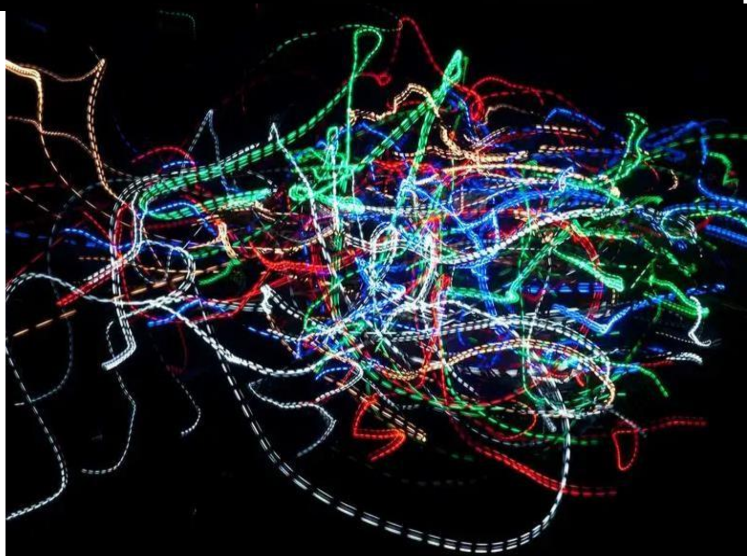
Quantum Entanglement



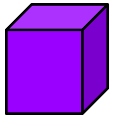
Quantum entanglement is when two or more particles become linked so that the state of one instantly affects the state of the other, no matter how far apart they are.

Entanglement

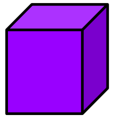




Suppose that two qubits are in states:



$\alpha|0\rangle + \beta|1\rangle$



$\alpha'|0\rangle + \beta'|1\rangle$

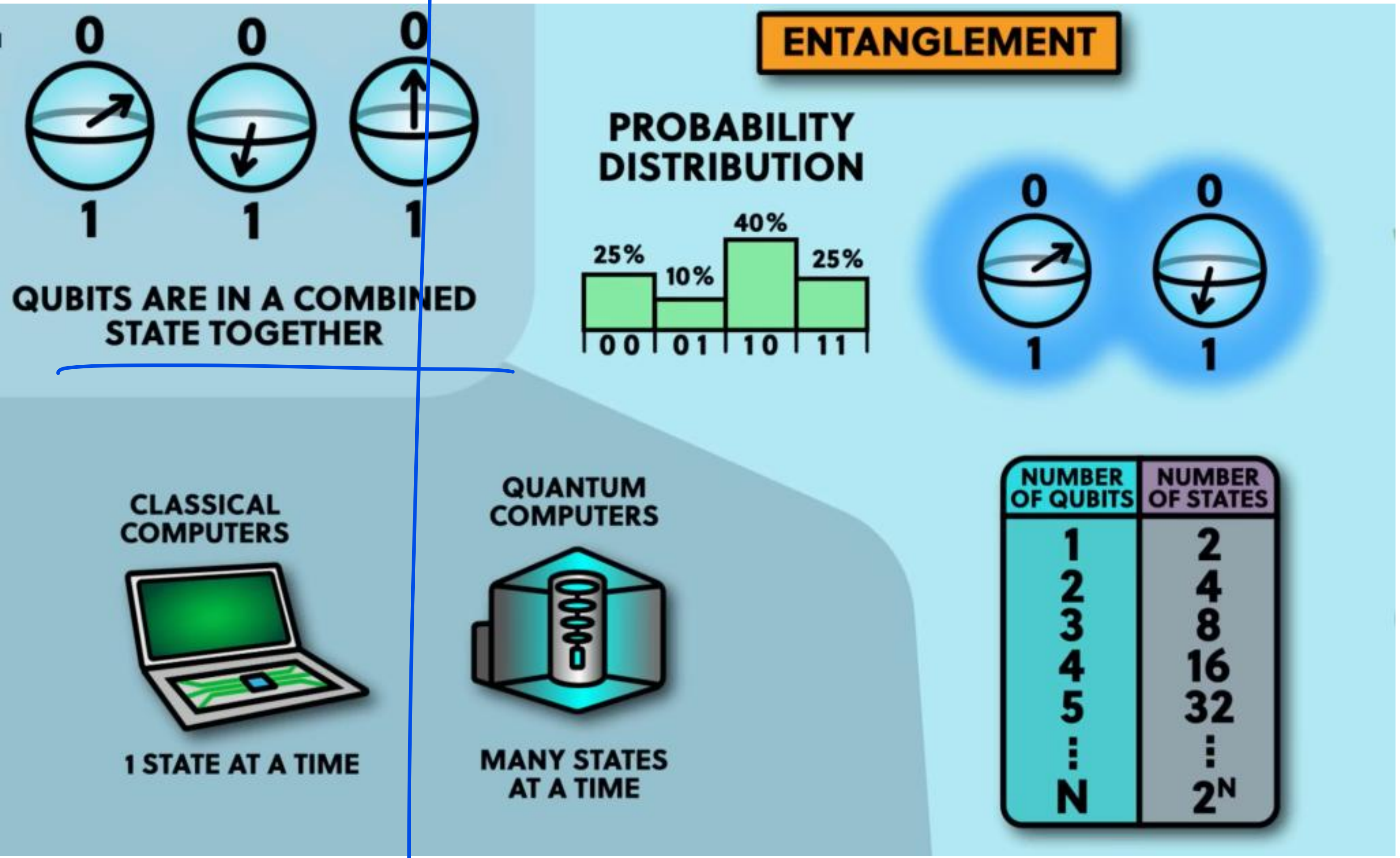
The state of the combined system is their ***tensor product***:

$$(\alpha|0\rangle + \beta|1\rangle)(\alpha'|0\rangle + \beta'|1\rangle) = \alpha\alpha'|00\rangle + \alpha\beta'|01\rangle + \beta\alpha'|10\rangle + \beta\beta'|11\rangle$$

Question: what are the states of the individual qubits for

1. $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$ an ***independent*** state
2. $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$ an ***entangled*** state

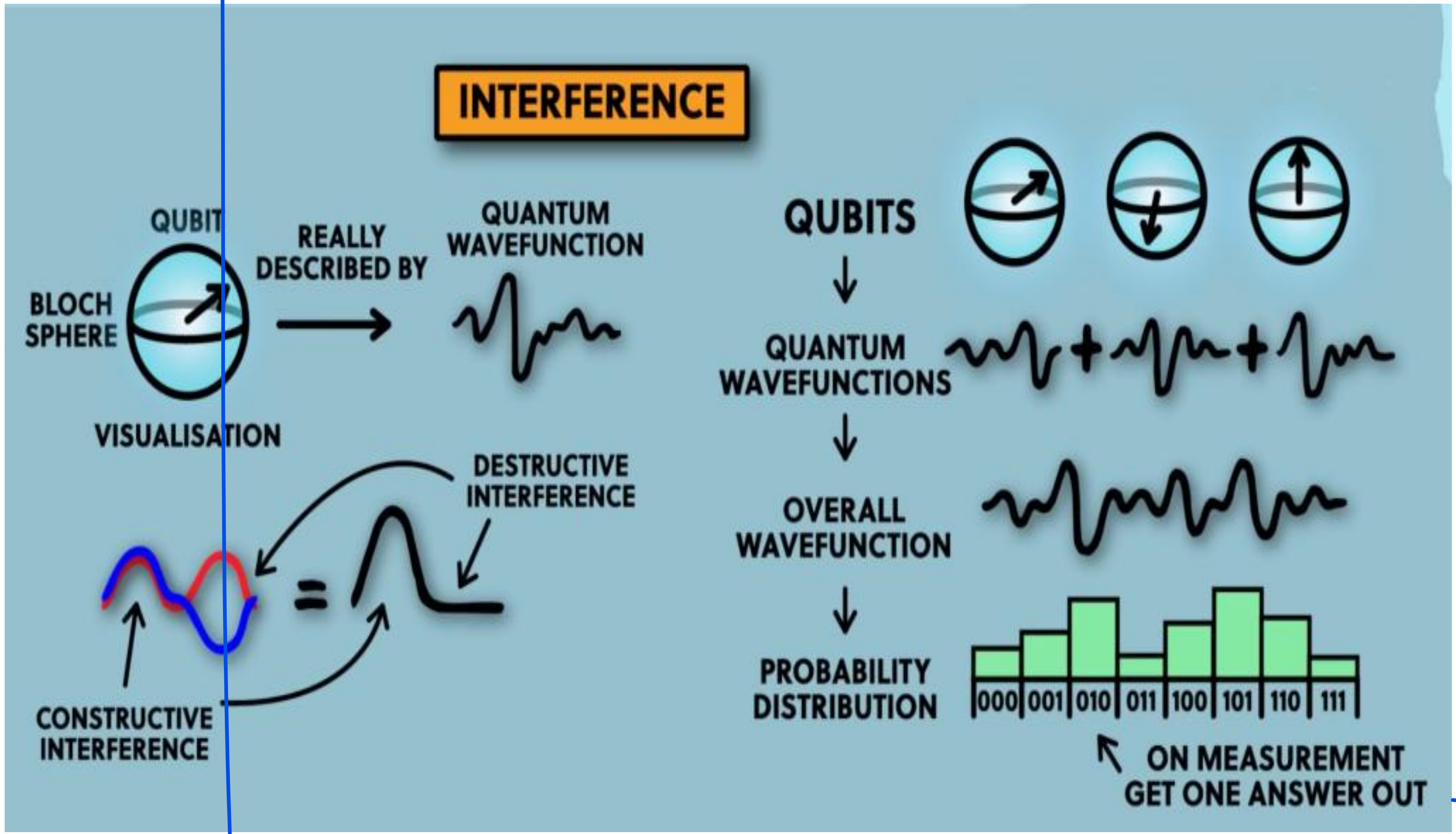
Entanglement



Feature	Description
Non-locality	Measuring one particle's state instantly determines the state of its entangled partner.
No Classical Analog	Entanglement is purely quantum, it cannot be explained by classical physics.
Shared State	The entire system is described by a single wavefunction, the subsystems don't have independent states.
Instant Correlation	Entangled particles always show correlated outcomes, even when separated by large distances.
Bell States	Quantum entanglement violates classical assumptions like locality and realism

Quantum entanglement is when two or more particles become linked so that the state of one instantly affects the state of the other, no matter how far apart they are.

