Assignment 1

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Question :

Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix}$

(ii) $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

(iii) $\begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$

(iv) $\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}$

Solution:

$$(i)\ \begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix}$$

$A = \begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix}$

then, $A' = \begin{pmatrix} 3 & 1 \\ 5 & 1 \end{pmatrix}$

let

$P = \frac{1}{2}(A + A')$

$P = \frac{1}{2}(\begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ 5 & 1 \end{pmatrix})$

$P = \frac{1}{2}(\begin{pmatrix} 6 & 6 \\ 6 & 2 \end{pmatrix})$

$P = \begin{pmatrix} 3 & 3 \\ 3 & 1 \end{pmatrix}$

now, $P' = \begin{pmatrix} 3 & 3 \\ 3 & 1 \end{pmatrix} = \text{P}$

therefore P is symmetric matrix

Now, let$Q = \frac{1}{2}(A - A')$

$$Q = \frac{1}{2}(\begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ 5 & 1 \end{pmatrix})$$

$$Q = \frac{1}{2}(\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix})$$

$$Q = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} = - \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} = Q'$$

therefore Q is a skew-symmetric matrix

now,

$$P + Q = (\begin{pmatrix} 3 & 3 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix})$$

$$P + Q = \begin{pmatrix} 3 & 5 \\ 1 & 1 \end{pmatrix} = A$$

thus A is written as $P + Q$ where P and Q are symmetric and skew-symmetric matrices respectively.

(ii) $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

$A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

then $A' = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

Now, $A + A' = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} + \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{pmatrix}$

let

$P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{pmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{pmatrix} = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

Now, P'=P Thus, P is a symmetric matrix

Now $A - A' = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} - \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

let, $Q = \frac{1}{2}(A - A')= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Q'= $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -$Q

thus, Q is a skew symmetric matrix

Representing A as sum of P and Q:

$A = P + Q = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} =$ A

(iii) $\begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$

$A = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$

$A' = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$

Now, $A + A' = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$

$A + A' = \begin{pmatrix} 6 & 1 & -5 \\ 1 & 4 & 4 \\ 5 & 4 & 4 \end{pmatrix}$

$P = \frac{1}{2}\ (A + A')$

$P = \frac{1}{2}\ (\begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix})$

$P = \begin{pmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{pmatrix}$

Now,P'= $\begin{pmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{pmatrix}$ =P

$Thus P is Symmetric matrix$

Now $(A - A') = \begin{pmatrix} 0 & 5 & 3 \\ -5 & 0 & 0 \\ -3 & -6 & 0 \end{pmatrix}$

therefore, $Q = \frac{1}{2}\ (A - A')$

$Q = \frac{1}{2} \begin{pmatrix} 0 & 5 & 3 \\ -5 & 0 & 0 \\ -3 & -6 & 0 \end{pmatrix}$ $Q = \begin{pmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 0 \\ -\frac{3}{2} & -3 & 0 \end{pmatrix}$

$Q' = \begin{pmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & 0 \\ \frac{3}{2} & 3 & 0 \end{pmatrix} = -Q$

therefore Q is skew-Symmetric Matrix

Now,

$P + Q = \begin{pmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{pmatrix} + \begin{pmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 0 \\ -\frac{3}{2} & -3 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$ = A

thus A is written as $P + Q$ where P and Q are symmetric and skew-symmetric matrices respectively.

(iv) $\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}$

then, $A' = \begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix}$

let

$P = \frac{1}{2}(A + A')$

$P = \frac{1}{2}(\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix})$

$P = \frac{1}{2}(\begin{pmatrix} 2 & 4 \\ 4 & 4 \end{pmatrix})$

$P = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$

now, $\quad P' = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} =$ P

therefore P is symmetric matrix

Now, let$Q = \frac{1}{2}(A - A')$

$Q = \frac{1}{2}(\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix})$

$Q = \frac{1}{2}(\begin{pmatrix} 0 & 6 \\ -6 & 0 \end{pmatrix})$

$Q = \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix} = Q'$

therefore Q is a skew-symmetric matrix

now,

$P + Q = (\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix})$

$P + Q = \begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} = A$

thus A is written as $P + Q$ where P and Q are symmetric and skew-symmetric matrices respectively.