Maharashtra State Board

Class X Mathematics – Geometry – Paper II

Board Paper 2019

Time: 2 hours **Maximum Marks: 40**

Note:

(i) All questions are compulsory

(ii) Use of calculator is not allowed

(iii) Figures to the right of questions indicate full marks.

(iv) Draw proper figures for answers wherever necessary

(v) The marks of construction should be clear and distinct. Do not erase them.

(vi) While writing any proof, drawing relevant figure is necessary. Also the proof should be consistent, with the figure.

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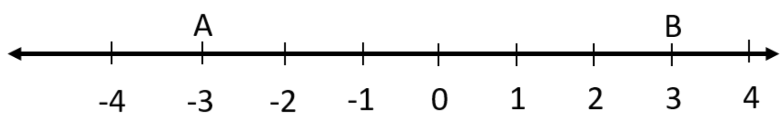
1. (A) Solve the following questions (Any *four*) : **4**

(i) If $\Delta ABC \sim \Delta PQR$ and $\angle A = 60°$, then $\angle P =$?

(ii) In right – angled ΔABC, if $\angle B = 90°$, $AB = 6$, $BC = 8$, then find AC.

(iii) Write the length of largest chord of a circle with radius 3.2 cm.

(iv) From the given number line, find d(A,B) :



(v) Find the value of $\sin 30° + \cos 60°$.

(vi) Find the area of a circle of radius 7 cm.

(B) Solve the following questions (Any *two*): **4**

(i) Draw seg AB of length 5.7 cm and bisect it.

(ii) In right-angled triangle PQR, if $\angle P = 60°$, $\angle R = 30°$ and $PR = 12$, then find the values of PQ and QR.

(iii) In a right circular cone, if perpendicular height is 12 cm and radius is 5 cm, then find its slant height.

2. (A) Choose the correct alternative : 4

(i) Δ ABC and ΔDEF are equilateral triangles. If A (Δ ABC): A (ΔDEF) = 1: 2 and AB = 4, then what is the length of DE?

(a) $2\sqrt{2}$ (b) 4

(c) 8 (d) $4\sqrt{2}$

(ii) Out of the following which is a Pythagorean triplet?

(a) (5,12,14) (b) (3,4,2)

(c) (8,15,17) (d) (5,5,2)

(iii) $\angle$ACB is inscribed in arc ACB of a circle with centre O. if $\angle ACB = 65°$, find m (arc ACB):

(a) 130° (b) 295°

(c) 230° (d) 65°

(iv) $1 + \tan^2\theta = ?$

(a) $\text{Sin}^2\theta$ (b) $\sec^2\theta$

(c) $\text{Cosec}^2\theta$ (d) $\cot^2\theta$

(B) Solve the following questions (Any two) : 4

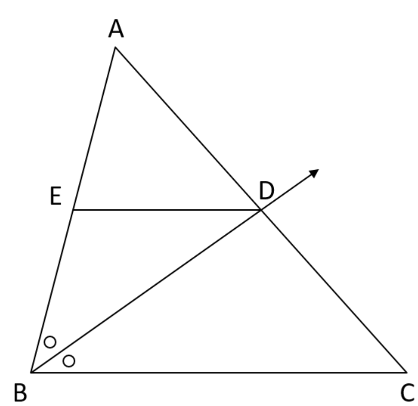
(i) Construct tangent to a circle with centre A and radius 3.4 cm at any point P on it.

(ii) Find slope of a line passing through the points A (3, 1) and B (5, 3).

(iii) Find the surface area of a sphere of radius 3.5 cm.

3. (A) Complete the following activities (Any two) : 4

(i)



In Δ ABC, ray BD bisects $\angle$ABC.

If A -D-C, A-E-B and seg ED $\parallel$ side BC, then prove that:

$$\frac{AB}{BC} = \frac{AE}{EB}$$

Proof:

In Δ ABC, ray BD is bisector of ∠ABC.

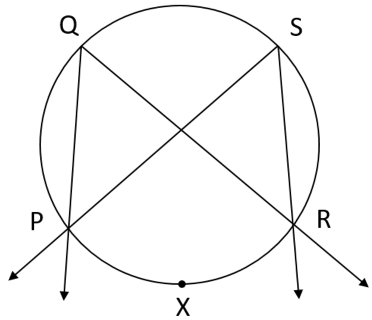
$\therefore \quad \frac{AB}{BC} = \boxed{\frac{\dots\dots\dots}{\dots\dots\dots}}$ (I) (by angle bisector theorem)

In Δ ABC, seg DE ∥ side BC.