Interference



Feature	Description
Superposition	Interference happens only when particles are in superposition states.
Probabilistic	It affects **probability amplitudes**, not final values directly.
Constructive	Amplitudes reinforce , increases likelihood of a result.
Destructive	Amplitudes cancel, suppresses certain outcomes.

Quantum interference is a fundamental phenomenon in quantum mechanics where **probability amplitudes, add or cancel** each other out, affecting the outcome of quantum measurements.

Quantum overview

Field	Description	Examples / Tools
Quantum Hardware / Devices	Builds physical quantum computers using various technologies	Superconducting qubits, trapped ions, photonics
Quantum Programming & Software	Develops tools and languages for programming quantum systems	Qiskit (IBM), Cirq (Google), Q# (Microsoft)
Quantum Algorithms	Designs efficient QC algorithms	Shor's algorithm, Grover's algorithm
Quantum Information Theory	Processing and transfer of quantum information	Qubits, entropy, entanglement, error correction
Quantum Error Correction	Protects quantum data from noise and decoherence	Shor code, surface code, stabilizer codes
Quantum Communication	Enables secure quantum data transmission	Quantum teleportation, QKD
Quantum Cryptography	Uses quantum Principles for Secure encryptions	BB84 protocol, post quantum cryptography
Quantum Simulation	Simulates quantum systems difficult for classical computers	Molecular simulation, quantum chemistry
Quantum Machine Learning	Merges quantum computing with ML techniques	Variational classifiers, quantum neural networks
Quantum Control & Calibration	Optimizes gate fidelity and operation in real devices	Gate error suppression
Quantum Physics	Provides mathematical foundation for quantum computing	Decoherence theory, entanglement entropy

Quantum Algorithms

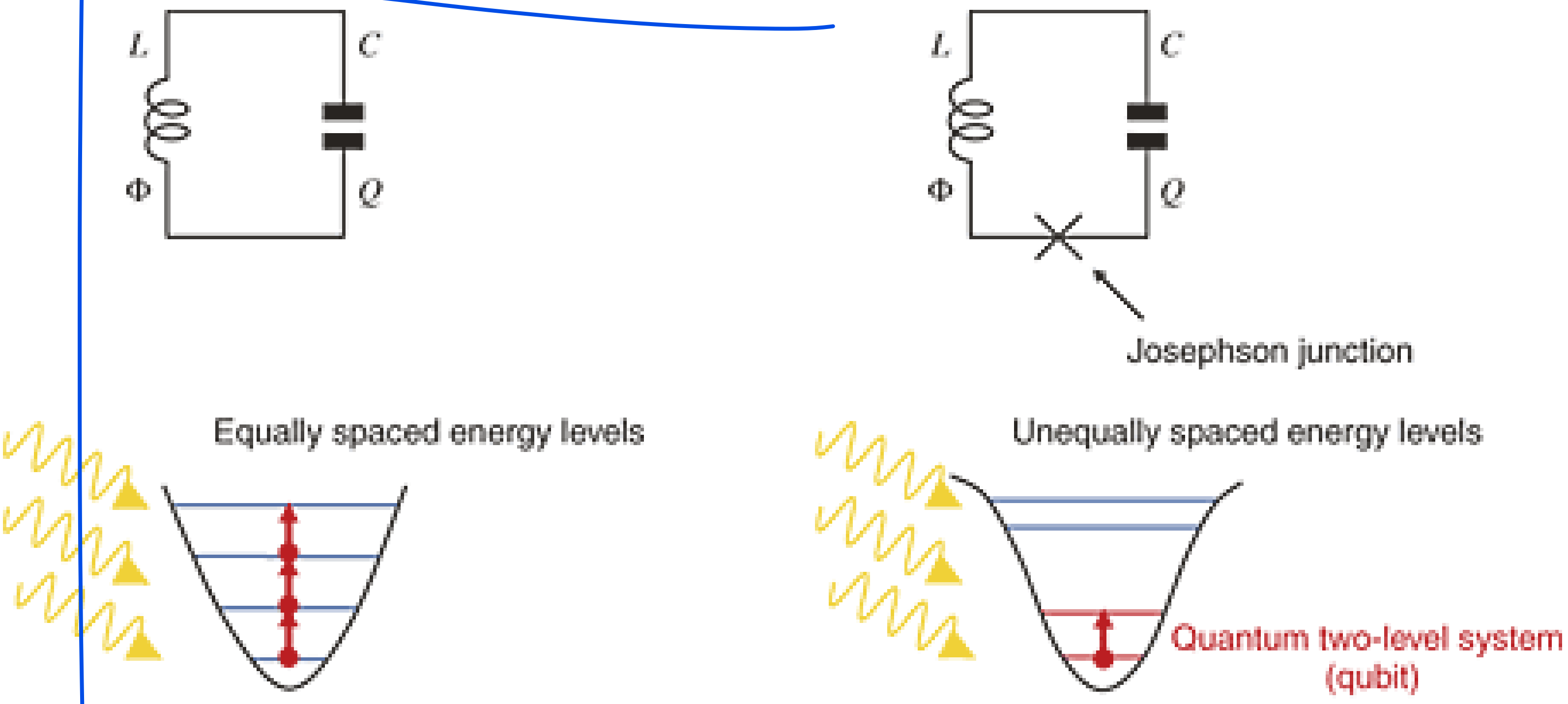
Quantum Algorithm	Purpose	Example
Shor's Algorithm	Integer factorization, exponential speedup over classical	Breaking RSA: factoring 15 → 3 × 5
Grover's Algorithm	Unstructured search, quadratic speedup	Finding a name in an unsorted phonebook - O(√n)
Quantum Fourier Transform (QFT)	Frequency analysis, used in many quantum algorithms	Shor's algorithm, Quantum Phase Estimation
Quantum Phase Estimation (QPE)	Estimate eigenvalues of a unitary operator	Finding energy levels of quantum systems
Variational Quantum Eigensolver (VQE)	Hybrid approach for finding ground-state energy of molecules	Estimating H_2 molecule energy
Deutsch-Jozsa Algorithm	Determines if a function is constant or balanced with a single query	Proving exponential advantage over classical
Simon's Algorithm	Solves a specific hidden period problem exponentially faster than classical	Foundation for Shor's algorithm
Amplitude Estimation	Estimates the probability of measuring a certain quantum state more efficiently	Used in finance, Monte Carlo simulations
Quantum Walk Algorithms	Solving problems via quantum analog of random walks	Element distinctness, graph traversal
Harrow-Hassidim-Lloyd (HHL)	Solving systems of linear equations exponentially faster than classical methods	Solving Ax = b in quantum machine learning

Quantum Hardware

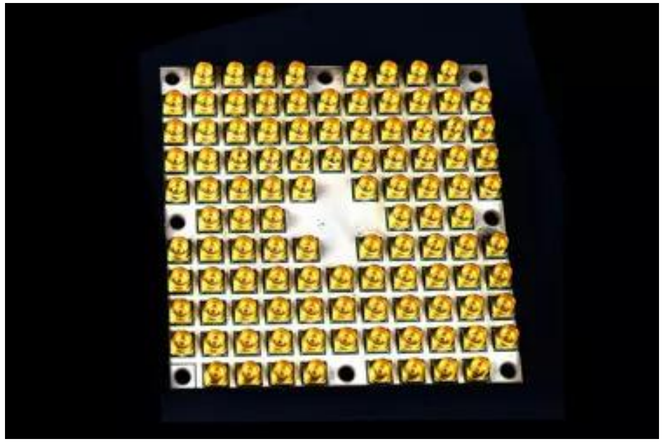
Feature	Superconducting Qubits	Trapped Ions	Photonic Qubits
Qubit Type	Electrical circuits (Josephson junctions)	Individual ions suspended in traps	Particles of light (photons)
Operating Temp	~15 millikelvin (ultra-cold)	Room temp (with vacuum)	Room temperature
Gate Speed	Very fast (nanoseconds)	Slower (microseconds)	Very fast (picoseconds), but gates are probabilistic
Gate Fidelity	High (improving rapidly)	Extremely high	Lower (currently improving)
Scalability	Good (2D chip integration)	Medium (scaling traps is complex)	Promising (optical chips, multiplexing)
Best Use Case	General-purpose quantum computing	High-precision quantum logic	Quantum communication, early computation
Major Companies	IBM, Google, Intel	IonQ, Quantinuum	Xanadu, PsiQuantum, ORCA Computing
Challenges	Requires dilution refrigerators	Complex laser systems, bulky setup	Photon loss, hard to store/interact photons
Maturity Level	Most mature and commercially deployed	Mature, especially for small systems	Emerging and rapidly evolving

