

Assignment 7

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Question

Draw a circle of radius **3** units. Take two points **P** and **Q** on one of its extended diameter each at a distance of **7** units from its centre. Draw tangents to the circle from these two points **P** and **Q**.

Answer

Taking the center of the circle at $(0,0)$

$$O = (0, 0)$$

Taking the diameter of the circle lie on the X axis
the extended diameter **PQ** would be

$$P = (-7, 0)$$

$$Q = (7, 0)$$

now, constructing circles with the mid point of **OP** and **OQ** as centers and **OP** and **OQ** as diameters respectively we obtain circle R and S as coordinates

$$R = (-3.5, 0)$$

$$S = (3.5, 0)$$

the intersection points of Circle **R(-3.5,0)** and **S(3.5,0)** with **O(0,0)** would be the points of contact of tangents through P and Q respectively
this point would be as

For tangents through P, solving the equations

$$x^2 + y^2 = 9 \tag{1}$$

$$(x + 3.5)^2 + y^2 = (3.5)^2 \tag{2}$$

we would obtain the point of contact for the tangents
similarly for tangents through Q, solving the equation (1) and

$$(x - 3.5)^2 + y^2 = (3.5)^2 \tag{3}$$

we would obtain the point of contact for the tangents
equation (2) can be rewritten as

$$x^2 + (3.5)^2 + 2 \times (3.5) \times x + y^2 = (3.5)^2 \quad (4)$$

$$\implies x^2 + y^2 + (3.5)^2 + 2 \times (3.5) \times x = (3.5)^2 \quad (5)$$

and equation (3) can be rewritten as

$$x^2 + (3.5)^2 - 2 \times (3.5) \times x + y^2 = (3.5)^2 \quad (6)$$

$$\implies x^2 + y^2 + (3.5)^2 - 2 \times (3.5) \times x = (3.5)^2 \quad (7)$$

substituting equation (1) in (5) we get

$$9 + (3.5)^2 + 2 \times (3.5) \times x = (3.5)^2$$

$$\implies 9 + 7x = 0$$

$$\implies x = \frac{-9}{7} = -1.286$$

substituting value of x in equation (1)

$$\left(\frac{-9}{7}\right)^2 + y^2 = 9$$

$$\implies y = \sqrt{9 - \left(\frac{-9}{7}\right)^2}$$

$$\implies y = \pm 2.710$$

Similarly substituting equation (1) in (7) we get $x = \frac{9}{7} = 1.286$
and substituting this in equation 1 we get

$$y = \pm 2.71$$

therefore the tangents from P would make contact with Circle O at

$$T_1(-1.286, 2.710)$$

$$T_2(-1.286, -2.710)$$

and tangents from Q would make contact at

$$T_3(1.286, 2.710)$$

$$T_4(1.286, -2.710)$$

joining this points we would obtain the asked tangents on circle O.

below is the constructed figure.

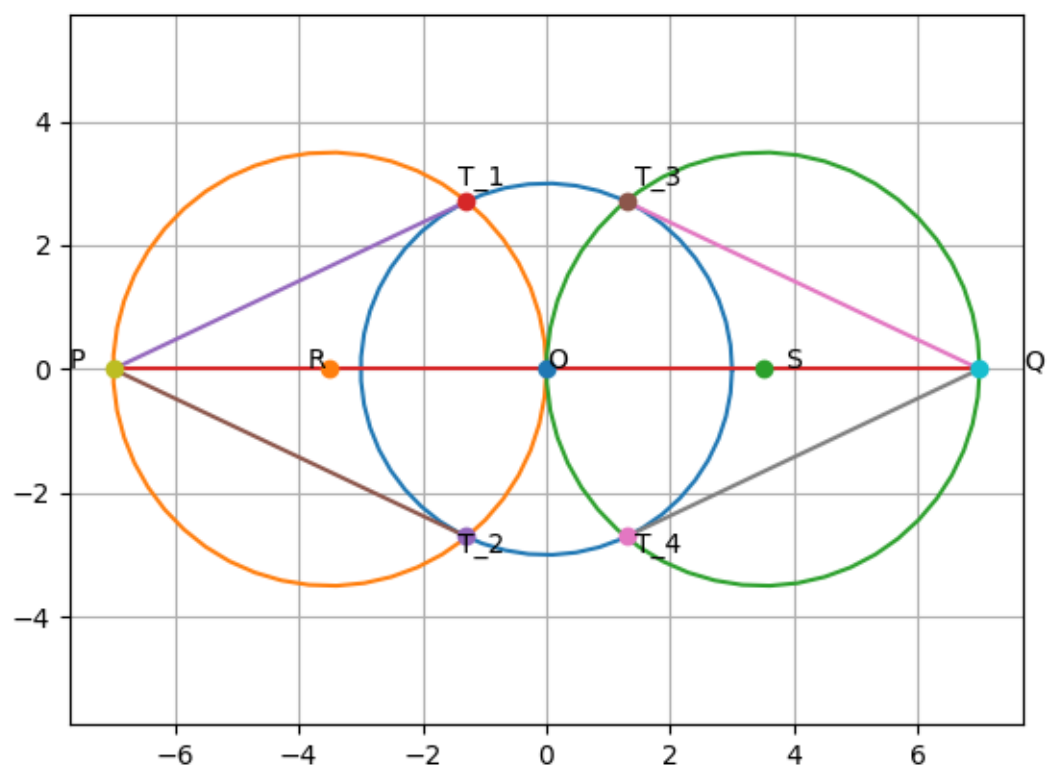


Figure 1: Python output