$\therefore \quad \frac{AE}{EB} = \frac{AD}{DC}$ (II) $\boxed{}$

$\therefore \quad \frac{AB}{\boxed{}} = \frac{\boxed{}}{EB}$........ (From I and II)

(ii)



Prove that, angles inscribed in the same arc are congruent.

Given: ∠PQR and ∠PSR are inscribed in the same arc.

Arc PXR is intercepted by the angles

To prove:

$$\angle PQR \cong \angle PSR$$

Proof

$m\angle PQR = \frac{1}{2} m(\text{arc } PXR)$ .... (I) $\boxed{}$

$m\angle\boxed{} = \frac{1}{2} m\ (\text{arc } PXR)$ ...... (II) $\boxed{}$

$\therefore \quad m\angle\boxed{} = m\angle PSR$ (from I and II)

$\therefore \quad \angle PQR \cong \angle PSR$ (Angles equal in measure are congruent)

(iii) How many solid cylinders of radius 6 cm and height 12 cm can be made by melting a solid sphere of radius 18 cm?

Activity: Radius of the sphere, r = 18 cm

For cylinder, radius R = 6 cm, height H = 12 cm

$\therefore$ Number of cylinders can be made = $\dfrac{\text{Volume of the sphere}}{\boxed{}}$

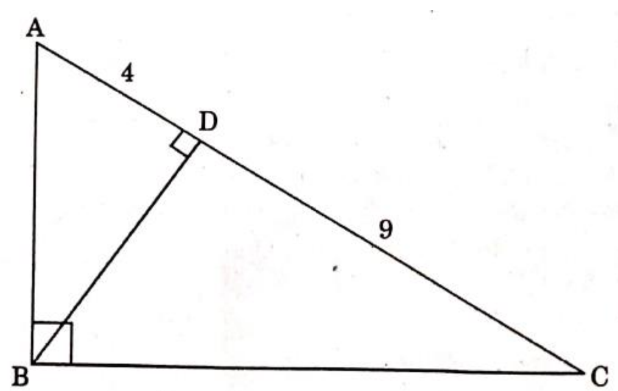
$$= \frac{\frac{4}{3}\pi r^3}{\boxed{}}$$

$$= \frac{\frac{4}{3} \times 18 \times 18 \times 18}{\boxed{}}$$

$$= \boxed{}$$

(B) **Solve the following questions (Any two):** **4**

(i)



In right-angled Δ ABC; BD ⊥ AC.

If AD = 4, DC = 9, then find BD.

(ii) Verify whether the following points are collinear or not :

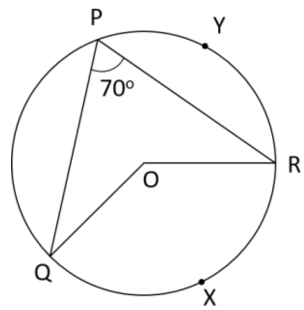
A (1,-3), B (2,-5), C (-4, 7).

(iii) if sec θ = $\dfrac{25}{7}$, then find the value of tan θ

4. Solve the following questions (Any three) : **9**

(i) In Δ PQR, seg PM is a median, PM = 9 and $PQ^2 + PR^2 = 290$. Find the length of QR.

(ii)



In the given figure, O is centre of circle. $\angle QPR = 70°$ and m (arc PYR) = 160°, then find the value of each of the following:

(a) m (arc QXR)

(b) $\angle QOR$

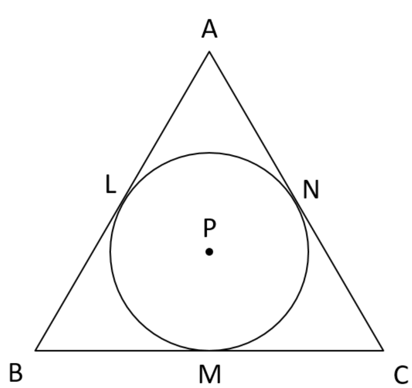
(c) $\angle PQR$

(iii) Draw a circle with radius 4.2 cm. Construct tangents to the circle from a point at a distance of 7 cm from the centre.