Superconducting Qubit Generation

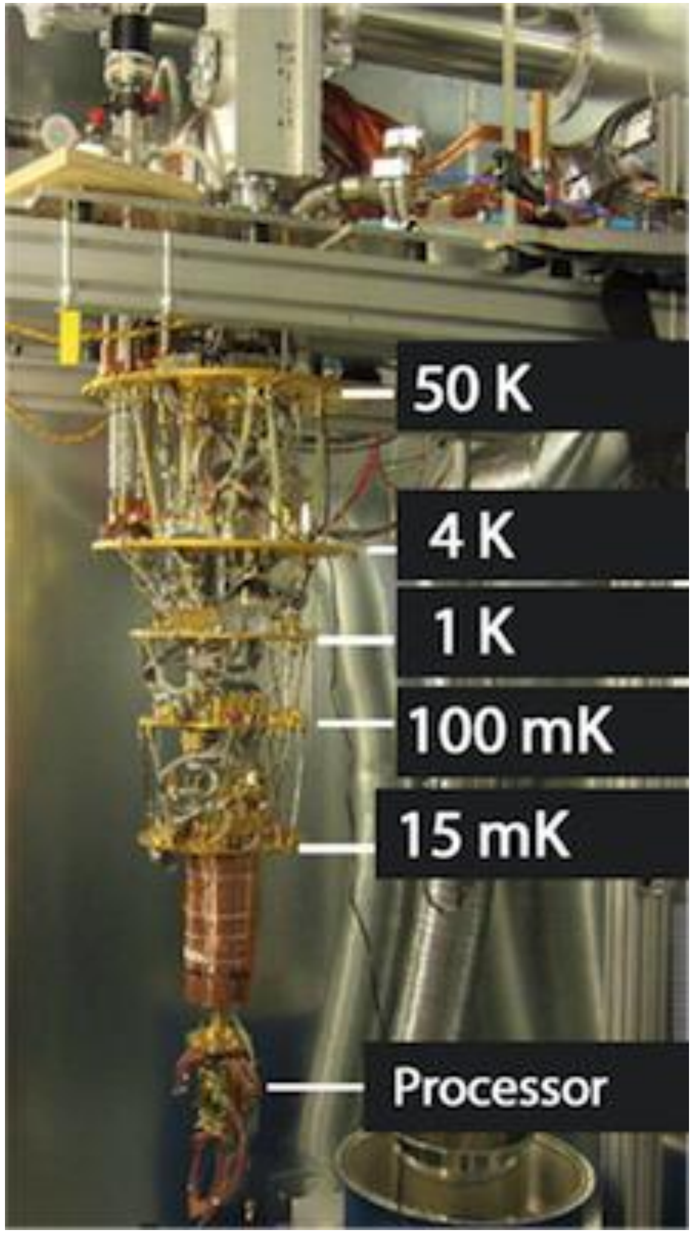
- Natural qubits are isotopes like ^{13}C and ^{29}Si
- "Non linear LC resonator" oscillation circuit forms man made qubit.
- A Josephson Junction(X) is used to restrict the dynamics to two levels.

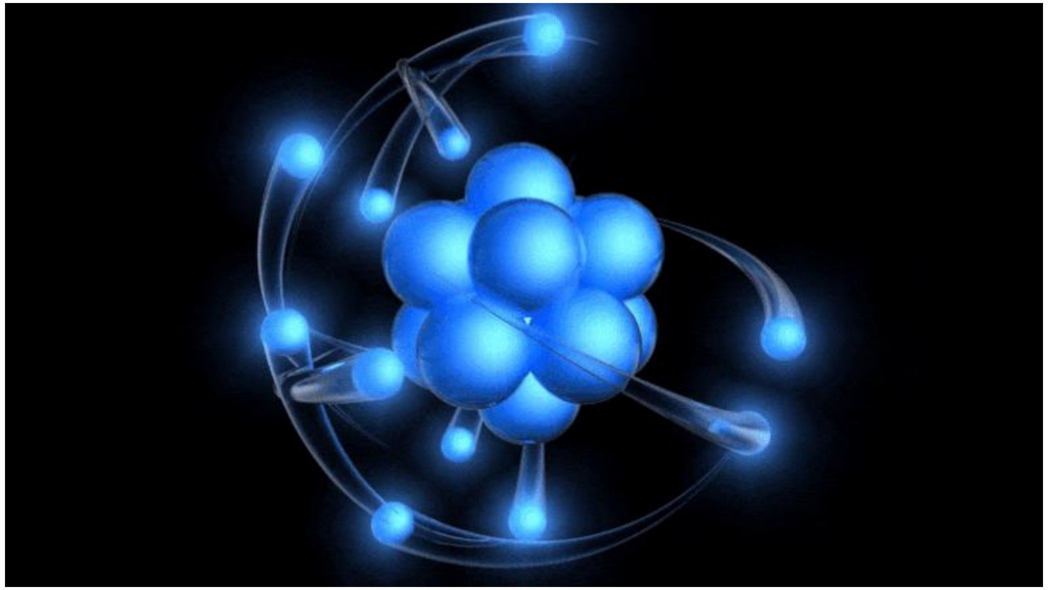


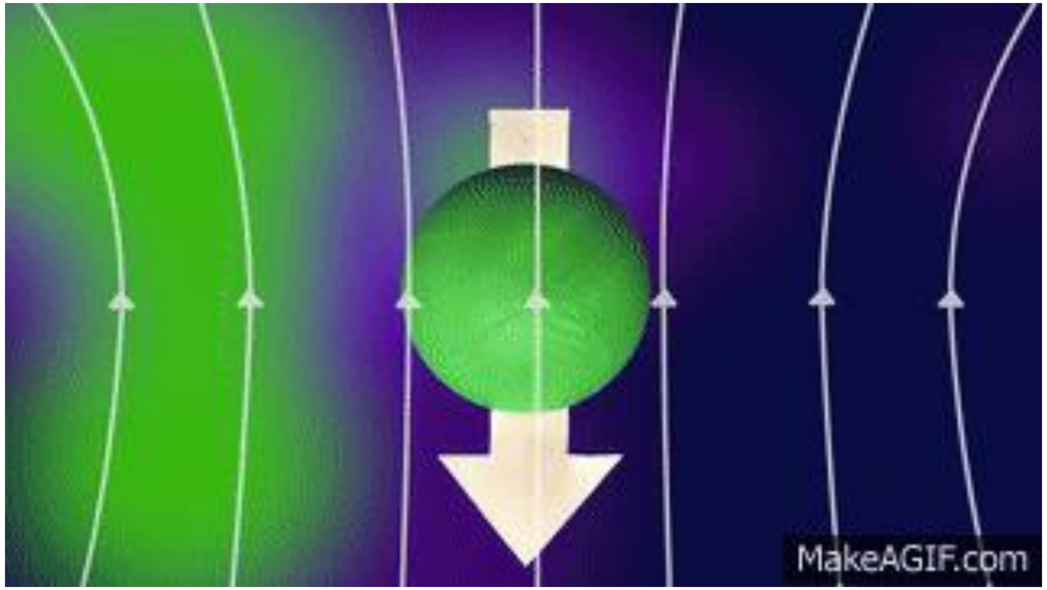
(a) LC-circuit without Josephson junction
(b) LC-circuit with Josephson junction



Creating a Quantum Qubit







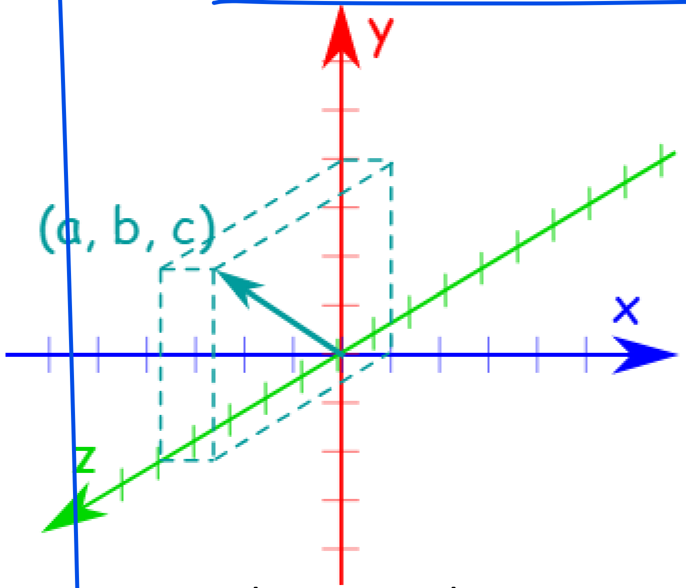
Quantum Errors

Quantum Error	Description
Decoherence	Loss of quantum behavior (like superposition or entanglement) due to interaction with the environment.
Dephasing (Phase Damping)	Loss of phase information between quantum states, destroys interference but not energy levels.
Amplitude Damping	Energy loss from excited state
Bit-flip Error	A qubit flips from $\|0\rangle \leftrightarrow \|1\rangle$
Phase-flip Error	Flips the phase of a qubit, changing superpositions without affecting probabilities.
Measurement Error	Wrong result is read out due to noise or imperfections in the measurement process.
Gate Error	Imperfect quantum gate application due to control inaccuracies or hardware limitations.

Dirac Notation (Bra-Ket)

Bra-ket notation < | > is a way of writing special vectors in quantum computing.
Example: <bra|ket> , <a|b>

Here is a vector in 3 dimensions:



We can write this as a column vector like this:

$$r = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Or we can write it as a "ket":

$$|r\rangle = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Bra-Ket

kets are special: $|\psi\rangle$

- The values (a, b and c above) are complex numbers (they may be real numbers, imaginary numbers or a combination of both)
- A ket is a quantum state
- Kets can have any number of dimensions, including infinite dimensions!

The "bra" $\langle\psi|$, is similar, but the values are in a row, and each element is the complex conjugate of the ket's elements.

Example: This ket:

$$|a\rangle = \begin{bmatrix} 2-3i \\ 6+4i \\ 3-i \end{bmatrix}$$

Has this bra:

$$\langle a| = \begin{bmatrix} 2+3i & 6-4i & 3+i \end{bmatrix}$$

The values are now in a row, and we also **changed the sign** (+ to −, and − to +) in the middle of each element.

