The Z-score formula is a powerful tool used in statistics to standardise and compare data. It allows us to understand how a particular data point relates to the average and spread of a dataset. By calculating the Z-score, we can determine how many standard deviations a value is away from the mean. This helps in identifying outliers, making comparisons, and conducting hypothesis testing.

In this maths formula article, we will delve into the intricacies of the Z-Score, providing a step-by-step guide to its calculation and exploring its significance in data analysis, hypothesis testing, and real-world applications.

Z Score Definition

A Z-score serves as a statistical measure indicating the deviation of a raw score from the mean of a distribution. It finds application in hypothesis testing through Z-tests and aids in determining the likelihood of a random variable falling within a specified range when constructing prediction intervals.

The Z-score is a statistical metric that quantifies how many standard deviations a score deviates from the mean of a distribution. Essentially, it measures the distance between a score and the mean. A positive Z-score indicates that the score is above the mean, while a negative Z-score signifies that the score is below the mean. Conversely, a Z-score of 0 indicates that the data point is equivalent to the mean.

Z Score Formula

In order to calculate a z-score, one must have information about the mean and standard deviation.

The z-score formula is utilised when both the population mean and population standard deviation are known, and it can be expressed in the following manner:

$z = \frac{x-\mu}{\sigma}$

Where:

- $\mu =$ population mean,
- $\sigma =$ population standard deviation, and
- $x =$ raw score.

When the population parameters are unknown, the z-score can be approximated using the sample mean and standard deviation. In such cases, the z-score formula undergoes modification, resulting in the following adaptation:

$z = \frac{x-\bar{x}}{S}$

Where:

- $\bar{x} =$ sample mean,
- $S =$ sample standard deviation, and
- $x =$ raw score.

How to Calculate Z Score?

To calculate the Z-Score, follow these steps:

- **Determine the data point of interest:** Identify the value you want to standardise or compare to a distribution.
- **Calculate the mean (μ) and standard deviation (σ) of the dataset:** Find the average value and the measure of variability for the dataset that the data point belongs to.
- **Subtract the mean from the data point:** Subtract the mean (μ) from the value of interest.
- **Divide the result by the standard deviation:** Divide the difference obtained in step 3 by the standard deviation (σ).
- **Obtain the Z-Score:** The result of the division in step 4 represents the Z-Score for the data point.

Z-Score Confidence Intervals

A confidence interval is a statistical tool utilised to indicate the likelihood of a specific parameter falling within a particular range. In normally distributed data, approximately 68% of the values will lie between one standard deviation above and below the mean, 25% between two standard deviations, and 99% between three standard deviations. The process of obtaining the Z-score through confidence intervals involves the following steps:

- Transform the confidence interval into decimals.
- Determine the alpha level using the confidence interval, $\alpha = 1-$ confidence interval.
- Divide the alpha level by 2 to obtain the actual alpha level.
- Subtract the alpha level from 1 to derive the required area.
- Utilise the z-score table to find the corresponding z-value based on the area obtained in the previous step.

Z-Score for 99 Confidence Interval

For a 99% confidence interval, the Z-score indicates that approximately 99% of the observations fall within the range of 3 standard deviations above and 3 standard deviations below the mean. This can be expressed as follows:

- 99% confidence level in decimals is 0.99.
- Alpha level: $\alpha = \frac{(1-0.99)}{2} = 0.005$.
- Area: $1 - 0.005 = 0.995$.
- Z-score for 99% confidence interval $= 2.57$.

Z-Score for 95 Confidence Interval

By employing identical steps, one can compute the Z-score for a 95% confidence interval, which will fall within the range of 2 and -2 on the normal distribution curve.

- 95% confidence level in decimals is 0.95.
- Alpha level: $\alpha = \frac{(1-0.95)}{2} = 0.025$.
- Area: $1 - 0.025 = 0.975$.
- Z-score for 95% confidence interval $= 1.96$.

Let's now explore a few examples to help us better understand the formula to find the z-score.

Z Score Formula Solved Examples

Example 1. The heights of a group of students are normally distributed with a mean of 165 cm and a standard deviation of 8 cm. What is the Z-Score for a student with a height of 180 cm?

Solution.

Given:

$X = 180$ cm

$\mu = 165$ cm

$\sigma = 8$ cm

Calculate the Z-Score using the formula:

$$Z = \frac{(X-\mu)}{\sigma}$$

$$\Rightarrow Z = \frac{(180-165)}{8}$$

$$\Rightarrow Z = \frac{15}{8}$$

$$\Rightarrow Z = 1.875$$

Answer: The Z-Score for a student with a height of 180 cm is 1.875.

Example 2. The average weight of a sample of 50 apples is 120 grams with a standard deviation of 10 grams. What is the Z-Score for an apple weighing 135 grams?

Solution.

Given:

$X = 135$ grams

$\mu = 120$ grams

$\sigma = 10$ grams

Calculate the Z-Score using the formula:

$$Z = \frac{(X-\mu)}{\sigma}$$

$$\Rightarrow Z = \frac{(135-120)}{10}$$

$\Rightarrow Z = \frac{15}{10}$

$\Rightarrow Z = 1.5$

Answer: The Z-Score for an apple weighing 135 grams is 1.5.

> **Example 3.** The test scores of a class are normally distributed with a mean of 75 and a standard deviation of 5. What percentage of students scored above 85?

Solution.

Given:

$X = 85$

$\mu = 75$ cm

$\sigma = 5$ cm

Calculate the Z-Score using the formula:

$Z = \frac{(X-\mu)}{\sigma}$

$\Rightarrow Z = \frac{(85-75)}{5}$

$\Rightarrow Z = \frac{10}{5}$

$\Rightarrow Z = 2$

Use a Z-Score table or calculator to find the percentage corresponding to a Z-Score of 2.

From the table, it is approximately 0.9772.

Answer: Approximately 97.72% of students scored above 85.

> **Example 4.** The mean score on a standardised test is 500 with a standard deviation of 100. What score corresponds to the 90th percentile?

Solution.

Given:

$\mu = 500$

$\sigma = 100$

Find the Z-Score corresponding to the 90th percentile using a Z-Score table or calculator.

The Z-Score is approximately 1.28.

Calculate the score using the formula:

$Z = \frac{(X-\mu)}{\sigma}$

$\Rightarrow 1.28 = \frac{(X-500)}{100}$

$\Rightarrow 128 = X - 500$

$\Rightarrow X = 628$

Answer: The score corresponding to the 90th percentile is 628.

Example 5. The temperatures in a city are normally distributed with a mean of $25°C$ and a standard deviation of $3°C$. What percentage of days have temperatures below $20°C$?

Solution.

Given:

$X = 20°C$

$\mu = 25°C$

$\sigma = 3°C$

Calculate the Z-Score using the formula:

$Z = \frac{(X-\mu)}{\sigma}$

$\Rightarrow Z = \frac{(20-25)}{3}$

$\Rightarrow Z = -\frac{5}{3}$

$\Rightarrow Z = -1.67$

Use a Z-Score table or calculator to find the percentage corresponding to a Z-Score of -1.67.

From the table, it is approximately 0.0475.

Answer: Approximately 4.75% of days have temperatures below $20°C$.

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