(iv) When an observer at a distance of 12 cm m from a tree looks at the top of the tree, the angle of elevation is 60°. What is the height of the tree? $\left(\sqrt{3} = 1.73\right)$

5. Solve the following questions (Any one) : **4**

(i)

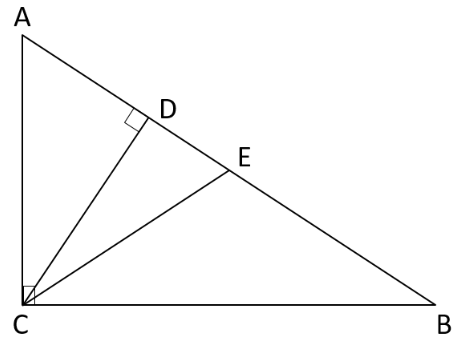


A circle with centre P is inscribed in the Δ ABC. Side AB, side BC and side AC touch the circle at points L, M and N respectively. Radius of the circle is r.

Prove that:

$$A(\Delta ABC)\frac{1}{2}(AB + BC + AC) \times r$$

(ii)



In Δ ABC, $\angle ACB = 90°$. Seg CD $\perp$ side AB and seg CE is angle bisector of $\angle ACB$.

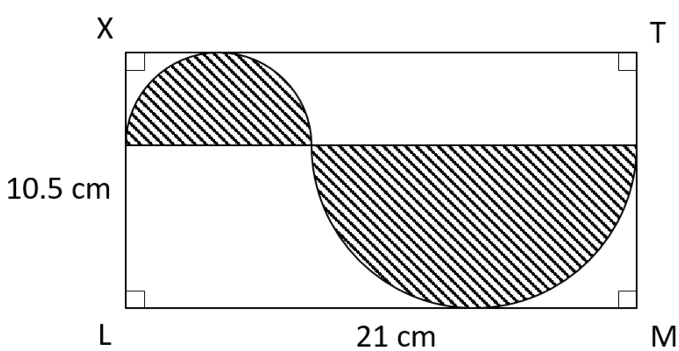
Prove that:

$$\frac{AD}{BD} = \frac{AE^2}{BE^2}$$

6. Solve the following questions (Any one) : **3**

(i) Show that the points (2, 0), (-2, 0) and (0, 2) are the vertices of triangle. Also state with reason the type of the triangle.

(ii)



In the above figure, □XLMT is a rectangle is a rectangle. LM = 21 cm, XL = 10.5 cm. diameter of the smaller semicircle is half the diameter of the larger semicircle. Find the area of non-shaded region.

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Board Paper 2019 – Solution

1. (A)

(i)

In $\Delta ABC \sim \Delta PQR$

$\angle A = \angle P \ldots (\text{c.p.c.t.})$

$\therefore \angle P = 60^{\circ}$

(ii)

In ΔABC

$\Rightarrow \angle B = 90^{\circ}$,

By Pythagoras theorem, we get

$\Rightarrow AC^2 = AB^2 + BC^2$

$\Rightarrow AC = \sqrt{AB^2 + BC^2}$

$\Rightarrow AC = \sqrt{6^2 + 8^2}$

$\Rightarrow AC = \sqrt{100} = 10$

(iii)

The largest chord of circle with radius 3.2 cm, will be its diameter.
Therefore, the length of diameter = twice the radius = $2 \times 3.2 = 6.4$ cm

(iv)

d(A,B) = (3) – (–3) = 6

(v)

$\sin 30^{\circ} + \cos 60^{\circ}$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

(vi)

Area of circle = πr^2

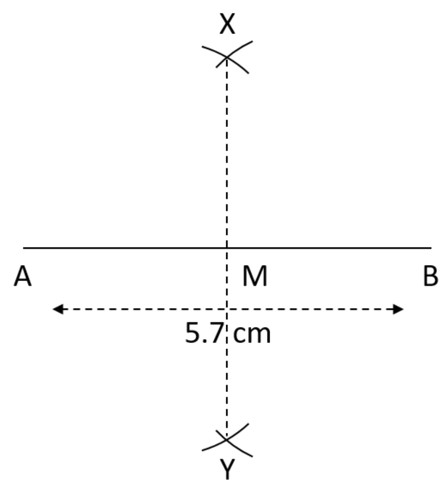
$= \frac{22}{7} \times 7^2$

$= 154 \text{ cm}^2$

1. (B)

(i)

a. Draw a line segment AB, measuring 5.7 cm
b. Take compass with the measure more than half of AB, put the steel end on A and make arcs above and below.
c. Take the same compass measurement and by keeping the steel head on B, cut the previously made arcs, name the point of intersection X and Y.
d. Join X Y and mark the point M, where the line XY cuts the line segment AB
e. M is the midpoint of the line AB and XY its bisector.



