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Shor code

→ Shor code is a quantum error that corrects one logical bit into 9 physical bits to protect it from bit flip and phase flip error.

→ The code works by using a repetative pattern of encoding and entanglement across the nine qubits to detect and correct errors without directly measuring logical bit.

Working of shor code
Step 1:- Transformation - encoding
Step 2:- Syndrome measurement
Step 3:- Error correction

Transformation:-

→ First apply modified version of the 3 bit repetation code which detect phase flip error and then we will encode each of the resulting three bits independently using the original 3 bit repetation code which detects phase flip errors

Step 1 → A single Qubit in unknown state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Create three copies

$$|0_L\rangle = |000\rangle, |1_L\rangle = |111\rangle$$

After encoding

$$|\psi\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$$

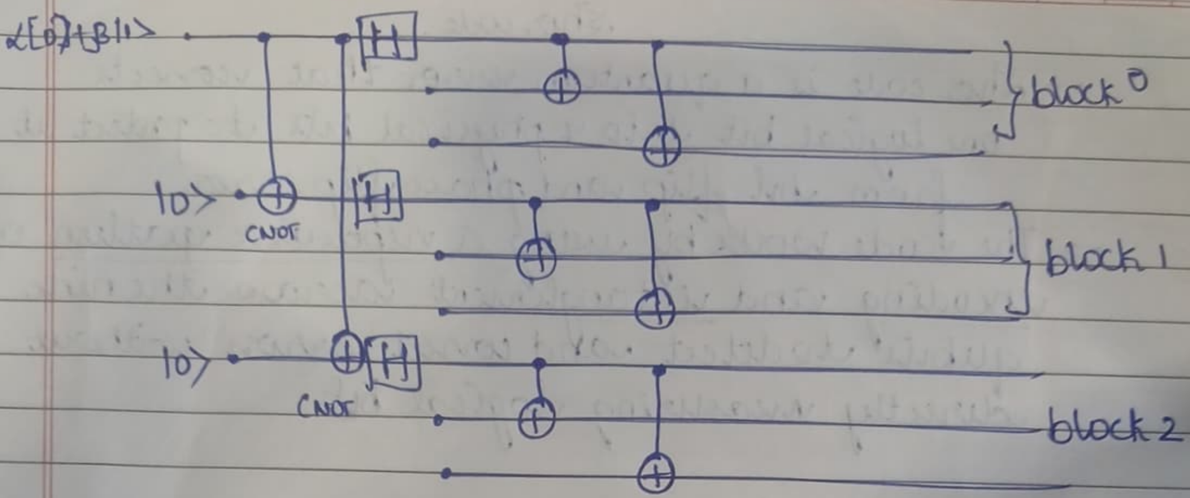
Step2 Encode each of the Three Qubits again

$$|000\rangle \rightarrow \frac{1}{2}|000\rangle + |111\rangle^{\otimes 3}$$

$$|111\rangle \rightarrow \frac{1}{2}|000\rangle + |111\rangle^{\otimes 3}$$

The nine Qubit encoded state

$$|\psi\rangle \rightarrow \frac{1}{2}\alpha(|000\rangle + |111\rangle)^{\otimes 3} + \frac{1}{2}\beta|000\rangle - |111\rangle^{\otimes 3}$$



→ Inner code – 3 bit repetation code
→ Outer code → First encoding

Syndrom detection – Parity check
Bit flip (Z-type) Stabalizers
Phase flip (X-type) stabalizers

Bit flip

$S_1 - Z_1 Z_2$
$S_2 - Z_2 Z_3$
$S_3 - Z_4 Z_5$
$S_4 - Z_5 Z_6$
$S_5 - Z_7 Z_8$
$S_6 - Z_8 Z_9$

Phase flip:

$S_7 = X_1 X_2 X_3 X_4 X_5 X_6$
$S_8 = X_4 X_5 X_6 X_7 X_8 X_9$

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UNIT 3

Fonco

Stean Quantum error correction

→ Quantum information is extremily fragile due to coherence and noise.

→ Quantum error correction provides methods to detect and correct errors without measuring or disturbing quantum information

→ One of the important QEC code is the steane code also called [7, 1, 3] CSS code.

→ It was proposed by Andrew steane 1996

→ It protects 1 logical qubit using 7 physical qubits and can correct any single qubit error.

→ Error types → Bit flip error $|0\rangle \longleftrightarrow |1\rangle$

Phase flip error $|+\rangle \longleftrightarrow |-\rangle$

Combined error $Y = iXZ$.