In "matrix language", changing a ket into a bra (or bra into a ket) is a "conjugate transpose":

- conjugate: 2−3i becomes 2+3i, etc...
- transpose: rows swap with columns

Bra-Ket Vector Notation

- Ket Vector |ψ⟩
  - column vector, representing a quantum state
  - $$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}, |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$
- Bra Vector ⟨ψ|
  - Complex conjugate transpose of Ket, A row Vector
  - <0| = [1 0], <+| = $\frac{1}{\sqrt{2}}$ [1 1]

  $$\langle\psi| = (|\psi\rangle)^\dagger$$

  e.g., If $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, then $\langle\psi| = \alpha^*\langle 0| + \beta^*\langle 1|$
- Inner Product ⟨ϕ|ψ⟩: Scalar value, scalar overlap
- Outer Product |ψ⟩⟨ϕ|: A matrix , An operator, A Gate
- Tensor product <ϕ| $\otimes$ |ψ⟩: Tensor product of <ϕ| and |ψ⟩

Hilbert Space

- Hilbert Space is a complex vector space with an inner product
  - Finite dimensional inner product space
- ⟨ϕ|ψ⟩ = c, where c is a complex vector
- The term ⟨ϕ|ψ⟩ is called a bracket

- In quantum computing
- Inner Product ⟨ϕ|ψ⟩: Scalar value, scalar overlap

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = x_1y_1 + x_2y_2 + x_3y_3 \,.$$

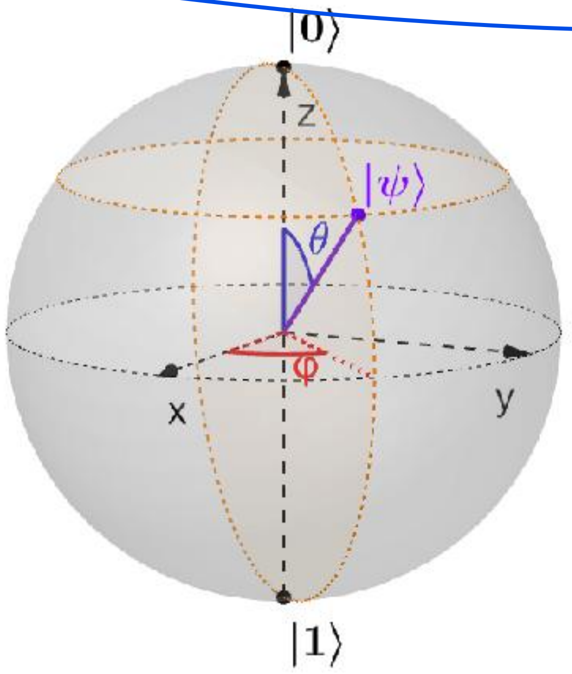
Quantum Qubit State Representation

- Qubit needs two more variable to denote its value based on its position in the Bloch sphere.

- Bra-Ket notation < | > is used to describe the value of a qubit as a vector in sphere.

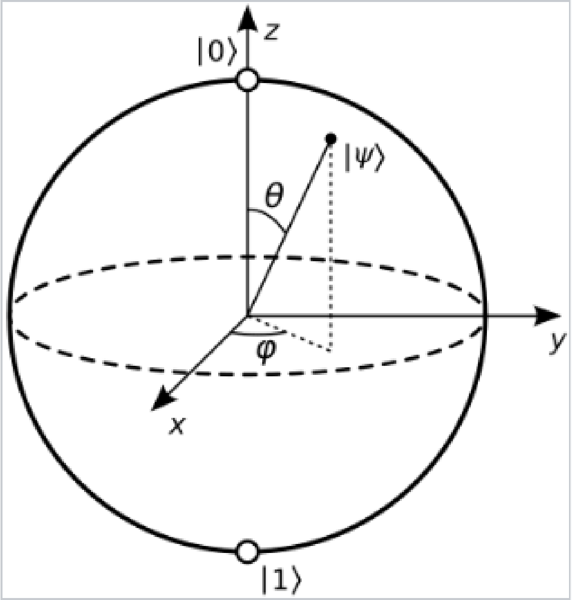
$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle =$$

$$\alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$



What is the Bloch Sphere?

- Motivation
  - How do we visualize a qubit
    - $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
    - $\alpha = a + ib$, $\beta = c + id$
  - The Fundamental Constraint
    - $|\alpha|^2 + |\beta|^2 = 1$



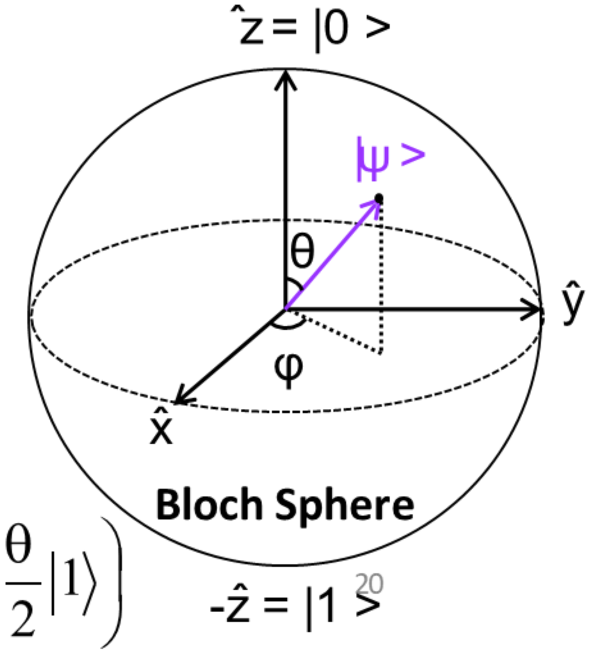
- The Bloch sphere is a **geometric representation** of a single qubit
- Every point on the surface represents a unique pure qubit state
  - Impure states can lie within the sphere (!!!)
- Any pure qubit state lies on the surface of the sphere
- Useful for visualizing superposition & quantum gates

- Since quantum systems evolve according to linear equations (the Schrödinger equation), linear combinations of solutions are also solutions. So, for the state of a qubit |0> and |1>, its superposition also describes the same state
- The general form of a qubit state can be represented by:

$$\alpha_0|0\rangle + \alpha_1|1\rangle$$

where α_0 and α_1 are complex numbers that specify the *probability amplitudes* of the corresponding states.

- $|\alpha\ |^2$ gives the probability that you will find the qubit in the “off” (0) state; $|\alpha\ |$ gives the probability that you will find the qubit in the “on” (1) state.
- Normalization condition: $|\alpha_0|^2 + |\alpha_1|^2 = 1$



$$|\Psi\rangle = e^{i\gamma}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right)$$

Why Bloch Sphere is Important

- Intuitive visualization tool for single-qubit states
- Helps understand:
    - Superposition
    - Phase
    - Effect of gates
    - Foundation for multi-qubit visualization & algorithms

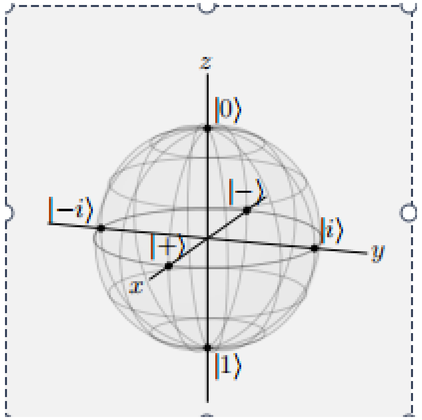
Some superposition names

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle),$$

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle),$$

$$|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle).$$



Classical Bit vs. Quantum Bit

CLASSICAL BITS:

- can be in two distinct states, 0 and 1
- can be measured completely
- are not changed by measurement
- can be copied
- can be erased

QUANTUM BITS:

- can be in state |0> or in state |1> or in any other state that is a linear combination of the two states
- can be measured partially with given probability
- are changed by measurement
- cannot be copied
- cannot be erased

Matrix – Tensor Product

$$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \\ ay \\ bx \\ by \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \otimes \begin{pmatrix} 2 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 7 \\ 6 \\ 15 \\ 21 \end{pmatrix}$$

Identity Matrix

Identity Matrix (I)

$XI = X$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}$$

$A^{-1} = 1/A$

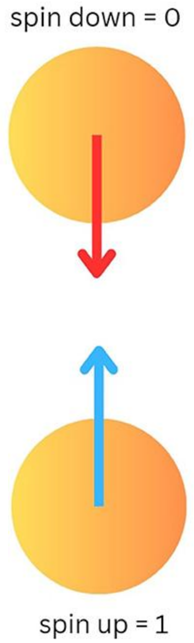
$AA^{-1} = I$ or $A^{-1}A = I$

Bra-ket Notation

Bra-ket $\langle A \mid E \rangle$

$|1\rangle$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ spin down

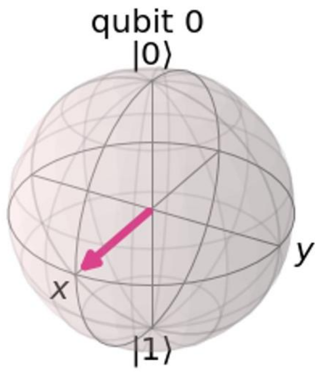
$|0\rangle$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ spin up



Bra-ket Notation in Superposition

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} (|0> + |1>)$$

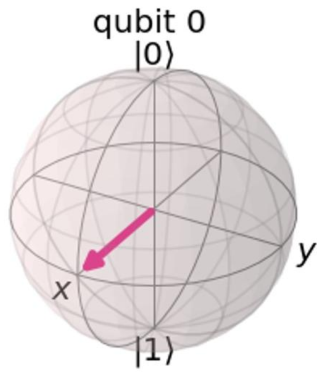


Total probability of a qubit in superposition must always equal 1

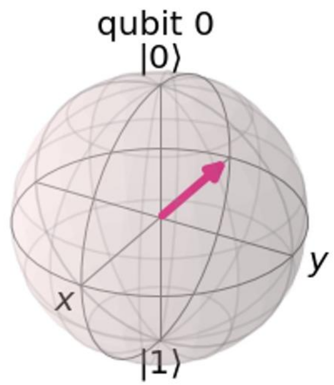
Bra-ket Notation in Superposition-Entanglement