(ii)

In ΔPQR

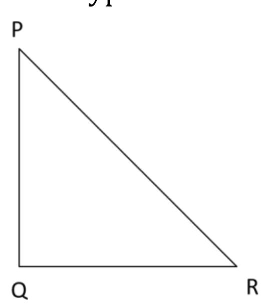
$\Rightarrow \angle P = 60^{o}$

$\Rightarrow \angle R = 30^{o}$

$\Rightarrow \angle Q = 180° - (\angle P + \angle R) = 90^{o}$ ... (Sum of all the angles of Δ is 180°)

$\therefore$ ΔPQR is 30° – 60° – 90° triangle.

PR is hypotenuse = 12 cm



$$\Rightarrow QR = \frac{\sqrt{3}}{2} PR \ldots (\text{side opp. to } 60°)$$

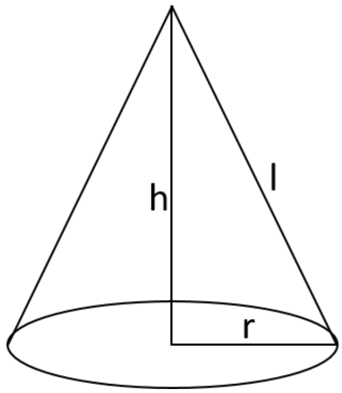
$$\Rightarrow QR = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3} \text{ cm}$$

$$\Rightarrow PQ = \frac{1}{2} PR \ldots (\text{side opp. to } 30°)$$

$$\Rightarrow PQ = \frac{1}{2} \times 12 = 6 \text{ cm}$$

(iii)

In a cone we have



Given that

h = 12cm

r = 5cm

So, by Pythagoras theorem, we can write

$$
\begin{aligned}
l &= \sqrt{r^2 + h^2} \\
&= \sqrt{5^2 + 12^2} \\
&= \sqrt{25 + 144} \\
&= \sqrt{169} \\
&= 13
\end{aligned}
$$

Therefore the slant height of a right circular cone is 13 cm.

2. (A)

(i)

Δ ABC and Δ DEF are equilateral triangles.

So, they are similar

By the theorem of areas of similar triangles, we have

$$\Rightarrow \frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \frac{1}{2} = \frac{4^2}{DE^2}$$

$$\Rightarrow DE^2 = 4 \times 2$$

$$\Rightarrow DE = 2\sqrt{2}$$

Correct option (a)

(ii)

A Pythagorean triplet should satisfy the condition that is

Square of large number = sum of squares of other two numbers

From the options given only option (c) satisfies the condition

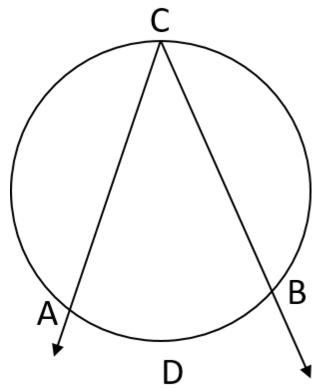
$17^2 = 289$

$8^2 + 15^2 = 289$

$\Rightarrow 8^2 + 15^2 = 17^2$

Correct option (c)

(iii)



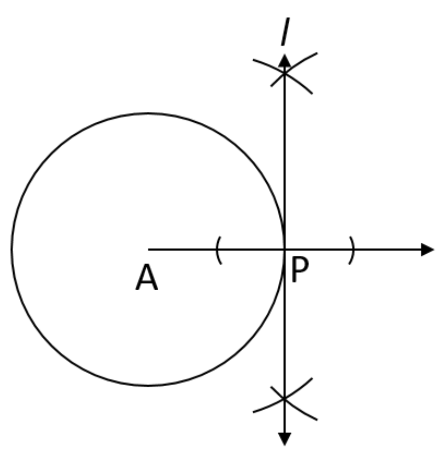
By inscribed angle theorem,
$2\angle ACB = m(\text{arc } ADB)$
$\Rightarrow m(\text{arc } ADB) = 2 \times 65^\circ = 130^o$
$\Rightarrow m(\text{arc } ADB) + m(\text{arc } ACB) = 360^o$... (Total sum of arc in a circle)
$\Rightarrow m(\text{arc } ACB) = 360^o - 130^o = 230^o$
Correct option (c)

(iv)
By trigonometric identities we know that,
$1 + \tan^2\theta = \sec^2\theta$
Correct option (b)

2. (B)

(i)

a. Take 3.4 cm in compass and draw a circle with center A.
b. Take any point P on it.
c. Draw a ray AP and extend it.
d. Now take an appropriate measurement in compass and make arcs on both sides of P.
e. From this arcs make arcs above and below and get intersection of the arcs.
f. Join the intersection of arcs, and extend them, this is the required tangent.



