Assignment 2

Swapnil Sirsat

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Question 1

If $A = \begin{pmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$ verify that
$(i)(A')' = A$
$(ii)(A+B)' = A' + B'$
$(iii)(kB)' = kB'$,where k is any constant.

Solution :

(i) $(A')' = A$

Given $A = \begin{pmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{pmatrix}$
Thus,
$A' = \begin{pmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{pmatrix}$
Now,
$(A')' = \begin{pmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{pmatrix}' = \begin{pmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{pmatrix} = A$
Hence, (A')' $= A$

(ii) $(A+B)' = A' + B'$

$A = \begin{pmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$
$A + B = \begin{pmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 5 & \sqrt{3}-1 & 4 \\ 5 & 4 & 4 \end{pmatrix}$
$(A+B)' = \begin{pmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{pmatrix}$

Now,

$A' = \begin{pmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{pmatrix}$ and $B' = \begin{pmatrix} 2 & 1 \\ -1 & 2 \\ 2 & 4 \end{pmatrix}$

$A' + B' = \begin{pmatrix} 5 & 5 \\ \sqrt{3} - 1 & 4 \\ 4 & 4 \end{pmatrix} = (A + B)'$

therefore, $A' + B' = (A + B)'$

$(iii)\ (kB)' = kB'$

$B = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$

$kB = \begin{pmatrix} 2k & -1k & 2k \\ 1k & 2k & 4k \end{pmatrix}$

$(kB)' = \begin{pmatrix} 2k & 1k \\ -1k & 2k \\ 2k & 4k \end{pmatrix}$

now, $B' = \begin{pmatrix} 2 & 1 \\ -1 & 2 \\ 2 & 4 \end{pmatrix}$

$kB' = \begin{pmatrix} 2k & 1k \\ -1k & 2k \\ 2k & 4k \end{pmatrix} = (kB)'$

therefore, $kB' = (kB)'$

Question 2

If $A = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 & -6 \end{pmatrix}$, verify that $(AB)' = B'A'$

Solution :

$A = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 & -6 \end{pmatrix}$

$A' = \begin{pmatrix} 2 & 4 & 5 \end{pmatrix}$ and $B' = \begin{pmatrix} 1 \\ 3 \\ -6 \end{pmatrix}$

now,

$AB = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} \begin{pmatrix} 1 & 3 & -6 \end{pmatrix}$

$AB = \begin{pmatrix} 2 & 6 & -12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{pmatrix}$

$(AB)' = \begin{pmatrix} 2 & 4 & 5 \\ 6 & 12 & 15 \\ -12 & -24 & -30 \end{pmatrix}$

Now,

$B'A' = \begin{pmatrix} 1 \\ 3 \\ -6 \end{pmatrix} \begin{pmatrix} 2 & 4 & 5 \end{pmatrix}$

$B'A' = \begin{pmatrix} 2 & 4 & 5 \\ 6 & 12 & 15 \\ -12 & -24 & -30 \end{pmatrix} = (AB)'$

therefore, $B'A' = (AB)'$