Assignment 8

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Question

Obtain the inverse of following matrix using elementary operations
$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$

Solution

We know that

$$AA^{-1} = I$$

therefore,

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_1 \leftrightarrow R_2, R_3 \rightarrow R_3 - 3R_1$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{pmatrix} A^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{pmatrix}$$

$$R_3 \rightarrow R_3 + 5R_2, R_1 \rightarrow R_1 - 2R_2$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix} A^{-1} = \begin{pmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{pmatrix}$$

$$R_3 \rightarrow \frac{1}{2}R_3$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{pmatrix}$$

$$R_2 \to R_2 - 2R_3, R_1 \to R_1 + R_3$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{pmatrix}$$

$$\implies IA^{-1} = A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{pmatrix}$$

Thus,

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{pmatrix}$$