(ii)

The slope of line passing through points (x_1, y_1) and (x_2, y_2) is given by

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

So, slope of AB is given by

$$\text{slope} = \frac{3-1}{5-3} = \frac{2}{2} = 1$$

(iii)

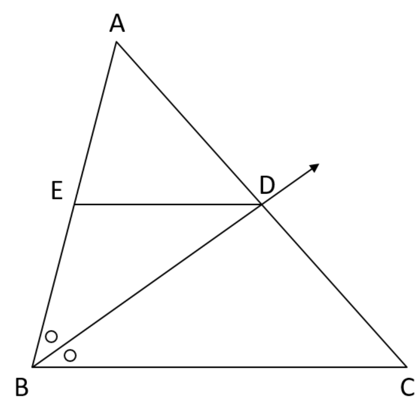
Surface area of sphere is given by $4\pi r^2$.

r = 3.5 cm

$$\text{S.A.} = 4 \times \frac{22}{7} \times 3.5^2 = 154 \text{ cm}^2$$

3. (A)

(i)



Proof:

In Δ ABC, ray BD is bisector of ∠ABC.

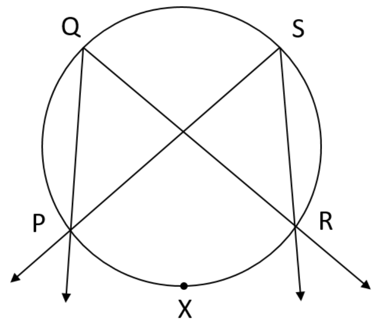
$$\therefore \quad \frac{AB}{BC} = \boxed{\frac{AD}{DC}} \quad \text{(I) (by angle bisector theorem)}$$

In Δ ABC, seg DE ∥ side BC.

$$\therefore \quad \frac{AE}{EB} = \frac{AD}{DC} \qquad \text{(II) } \boxed{\text{by Basic proportionality theorem}}$$

$$\therefore \quad \frac{AB}{\boxed{BC}} = \frac{\boxed{AE}}{EB} \text{.......} \quad \text{(From I and II)}$$

(ii)



Proof

$$m\angle PQR = \frac{1}{2} m(\text{arc } PXR) \text{ .... (I) } \boxed{\text{inscribed angle theorem}}$$

$$m\angle \boxed{PSR} = \frac{1}{2} m\ (\text{arc } PXR) \text{ ...... (II) } \boxed{\text{inscribed angle theorem}}$$

$\therefore$ $m\angle \boxed{PQR} = m\angle PSR$ (from I and II)

$\therefore$ $\angle PQR \cong \angle PSR$ (Angles equal in measure are congruent)

(iii)

Activity: Radius of the sphere, r = 18 cm

For cylinder, radius R = 6 cm, height H = 12 cm

$\therefore$ Number of cylinders can be made $= \dfrac{\text{Volume of the sphere}}{\boxed{\text{Volume of a soild cylinder}}}$

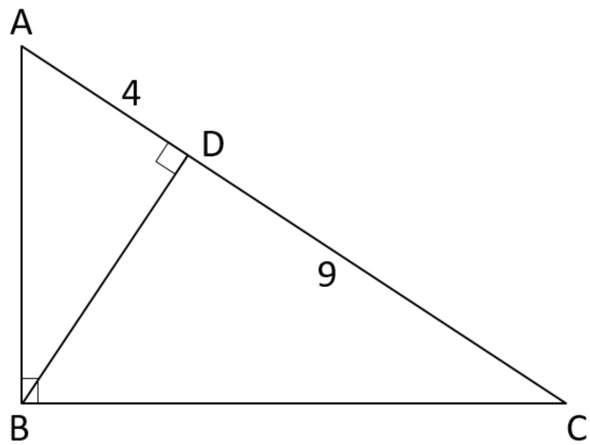
$$= \frac{\frac{4}{3}\pi r^3}{\boxed{\pi r^2 h}}$$

$$= \frac{\frac{4}{3} \times 18 \times 18 \times 18}{\boxed{6 \times 6 \times 12}}$$

$$= \boxed{18 \text{ cylinders}}$$

3. (B)

(i)



We are given a right angled triangle ABC, right angle at B and BD perpendicular to hypotenuse AC.
So, by similarity in right angled triangles we can say that

