Maharashtra State Board

Class X Mathematics – Geometry – Paper II

Board Paper 2018

Time: 2 hours **Maximum Marks: 40**

Note:

(i) Solve all questions. Draw diagrams wherever necessary

(ii) Use of calculator is not allowed

(iii) Figures to thy right indicate full marks.

(iv) Marks of constructions should be distinct. They should not be rubbed off.

(v) Diagram is essential for writing the proof of the theorem.

1. Attempt any five sub-questions from the following. **5**

(i) ΔDEE ~ ΔMNK. If DE = 5 and MN = 6, then find the value of $\frac{A(\Delta\ DEF)}{A(\Delta MNK)}$.

(ii) If two circles with radii 8 cm and 3 cm respectively touch externally, then find the distance between their centres.

(iii) Find the length of the altitude of an equilateral triangle with side 6 cm.

(iv) If $\theta = 45^{\circ}$, then find tan θ.

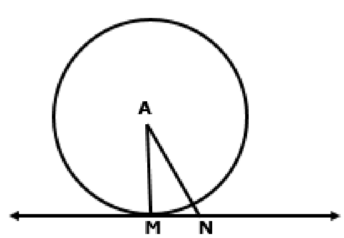
(v) Slope of a line is 3 and y intercept is –4. Write the equation of a line.

(vi) Using Euler's formula, find V, if E = 30, F = 12.

2. Attempt any four sub-questions from the following: **8**

(i) The ratio of the areas of two triangles with the common base is 10 : 7. Height of the larger triangle is 15 cm, then find the corresponding height of the smaller triangle.

(ii) In the following figure, point 'A' is the centre of the circle. Line MN is tangent at point M. If AN= 16 cm and MN = 8 cm. Determine the radius of the circle.



(iii) Draw $\angle XYZ$ of measure 50° and bisect it.

(iv) If $\cos\theta = \frac{24}{25}$, where θ is an acute angle. Find the value of $\sin\theta$.

(v) The volume of a cube is 216 cm^3. Find its side.

(vi) The radius and slant height of a cone are 10 cm and 30 cm respectively. Find the curved surface area of that cone ($\pi = 3.14$)

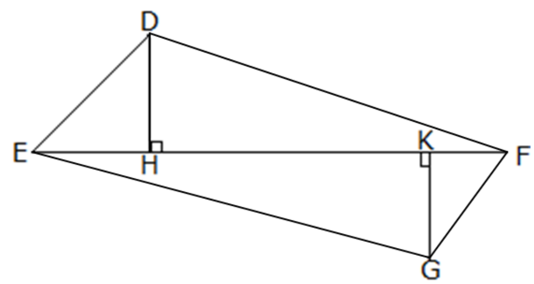
3. Attempt any three sub-questions from the following: 9

(i) In the following figure, seg DH ⊥ seg EF and seg GK ⊥ seg EF. If DH = 12 cm, GK = 20 cm and $\Delta(\Delta DEF) = 300$ cm^2, then find:

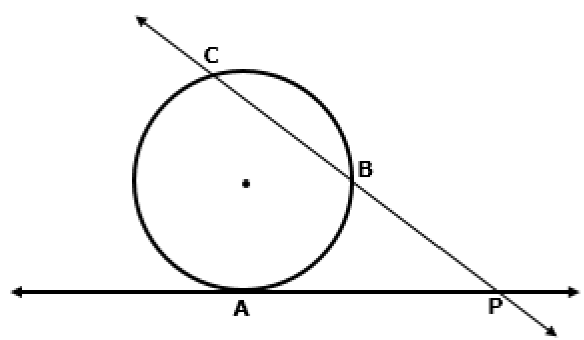
(i) EF

(ii) A(ΔGEF)

(iii) A(▭DFGE)



(ii) In the following figure, ray PA is tangent to the circle at A and PBC is a secant. If AP = 18, BP = 10, then find BC.



(iii) Draw the circle with centre C and radius 3.3 cm. Take a point B at a distance 6.6 cm from the centre C. Draw tangents to the circle from the point B.

(iv) Show that: $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A.$

(v) Write the equation of the line passing through A (−2, −3) and B (−4, 7) in the form of ax + by + c = 0.

4. Attempt any two sub-questions from the following: **8**

(i) Prove that, "the lengths of the two tangent segments to a circle drawn from an external point are equal".

(ii) A tree is broken by the wind. The top of that tree struck the ground at an angle of 30^{o} and at a distance of 30 m from the root. Find the height of the whole tree. $(\sqrt{3} = 1.73)$

(iii) A (5, 4), B (–3, –2) and C (1, –8) are the vertices of triangle ABC. Find the equation of median AD.

5. Attempt any two sub-questions from the following: **10**

(i) Prove that, in a right-angle triangle, the square of hypotenuse is equal to the sum of the square of remaining two sides.

(ii) ΔSHR ~ ΔSVU, in ΔSHR, SH = 4.5 cm, HR = 5.2 cm, SR = 5.8 cm and $\frac{SH}{SV} = \frac{3}{5}$,

Construct ΔSVU.