$\begin{pmatrix} x \\ y \end{pmatrix}$ $|x|^2 + |y|^2 = 1$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $|1>$ $0^2 + 1^2 = 1$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|0>$ $1^2 + 0^2 = 1$

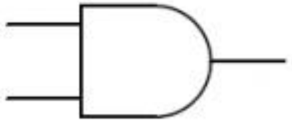
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ $\frac{1}{\sqrt{2}}(|0> + |1>) = |+>$ $(\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 = 1$



$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$ $\frac{1}{\sqrt{2}}(|0> - |1>) = |->$ $(\frac{1}{\sqrt{2}})^2 + (\frac{-1}{\sqrt{2}})^2 = 1$

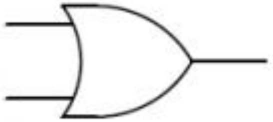


Classical Gates



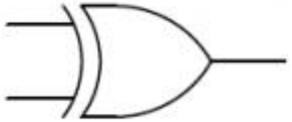
AND

A	B	Output
0	0	0
0	1	0
1	0	0
1	1	1



OR

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1

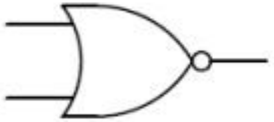


XOR

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0

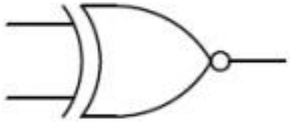
NAND

A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0



NOR

A	B	Output
0	0	1
0	1	0
1	0	0
1	1	0



XNOR

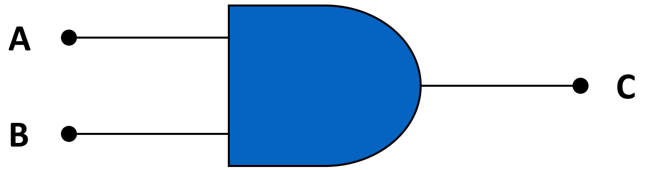
A	B	Output
0	0	1
0	1	0
1	0	0
1	1	1

Operations on Qubits - Reversible Logic

- Due to the nature of quantum physics, the destruction of information in a gate will cause heat to be evolved which can destroy the superposition of qubits.

Ex.

The AND Gate



Input		Output
A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

In these 3 cases, information is being destroyed

- This type of gate cannot be used. We must use ***Quantum Gates***.

Quantum Gates

- Quantum gates act on Qubits
- Quantum gates transform the state of the Qubit into other states.
- Quantum gates are linear maps that keep the total probability equal to 1
- Quantum Gates are similar to classical gates, but do not have a degenerate output. i.e. their original input state can be derived from their output state, uniquely. ***They must be reversible.***
- This means that a deterministic computation can be performed on a quantum computer only if it is reversible. Luckily, it has been shown that any deterministic computation can be made reversible.(Charles Bennet, 1973)
- Classical reversible logic gates are valid quantum gates.
- In contrast, irreversible gates are not valid quantum gates.

Quantum Gates are Irreversible

- Consider a quantum gate that performs the following map

$$U|0\rangle = \frac{\sqrt{2}-i}{2}|0\rangle - \frac{1}{2}|1\rangle,$$
$$U|1\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{2}+i}{2}|1\rangle.$$

A quantum gate must be *linear*, meaning we can distribute it across superpositions:

$$\begin{aligned} U(\alpha|0\rangle+\beta|1\rangle) &= \alpha U|0\rangle + \beta U|1\rangle \\ &= \alpha\left(\frac{\sqrt{2}-i}{2}|0\rangle - \frac{1}{2}|1\rangle\right) + \beta\left(\frac{1}{2}|0\rangle + \frac{\sqrt{2}+i}{2}|1\rangle\right) \\ &= \left(\alpha\frac{\sqrt{2}-i}{2} + \beta\frac{1}{2}\right)|0\rangle + \left(-\alpha\frac{1}{2} + \beta\frac{\sqrt{2}+i}{2}\right)|1\rangle. \end{aligned}$$

For this to be a valid quantum gate, the total probability must remain 1. Assuming the original state was normalized, i.e., $|\alpha|^2 + |\beta|^2 = 1$, we can calculate the total probability by summing the norm-square of each amplitude to see if it is still 1:

$$
\begin{aligned}
&\left|\alpha\frac{\sqrt{2}-i}{2} + \beta\frac{1}{2}\right|^2 + \left|-\alpha\frac{1}{2} + \beta\frac{\sqrt{2}+i}{2}\right|^2 \\
&= \left(\alpha\frac{\sqrt{2}-i}{2} + \beta\frac{1}{2}\right)\left(\alpha^*\frac{\sqrt{2}+i}{2} + \beta^*\frac{1}{2}\right) \\
&\quad + \left(-\alpha\frac{1}{2} + \beta\frac{\sqrt{2}+i}{2}\right)\left(-\alpha^*\frac{1}{2} + \beta^*\frac{\sqrt{2}-i}{2}\right) \\
&= |\alpha|^2\frac{(\sqrt{2}-i)(\sqrt{2}+i)}{4} + \alpha\beta^*\frac{\sqrt{2}-i}{4} + \beta\alpha^*\frac{\sqrt{2}+i}{4} + |\beta|^2\frac{1}{4} \\
&\quad + |\alpha|^2\frac{1}{4} - \alpha\beta^*\frac{\sqrt{2}-i}{4} - \beta\alpha^*\frac{\sqrt{2}+i}{4} + |\beta|^2\frac{(\sqrt{2}+i)(\sqrt{2}-i)}{4} \\
&= |\alpha|^2\frac{3}{4} + |\beta|^2\frac{1}{4} + |\alpha|^2\frac{1}{4} + |\beta|^2\frac{3}{4} \\
&= |\alpha|^2 + |\beta|^2 \\
&= 1.
\end{aligned}
$$

So, U is a valid quantum gate. Then,

Quantum Gates

Gate	Effect on Amplitude / Phase
X	Flips amplitudes |0> , |1>
Z	Adds a phase of π (i.e., -1)
Y	Flips and adds a phase (combination of X and Z)
H (Hadamard)	Creates superposition, redistributes amplitudes, adds ± phases
T	Adds a phase of π/4 (45°) to
S	Adds a phase of π/2 (90°) to
RX/RY/RZ	Rotates the qubit on the Bloch sphere, changing amplitude and/or phase

Quantum Gates – Pauli X

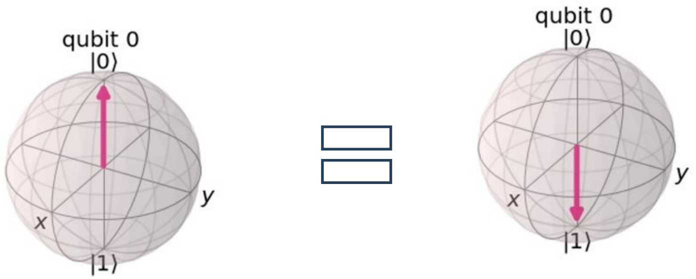
X (NOT) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$X|1> = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0>$$

$$X|0> = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1>$$

Pauli-X Matrix

$$\boxed{X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$$



Quantum Gates – Pauli Y

Pauli-Y Matrix

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$Y|1\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -|i\rangle$$

$$Y|0\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = |i\rangle$$

Quantum Gates – Pauli Z

Pauli-Z Matrix

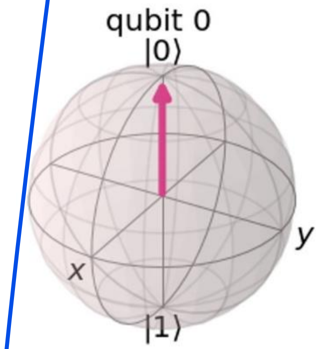
$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Z|1> \;=\; \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \;=\; \begin{pmatrix} 0 \\ -1 \end{pmatrix} \;=\; -|1>$$

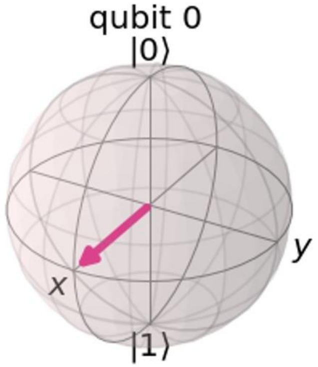
$$Z|0> \;=\; \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \;=\; \begin{pmatrix} 1 \\ 0 \end{pmatrix} \;=\; |0>$$

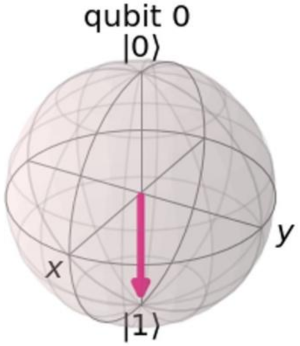
Quantum Gates – Hadamard Gate

$$\text{H (Hadamard)} \quad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

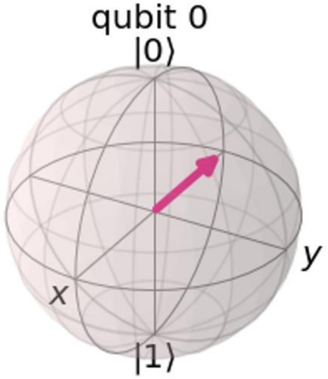


$$H\,|0\rangle \quad \frac{1}{\sqrt{2}}(\,|0\rangle + |1\rangle) \quad = |+\rangle$$





$$H\,|1\rangle \quad \frac{1}{\sqrt{2}}(\,|0\rangle - |1\rangle) \quad = |-\rangle$$



Application of gates

X –gate Applications:

- Works like a classical NOT (bit-flip).
- Used for inverting qubit values,
- building block for universal quantum gates.

Z gate Application:

- Flips the phase of $|1\rangle$.
- Crucial in phase estimation algorithms, error correction (Stabilizer codes), and building controlled operations

Y gate Application:

- Bit and phase flip together.
- Used in certain error-correction codes and Bloch sphere rotations.