$\Delta ABC \sim \Delta ADB$

$\frac{AB}{AD} = \frac{BC}{DB} = \frac{AC}{AB} \ldots (\text{c.p.s.t.})$

$\Rightarrow \frac{AB}{4} = \frac{BC}{DB}$

$\Rightarrow AB = \frac{BC}{DB} \times 4 \ldots (1)$

Also

$\Delta ABC \sim \Delta BDC$

$\frac{AB}{BD} = \frac{BC}{DC} = \frac{AC}{BC} \ldots (\text{c.p.s.t.})$

$\Rightarrow \frac{AB}{BD} = \frac{BC}{9}$

$\Rightarrow \frac{\frac{BC}{DB} \times 4}{BD} = \frac{BC}{9} \ldots (\text{from } 1)$

$\Rightarrow \frac{BC \times 4}{BD^2} = \frac{BC}{9}$

$\Rightarrow 4 \times 9 = BD^2$

$\Rightarrow BD = 2 \times 3 = 6$

(ii)

If the sum of any two distances out of d(A,B), d(B,C) and d(A,C) is equal to the third, then the three points A, B and C are collinear.
Therefore, we will find d(A,B), d(B,C) and d(A,C).
Given A(1,-3), B(2,-5), C(-4,7)

$$\therefore d(AB) = \sqrt{(1-2)^2 + (-3+5)^2} = \sqrt{5}$$

$$\therefore d(AC) = \sqrt{(1+4)^2 + (-3-7)^2} = 5\sqrt{5}$$

$$\therefore d(BC) = \sqrt{(2+4)^2 + (-5-7)^2} = 6\sqrt{5}$$

So,

$$d(BC) = d(AB) + d(AC)$$

Therefore A, B and C are collinear

(iii)

Given that

$$\sec\theta = \frac{25}{7}$$

Using trigonometric identity

$$1 + \tan^2\theta = \sec^2\theta$$

we get,

$$\tan^2\theta = \sec^2\theta - 1$$

$$= \left(\frac{25}{7}\right)^2 - 1$$

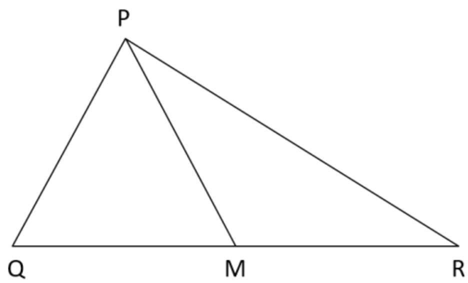
$$= \frac{576}{49}$$

$$\Rightarrow \tan^2\theta = \frac{576}{49}$$

$$\therefore \tan\theta = \frac{24}{7}$$

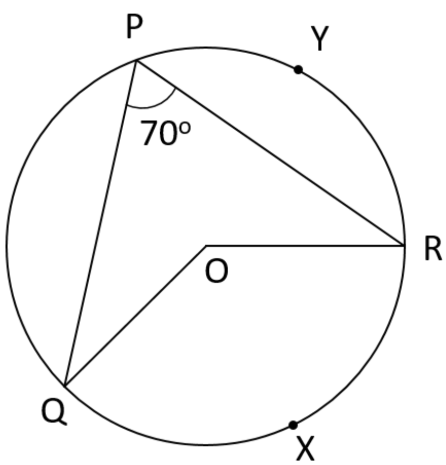
4.

(i)



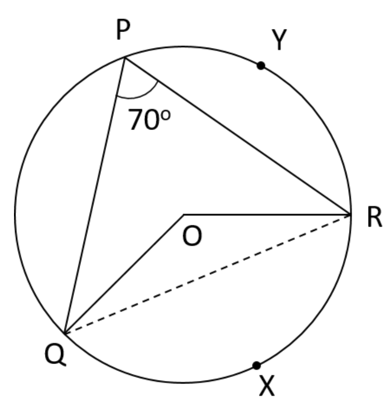
In ΔPQR, we have PM as median
so, by Apollonius theorem we have,
$PQ^2 + PR^2 = 2PM^2 + 2QM^2$
$\therefore\ 290 = 2(9)^2 + 2QM^2$
$\therefore\ 2QM^2 = 128$
$\therefore\ QM^2 = 64$
$\therefore$ QM = 8 units

(ii)



Given that
$\angle QPR = 70°$ and m (arc PYR) = 160°

(a) m(arc QXR) = $2\angle QPR$ ... inscribed angle theorem
m(arc QXR) = 2(70°)
m(arc QXR) = 140°