Steane code detects and corrects all three errors

(1)

→ Steane code is a CSS (Calder-Shor - Steane) code constructed using two classical linear codes

Hamming [7,4]   dual code

↳ Hamming encode classical 4 bits into 7 bits

↳ Detects and corrects 1 classical bit/flip error.

↳ Defined by parity check matrix

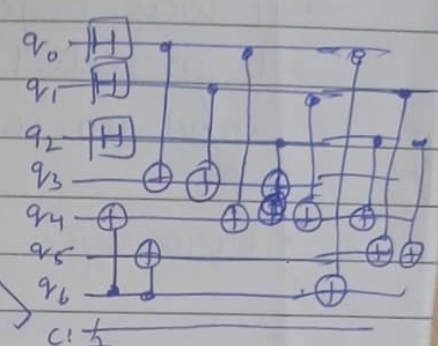
$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Steane uses this code to built the quantum stabalizers

→ Encoding
Structure of Steane code
[7, 1, 3]
7 → 7 Physical bits
1 → 1 Logical bit
3 → Distance 3 → can correct one error.

Logical States
Logical states are superposition of all valid Hamming codewords

$$|0_L\rangle = \frac{1}{\sqrt{8}} \sum_{c \in C} |c\rangle$$

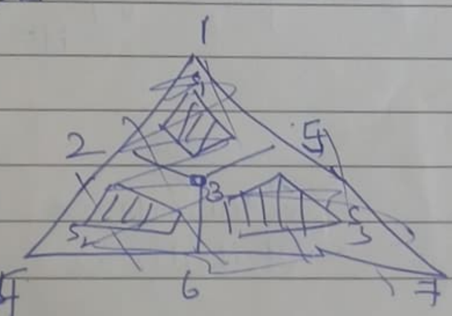


$$|1_L\rangle = \frac{1}{\sqrt{8}} \sum_{c \in C} |c \oplus 1111111\rangle$$

Ensures both bit flip & Phase flip protection
→ CNOT / Hadamard / Hamming code → Circuit

→ Syndrome
To detect errors ancillary qubits are used to measure stabalizers
X type stabalizers – Detect Z phase errors
Z type stabalizers - Detect X bit errors
6 stabalizers

X type stabalizers
$S_1 - X_1 X_2 X_3 X_5$
$S_2 - X_1 X_3 X_4 X_6$
$S_3 - X_2 X_3 X_4 X_7$



Z type stabalisers
$S_4 - Z_1 Z_2 Z_3 Z_5$
$S_5 - Z_1 Z_3 Z_4 Z_6$
$S_6 - Z_3 Z_3 Z_4 Z_7$
Derieved from Hamming code

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Syndrome table

Qubit	M1	M2	M3	Syndrome
1	1	0	0	100
2	0	1	0	010
3	0	0	1	001
4	1	1	0	110
5	0	1	1	010
6	1	0	1	101
7	1	1	1	111
no error	0	0	0	000

If we measure 100 → flip Qbit 1
If we measure 010 → flip Qubit 5
↑ same table for phase error

<u>Error correction:-</u>

→ Apply the inverse operation to restore the correct encoded state

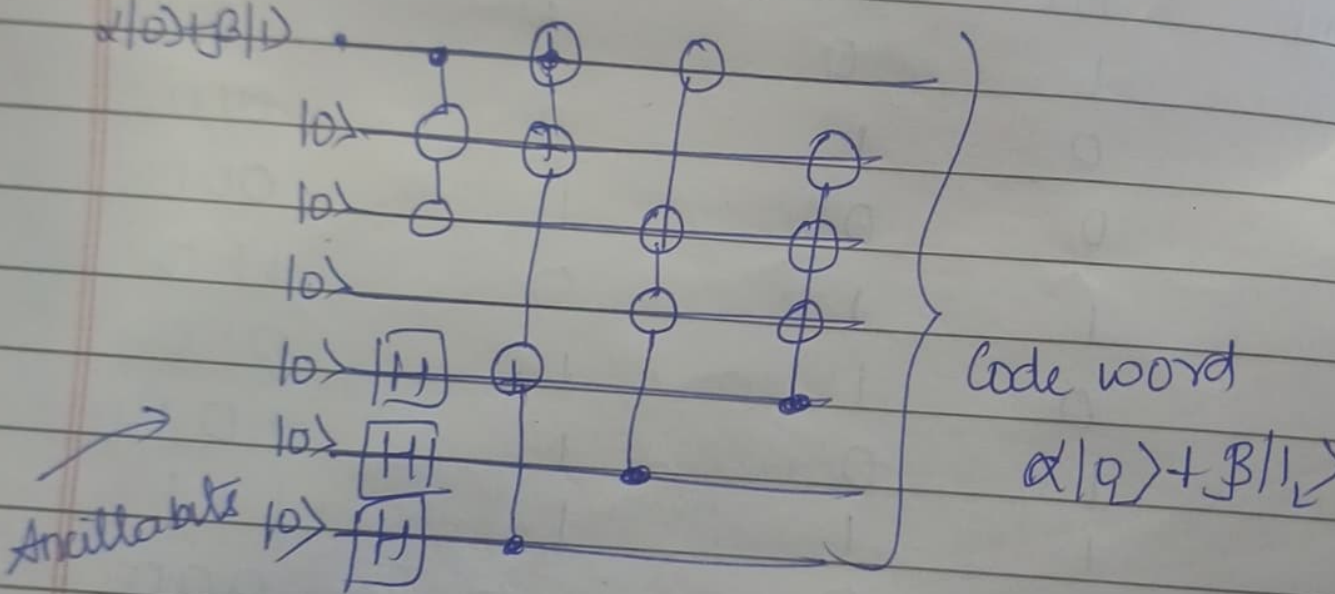
→ If a bit flip is detected apply X on faulty bit