H Gate Application:

- Creates superposition from a classical state.
- Used at the start of many algorithms (Deutsch–Jozsa, Grover, Shor).

Multi Qubit

- A single bit has 2 possible states 0 and 1.
- A qubit has 2 possible states |0> & |1> and complex amplitude.
- Two bits have 4 possible states 00 01 10 11
- Two qubits requires four complex amplitudes to represent its states.

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \otimes \begin{bmatrix}\gamma\\ \delta\end{bmatrix} = \begin{bmatrix}\alpha\begin{bmatrix}\gamma\\ \delta\end{bmatrix}\\ \beta\begin{bmatrix}\gamma\\ \delta\end{bmatrix}\end{bmatrix} = \begin{bmatrix}\alpha\gamma\\ \alpha\delta\\ \beta\gamma\\ \beta\delta\end{bmatrix}$$

TP

Multi Qubit – Superposition and Entanglement

$$\begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \qquad \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle \right)$$

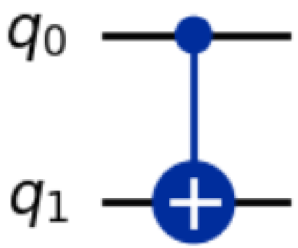
|00> |01> |10> |11>

Quantum Gates – CNOT Gate Matrix

c t

Input (t,c) output(t,c)

Performs X gate on the second qubit (target) If the state of the first qubit(control) is |1>



c

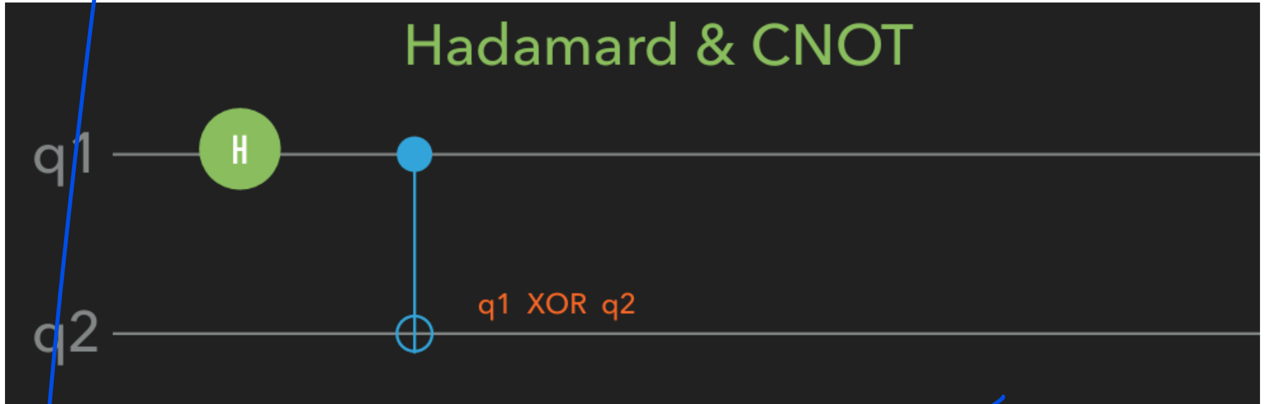
t

$$\text{CNOT}\,|00\rangle = |00\rangle$$
$$\text{CNOT}\,|01\rangle = |11\rangle$$
$$\text{CNOT}\,|10\rangle = |10\rangle$$
$$\text{CNOT}\,|11\rangle = |01\rangle$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Quantum Gates – Hadamard and CNOT

$\begin{bmatrix}1\\0\end{bmatrix}$ $\begin{bmatrix}1\\0\end{bmatrix}$



$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} * \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}$$

Hadamard * |0> (q1) = |+> (q1)

$$\begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\\ 0\\ 0\end{bmatrix}$$

|0> ⊗ |+> = |0+>

q2 ⊗ q1 = |0+>

$$\begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\\ 0\\ 0\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ 0\\ 0\\ \frac{1}{\sqrt{2}}\end{bmatrix}$$

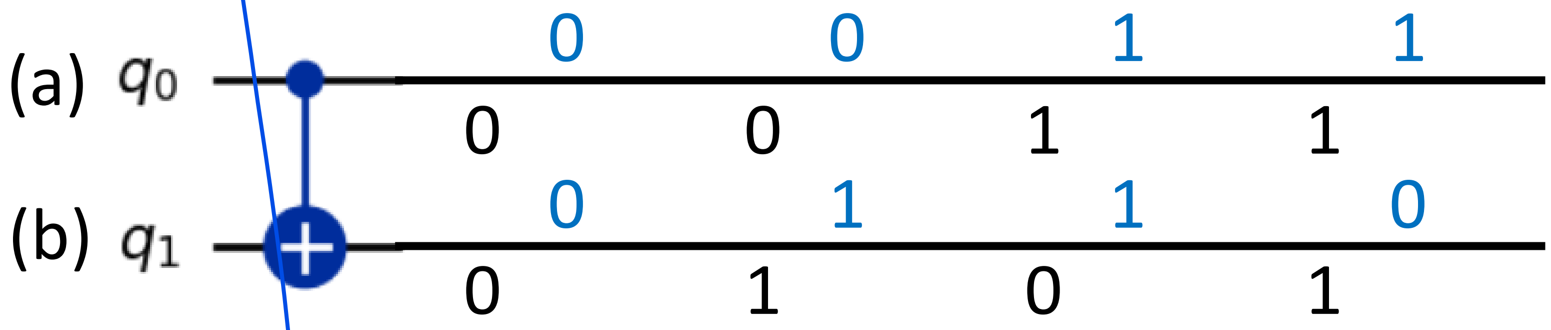
CNOT * |0+> = $\frac{1}{\sqrt{2}}$(|00> + |11>

The Amazing H-Gate

- After a qubit in state $|0\rangle$ or $|1\rangle$ has been acted upon by a H gate, the state of the qubit is an equal superposition of $|0\rangle$ and $|1\rangle$. Thus, the qubit goes from a deterministic state to a truly random state, i.e., if the qubit is now measured, we will measure $|0\rangle$ or $|1\rangle$ with equal probability.
- We see that H is its own inverse, that is, $H^{-1} = H$ or $H^2 = I$. Therefore, by applying H twice to a qubit we change nothing. This is amazing!
- By applying a randomizing operation to a random state produces a deterministic outcome!
- **One of the most important gates in quantum computing!**

Quantum Gates – CNOT Gate Matrix

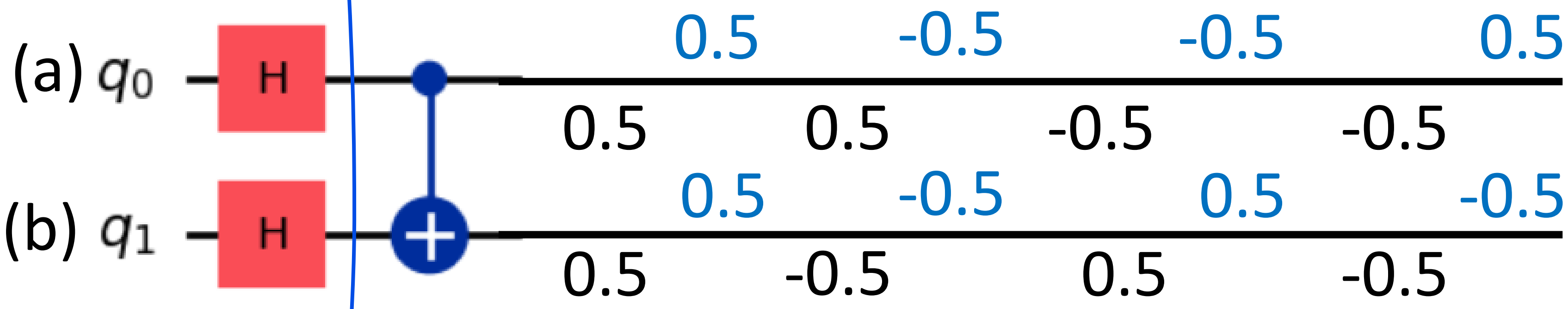
cx(a,b) without superposition



(a) q0 = 1

(b) q1 = (0,1) flips (1,0)

cx(a,b) with superposition



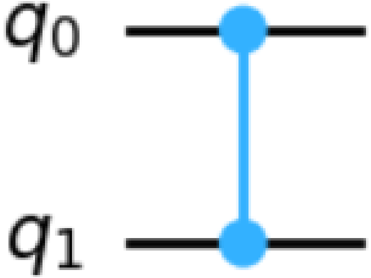
(a) q0 = (0.5,-0.5) flips (-0.5,0.5)

(b) q1 = -0.5

Quantum Gates – Controlled Z Gate

CZ gate applies Z gate to ‘target’ when the ‘control’ is 1.