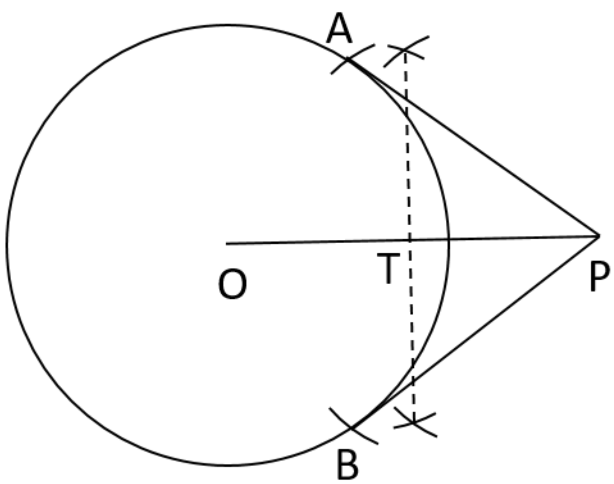


(b) $\angle QOR$ = m(arc QXR) ... measure of arc is equal to the measure of its central angle
$\angle QOR = 140°$

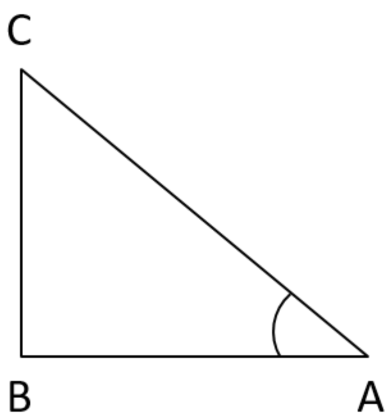
(c) $\angle PQR = \frac{1}{2}$ m(arc PYR) ... inscribed angle theorem
$\angle PQR = \frac{1}{2}(160°)$
$\angle PQR = 80°$

(iii)

(a) Take 4.2 cm in compass as a radius and draw a circle with center O.
(b) Take a point P such that d(OP) = 7cm
(c) Join OP and find its midpoint using perpendicular bisector, name it T.
(d) Take measure OT in compass, keep steel head at T and make 2 arcs on circle, above and below.
(e) Mark the intersection A and B, then join AP and BP which are the required tangents



(iv)



Let BC represent the height of tree,
Observer is at A, such that $\angle CAB = 60^{o}$
BA = 12 m
So in ΔABC

$$\tan 60^{\circ} = \frac{CB}{AB}$$

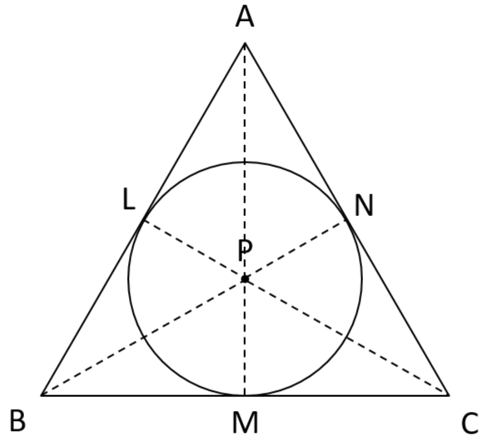
$$\Rightarrow \sqrt{3} = \frac{CB}{12}$$

$$\Rightarrow CB = 12\sqrt{3}\,\text{m} = 20.76\ \text{m}$$

5

(i)

Construction: join PA, PL, PN, PC, PM and PB



Now, AB, AC and BC can be considered tangents to the in circle, since it touches the circle at a single point.
PL, PM and PN being the radius of this circle
Also, tangent is perpendicular to radius
Hence,
$PL \perp AB$
$PN \perp AC$
$PM \perp BC$
Now in ΔAPB,

Area (ΔAPB) = $\frac{1}{2} \times PL \times AB$

$= \frac{1}{2} \times r \times AB$ ... (1)

Similarly in ΔAPC and ΔBPC, we can say

Area (ΔAPC) = $\frac{1}{2} \times r \times AC$ ...(2)

Area (ΔBPC) = $\frac{1}{2} \times r \times BC$ ...(3)

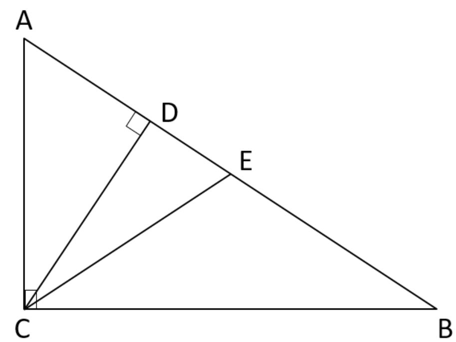
Adding (1), (2) and (3)
Area (ΔAPB) + Area (ΔAPC) + Area (ΔBPC) =