→ If a phase flip is detected apply Z on faulty bit

→ If combined error is detected apply Y

→ After correction the logical qubit returns to $|\psi_L\rangle$

Encoder for Steane's code



Hamming code.

Steam code :- [7,1,3] CSS quantum error correcting code.

7 → Physical bit

1 → logical bit

3 → distance [corrects one qubit error)

Build from Hamming code

X, Z & Y errors

Hamming code:- encode 4 bits to 7 bits

Parity check matrix :-

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Parity checks

bit position

Stean uses this matrix for X & Z stabalizers

4 bit message → a, b, c, d } original

z_1, z_2, z_3 } Parity check bits

[7,4]

7 → coded bits

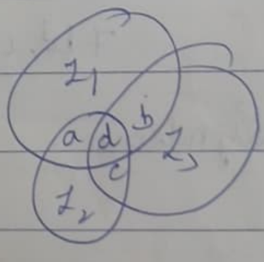
4 → Logical bits

$z_1 = a+b+d$

$z_2 = a+c+d$

$z_3 = b+c+d$

} mod2 → XOR



Bit position	1	2	3	4	5	6	7
Party bit	P1	P2	D1	P3	D2	D3	D4
arrangement	↓ LSB1	↓ LSB2		↓ LSB3			

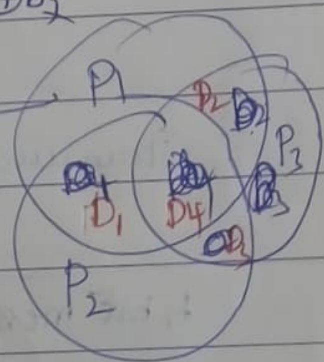
2^2 2^1 2^0
4 2 1

Position	Binary
1	001
2	010
3	011
4	100
5	101
6	110
7	111

P1 → 1 3 5 7 → $P_1 = b_1 \oplus b_3 \oplus b_5 \oplus b_7$

P2 → 2 3 6 7 → $P_2 = b_2 \oplus b_3 \oplus b_6 \oplus b_7$

P_3 → 4 5 6 7 → $P_3 = b_4 \oplus b_5 \oplus b_6 \oplus b_7$



$P_1 = D_1 \oplus D_2 \oplus D_4$

$P_2 = D_1 \oplus D_3 \oplus D_4$

$P_3 = D_2 \oplus D_3 \oplus D_4$

1 2 3 4 5 6 7

7 bit code → $P_1\ P_2\ D_1\ P_3\ D_2\ D_3\ D_4$

Syndrome: 3 bit no

000 No error

101 → bit 5 error

011 → bit 3 error

110 – bit 6 error

Creating Global Leaders | CBSE

Date :

Name : Kum / Master

Max. Marks :

Time :

I.D. No. : Subject : Class / Sec :

Signature of Invigilator (With date)

Sign. of Evaluator (With date)

Ex: $D_1 = 1,\ D_2 = 0,\ D_3 = 1,\ D_4 = 1$

$1 \oplus 0\ 0 \bmod 2$

$= 0 + 1$

$P_1 = 1 \oplus 0 \oplus 1 = 0$

$P_2 = 1 \oplus 1 \oplus 1 = 1$

$P_3 = 0 \oplus 1 \oplus 1 = 0$

Codeword = $\overset{P_1\,P_2\,D_1\,P_3\,D_2\,D_3\,D_4}{0110011}$

syndrom = 101 = 5 bit flip

0110011

0100011

Stean – 7 1 3

P ← ↓ Loo → 3

bit positions

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Part