Input (t,c)	output(t,c)
00	00
01	01
10	10
11	-11



Cit

F

X $\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$ $\begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}$ Z

π

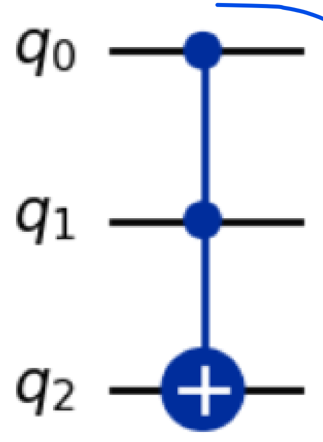
$\begin{matrix} 0 & -i \\ i & 0 \end{matrix}$ Y

π, F

Toffoli gate (Controlled-Controlled-NOT)

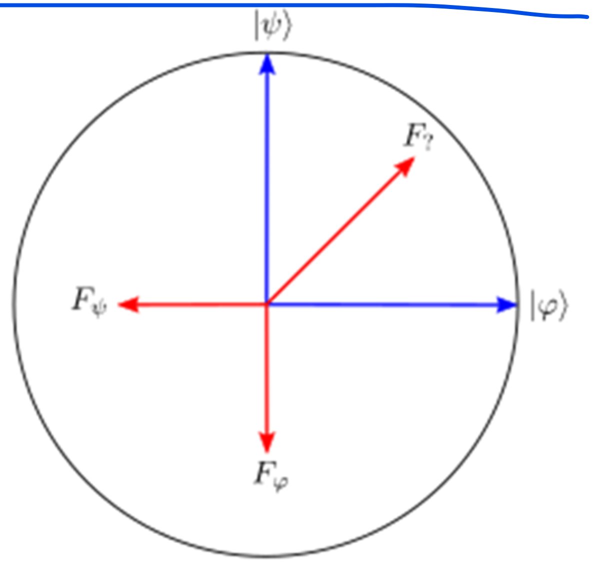
- A three qubit gate with 2 controls and 1 target.
- It performs an X gate on the target only is both controls are in |1>

Input			Output		
A	B	C	P	Q	R
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0



Bell States (EPR Pairs)

- Definition: The **Bell states** are a set of four special two-qubit states that are **orthonormal** (mutually independent and normalized) and **maximally entangled**.
- Also known as EPR pairs or states. EPR – Einstein, Podolsky and Rosen, who first point out the strange properties.
- They are written as, often in direc notation:



$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

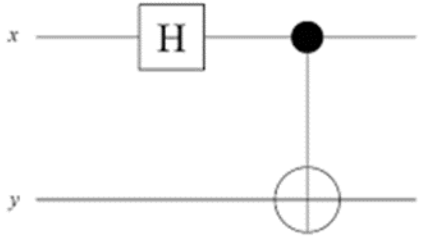
- These four states together form the Bell basis for the 2-qubit Hilbert space.

Bell States (EPR Pairs)

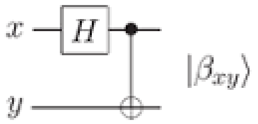
- Why are they special?
- Entanglement: Neither qubit has a well-defined individual state. The two qubits exist in a correlated superposition.
- Maximal correlation: Measurement outcomes are perfectly correlated (or anti-correlated), no matter how far apart the qubits are.
- Orthonormal: $\langle\Phi^+|\Phi^-\rangle=0$, etc. They span the entire 2-qubit state space.

How to create Bell states

- Although there are many possible ways to create entangled Bell states through quantum circuits, the simplest takes a computational basis as the input, and contains a Hadamard gate and a CNOT gate



In	Out
$\lvert 00\rangle$	$(\lvert 00\rangle + \lvert 11\rangle)/\sqrt{2} \equiv \lvert\beta_{00}\rangle$
$\lvert 01\rangle$	$(\lvert 01\rangle + \lvert 10\rangle)/\sqrt{2} \equiv \lvert\beta_{01}\rangle$
$\lvert 10\rangle$	$(\lvert 00\rangle - \lvert 11\rangle)/\sqrt{2} \equiv \lvert\beta_{10}\rangle$
$\lvert 11\rangle$	$(\lvert 01\rangle - \lvert 10\rangle)/\sqrt{2} \equiv \lvert\beta_{11}\rangle$

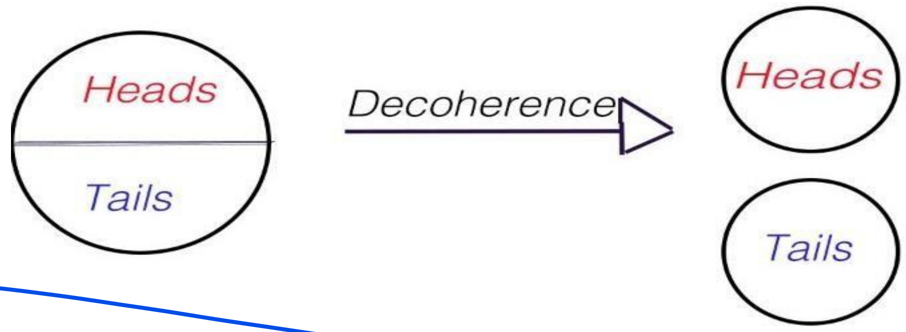


- If qubit x starts with a state of $\lvert 0\rangle$, the Hadamard gate puts it into a superposition of $\lvert 0\rangle$ and $\lvert 1\rangle$.
-
- The CNOT gate flips qubit y depending on the state of qubit x, but since qubit x is in a superposition of $\lvert 0\rangle$ and $\lvert 1\rangle$ this makes the final state of qubit y dependent on what the final state of qubit x turns out to be — thus the two qubits are part of an entangled state.

Applications of Bell States

- Quantum teleportation. A Bell pair shared between Alice and Bob enables teleporting an unknown qubit state from Alice to Bob when Alice performs a Bell measurement and sends two classical bits. Teleportation consumes one Bell pair (one ebit) and two classical bits. ✓
- Superdense coding. With one Bell pair, Alice can send two classical bits by sending a single qubit to Bob after applying one of four local unitaries (I, X, Z, XZ). Bob performs a Bell measurement and recovers two classical bits — doubling classical capacity.
- Bell inequality tests / foundations. Bell states (e.g. $|\Phi^+\rangle$) achieve maximal CHSH violation (quantum value 2222), demonstrating incompatibility with local hidden-variable theories.
- Entanglement distribution & entanglement swapping. Bell pairs are the basic resource for quantum repeaters and entanglement swapping protocols. ✓
- Quantum key distribution (entanglement-based protocols). E91 protocol uses Bell states to establish secure keys. ✓
- Quantum error correction & entanglement-assisted codes. Bell pairs are used as ancillas and to distribute entanglement across logical qubits. ✓
- Measurement-based (cluster-state) computation: Bell states are building blocks for larger entangled resource states.

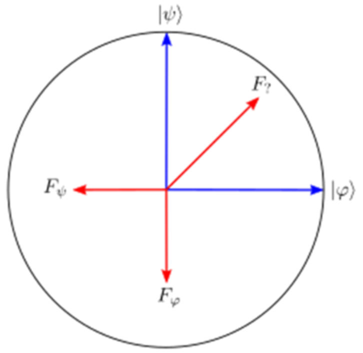
Decoherence



- Quantum decoherence is the loss of superposition, because of the spontaneous interaction between a quantum system and its environment.
- Decoherence process destroys the superposition and the system collapses randomly into one of the states that constitute the superposition state.
- Decoherence can be viewed as the loss of information from a system into the environment.
- The reason why quantum computers still have a long way to go because superposition and entanglement are extremely fragile states.
- Preventing decoherence remains the biggest challenge in building quantum computers.

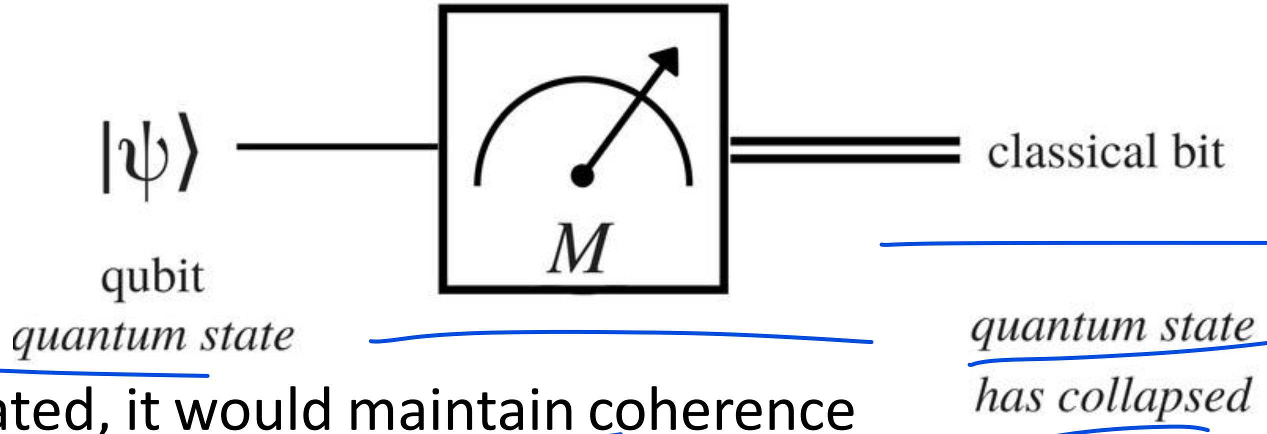
What is Measurement?

- Measurement is the process of **extracting classical information** from a quantum system (qubit).
- A qubit can be in a **superposition**:
  - $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle, |\alpha|^2+|\beta|^2=1$



- When we measure in the **computational basis** ($|0\rangle, |1\rangle$):
  - Outcome = $|0\rangle$ with probability $|\alpha|^2$.
  - Outcome = $|1\rangle$ with probability $|\beta|^2$.
- After measurement, the qubit **collapses** into the observed state (no longer in superposition).
- **So, measurement turns quantum information into classical bits.**

Measurement



- If a quantum system were perfectly isolated, it would maintain coherence indefinitely, but it would be impossible to manipulate or investigate it.
- A quantum measure is a decoherence process.
- When a quantum system is measured, the wave function $|\psi\rangle$ collapses to a new state according to a probabilistic rule.
- If $|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$, after measurement, either $|\psi\rangle = |0\rangle$ or $|\psi\rangle = |1\rangle$, and these alternatives occur with certain probabilities of $|\alpha_0|^2$ and $|\alpha_1|^2$ with $|\alpha_0|^2 + |\alpha_1|^2 = 1$.
- A quantum measurement never produces $|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.
- *Example*: Two qubits: $|\psi\rangle = 0.316|00\rangle + 0.447|01\rangle + 0.548|10\rangle + 0.632|11\rangle$

  The probability to read the rightmost bit as 0 is $|0.316|^2 + |0.548|^2 = 0.4$

Purpose of Measurement

- Why do we measure qubits?
- Readout of results: At the end of a quantum algorithm, we need classical bits as answers.
- Probabilistic outcomes: By repeating measurements, we estimate probabilities and amplitudes.
- Intermediate feedback: In some algorithms (e.g., variational quantum eigensolver, error correction), measurement results guide further quantum or classical operations.