$\frac{1}{2} \times r \times AB + \frac{1}{2} \times r \times AC + \frac{1}{2} \times r \times BC$

Now,
Area (ΔAPB) + Area (ΔAPC) + Area (ΔBPC) = Area (ΔABC)

Area (ΔABC) = $\frac{1}{2} \times r \times \left(AB + AC + BC\right)$

(ii)



In ΔABC, $CD \perp AD$

So by the property of similar triangles in a right angled triangle we can say that

$\Delta\ ACB \sim \Delta\ ADC$

$$\frac{AC}{AD} = \frac{CB}{DC} = \frac{AB}{AC} \text{ ... (c.p.s.t.)}$$

$$\Rightarrow \frac{AC}{AD} = \frac{AB}{AC}$$

$$\Rightarrow AC^2 = AB \times AD$$

Also,

$\Delta\ ACB \sim \Delta\ CDB$

$$\frac{AC}{CD} = \frac{CB}{DB} = \frac{AB}{CB} \text{...(c.p.s.t.)}$$

$$\Rightarrow \frac{CB}{DB} = \frac{AB}{CB}$$

$$\Rightarrow CB^2 = AB \times DB$$

Also, CE is angle bisector

hence,

$$\frac{AC}{CB} = \frac{AE}{EB}$$

Squaring on both the sides, we get

$$\therefore \frac{AC^2}{CB^2} = \frac{AE^2}{EB^2}$$

$$\therefore \frac{AB \times AD}{AB \times DB} = \frac{AE^2}{EB^2}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AE^2}{EB^2}$$

6.

(i)

Let A(2,0), B(-2,0) and C(0,2)

So by distance formula, we get

$$d(AB) = \sqrt{(2+2)^2 + (0-0)^2} = 4\,\text{cm}$$

$$d(AC) = \sqrt{(2-0)^2 + (0-2)^2} = 2\sqrt{2}\,\text{cm}$$

$$d(BC) = \sqrt{(-2-0)^2 + (0-2)^2} = 2\sqrt{2}\,\text{cm}$$

$$d(AB) + d(AC) > d(BC)$$

$$d(AB) + d(BC) > d(AC)$$

$$d(BC) + d(AC) > d(AB)$$

As, sum of any two distance is greater than the third, points A, B and C denote a triangle

Also,

$$d(AC) = d(BC)$$

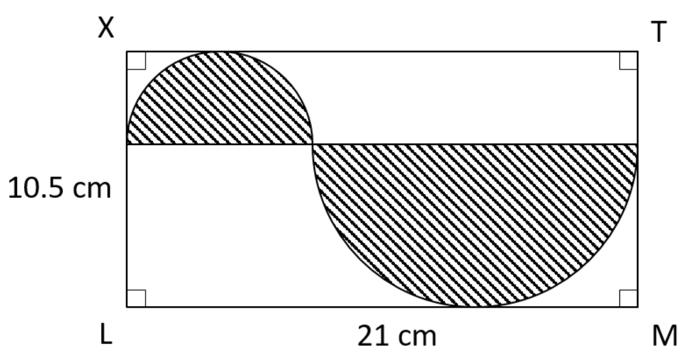
$$AB^2 = 16$$

$$AC^2 + BC^2 = 8 + 8 = 19$$

$$AB^2 = AC^2 + BC^2$$

So, we can say ΔABC is isosceles right angled triangle.

(ii)



Let the diameter of smaller circle be d.

then the diameter of larger circle will be 2d.

hence

d + 2d = 21

$\Rightarrow$ d = 7 cm

Hence area of smaller semi-circle is given by

$$\text{Area} = \frac{\pi}{8} d^2$$
$$= \frac{22}{7} \times \frac{1}{8} \times 7^2$$
$$= 19.25\ \text{cm}^2$$

And area of larger semi-circle is given by

$$\text{Area} = \frac{\pi}{8} (2d)^2$$
$$= \frac{22}{7} \times \frac{1}{8} \times 14^2$$
$$= 77\ \text{cm}^2$$

Now, area of non-shaded region = Area of rectangle – Area of shaded region

$\therefore$ Area of shaded region = $l \times b$ – (Area of two semi circles)

=220.5 – (19.25 + 77) .... ($l \times b = 21 \times 10.5 = 220.5$)

=124.25 cm^2