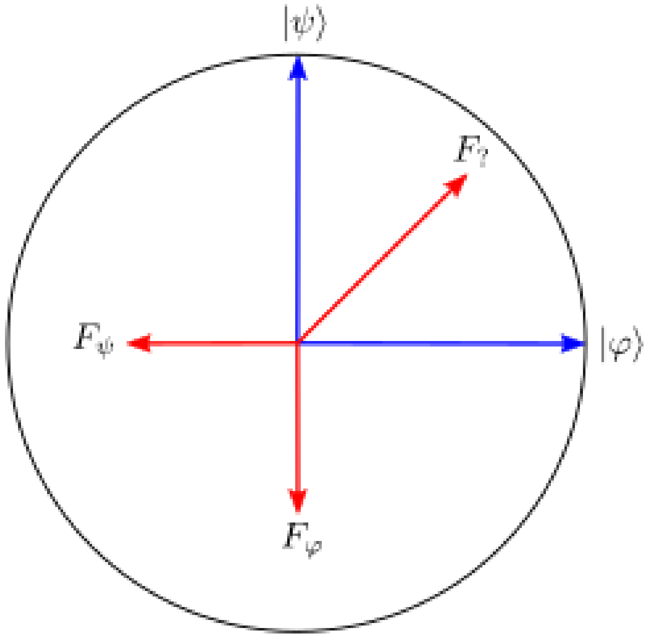
Key Characteristics of Measurement

- Irreversible: Once measured, superposition is destroyed.
- Probabilistic: Results are not deterministic, unless the state is already $|0\rangle$ or $|1\rangle$.
- ~~Basis-dependent: Usually done in the~~ computational basis, but we can measure in other bases by applying gates first (e.g., apply Hadamard → measure in X-basis).

Applications

- Final readout: Get answers from quantum algorithms.
- Sampling distributions: Algorithms like QFT, Grover, or QAOA rely on measurement statistics.
- Error correction: Measure ancilla qubits to detect errors without disturbing data qubits.
- Quantum teleportation: Measurement outcomes are sent classically to apply corrections.

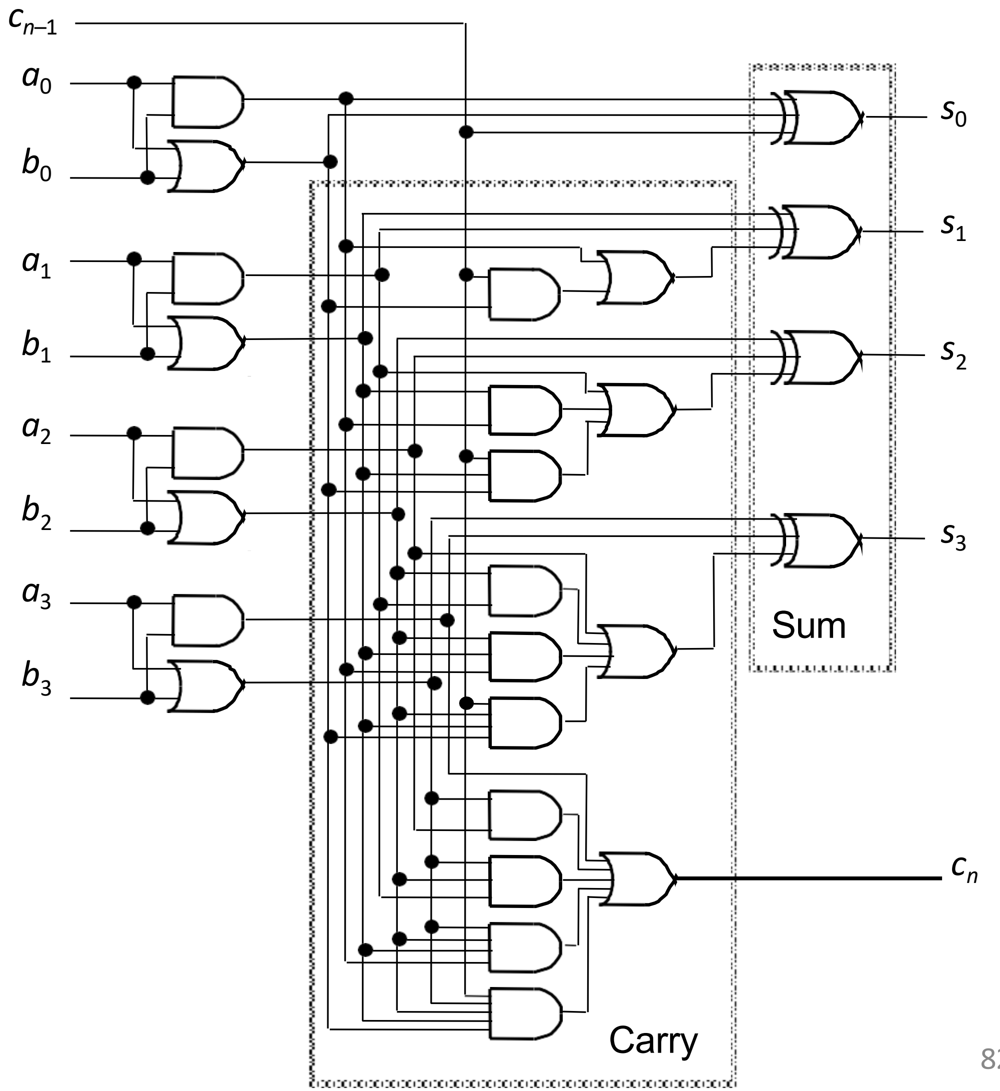
How Measurement Works



$|\psi\rangle$
$F_?$
F_ψ
$|\varphi\rangle$
F_φ

Quantum vs. Classical Circuits

- Classical Logic Circuits
  - Circuit behavior is governed implicitly by classical physics
  - Signal states are simple bit vectors, e.g. $X = 01010111$
  - Operations are defined by Boolean Algebra
  - No restrictions exist on copying or measuring signals
  - Small well-defined sets of universal gate types, e.g. {NAND}, {AND,OR,NOT}, {AND,NOT}, etc.
  - Well developed CAD methodologies exist
  - Circuits are easily implemented in fast, scalable and macroscopic technologies such as CMOS

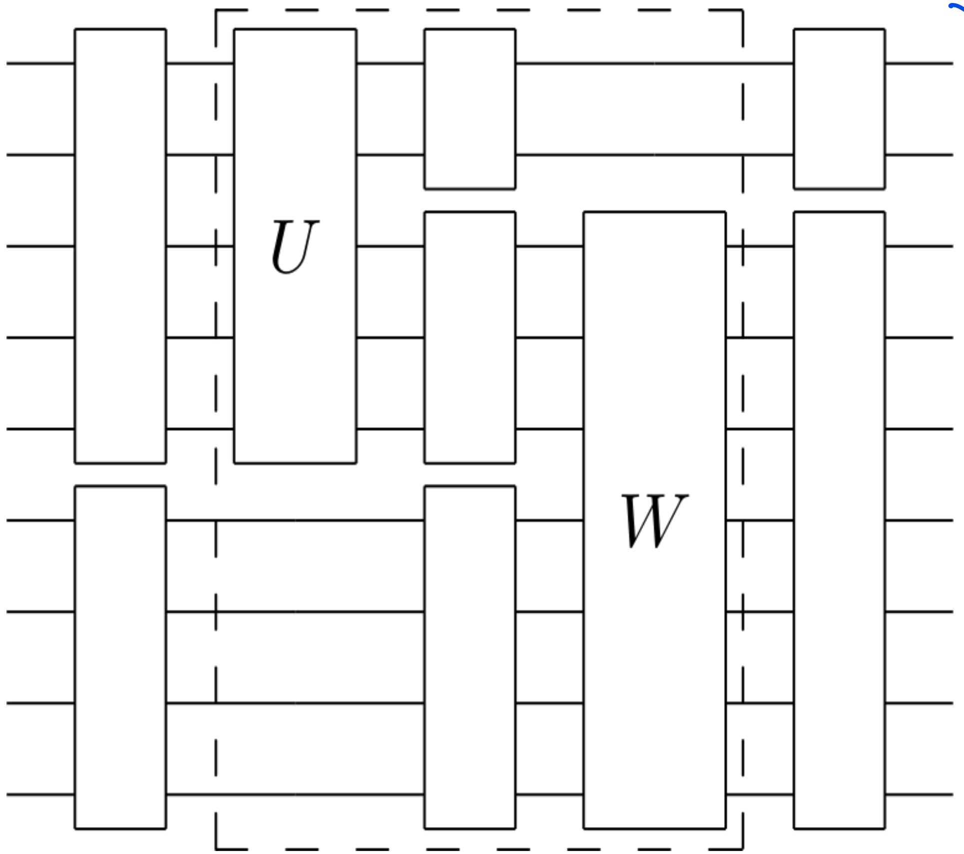


Quantum vs. Classical Circuits

- Quantum Logic Circuits
  - Circuit behavior is governed explicitly by quantum mechanics
  - Signal states are vectors interpreted as a superposition of binary "qubit" vectors with complex-number coefficients

$$|\Psi\rangle = \sum_{i=0}^{2^n-1} c_i \left| i_{n-1} i_{n-1} \ldots i_0 \right\rangle$$

  - Operations are defined by linear algebra over Hilbert Space and can be represented by unitary matrices with complex elements
  - Severe restrictions exist on copying and measuring signals
  - Many universal gate sets exist but the best types are not obvious
  - Circuits must use microscopic technologies that are slow, fragile, and not yet scalable, e.g., NMR



- https://www.youtube.com/watch?v=fkAAbXPEAtU&t=263s

Thank You