(iii) If 'V' is the volume of a cuboid of dimensions α × b × c and 'S' is its surface area, then prove that:

$$\frac{1}{V} = \frac{2}{S}\left[\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right]$$

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Class X Mathematics – Geometry – Paper II
Board Paper 2018 – Solution

1.

i. Given that $\Delta DEF \sim \Delta MNK$, DE = 5 and MN = 6

We know that the ratios of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\frac{A(\Delta DEF)}{A(\Delta MNK)} = \frac{DE^2}{MN^2}$$

$$\frac{A(\Delta DEF)}{A(\Delta MNK)} = \frac{5^2}{6^2}$$

$$\frac{A(\Delta DEF)}{A(\Delta MNK)} = \frac{25}{36}$$

ii. Let r_1 and r_2 be the radius of given circles.

$r_1 = 8$ cm and $r_2 = 3$ cm.

Distance between their centres = $r_1 + r_2$

= 8 + 3

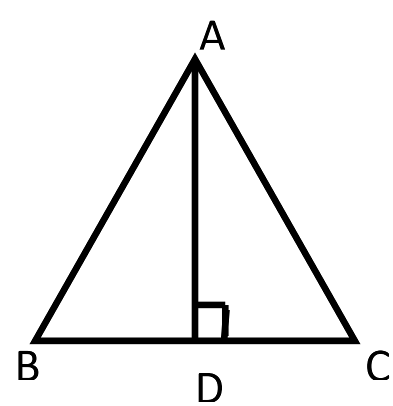
= 11 cm

iii. Let ABC be an equilateral triangle with side 6 cm.

Hence, ΔADC is 30°-60°-90°.

Because $\angle ACD = 60°$ ....(angles of equilateral triangle)

AD is opposite to 60°.



$$AD = \frac{\sqrt{3}}{2} \times AB$$

$$AD = \frac{\sqrt{3}}{2} \times 6$$

$$AD = 3\sqrt{3} \text{ cm}$$

Hence, altitude of equilateral triangle is $3\sqrt{3}$ cm.

iv. Given that $\theta = 45°$

$\tan 45° = 1$

v. Given that slope of a line = m = 3 and y-intercept = c = −4

Then, equation of line is y = mx + c

$\therefore$ y = 3x − 4

vi. Given that E = 30, F = 12.

Using Euler's formula,

F + V = E + 2

12 + V = 30 + 2

V = 32 − 12

V = 20

2.

i. Let A_1 and A_2 be the areas of the larger and smaller triangles respectively.

$$\frac{A_1}{A_2} = \frac{10}{7}$$

Let the corresponding heights be h_1 and h_2 respectively.

$$\frac{A_1}{A_2} = \frac{h_1}{h_2}$$

$$\therefore \frac{10}{7} = \frac{15}{h_2} \qquad (\because h_1 = 15 \text{ cm})$$

$$\therefore h_2 = \frac{15 \times 7}{10}$$

$$\therefore h_2 = 10.5 \text{ cm}$$

ii. From the given figure,

AM is perpendicular to MN (tangent is perpendicular to radius)

In ΔAMN,,

$$AM^2 + MN^2 = AN^2 \quad \text{(Pythagoras theorem)}$$

$$\therefore AM^2 + 8^2 = 16^2$$

$$\therefore AM^2 = 256 - 64$$

$$\therefore AM^2 = 192$$

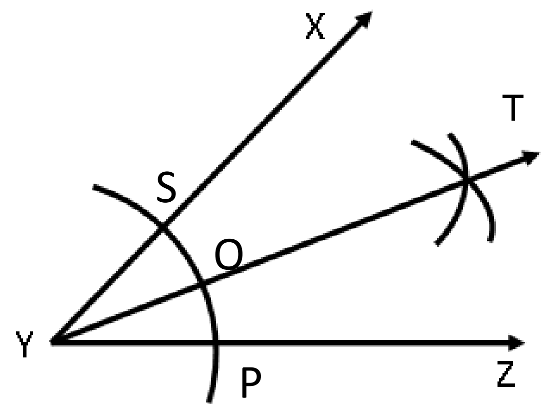
$$\therefore AM = \sqrt{192} = \sqrt{64 \times 3}$$

$$\therefore AM = 8\sqrt{3} \text{ cm}$$

Radius of a circle is $8\sqrt{3}$ cm.

iii. Steps of construction:

1) Draw an $\angle XYZ = 50^\circ$.
2) Now place the point of a compass on point Y and with any convenient radius draw an arc to cut rays YX and YZ. Name the points of intersection as S and P respectively.
3) Place the point of the compass at S and taking a convenient radius, draw an arc inside the angle. Using the same distance, draw another arc inside the angle from the point P, to cut the previous arc.
4) Name the point of intersection as point T. Now draw rat YT. Ray YT is the bisector of $\angle XYZ$.



iv. Given that $\cos\theta = \frac{24}{25}$

Using $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \left(\frac{24}{25}\right)^2 = 1$$

$$\therefore \sin^2\theta = 1 - \left(\frac{24}{25}\right)^2$$

$$\therefore \sin^2\theta = 1 - \frac{576}{625}$$

$$\therefore \sin^2\theta = \frac{49}{625}$$

$$\therefore \sin\theta = \pm\frac{7}{25}$$

$$\therefore \sin\theta = \frac{7}{25} \qquad (\cos \text{ is positive}, \theta \text{ is acute angle})$$

v. Given that Volume of a cube = 216 cm^3

Let x be the side of a cube.

$\therefore x^3 = 216$

$\therefore x^3 = 6^3$

$\therefore x = 6$ cm

Hence, side of a cube is 6 cm.

vi. Given that r = 10 cm and l = 30 cm of a cone.

Hence, Curved surface area of a cone

$= \pi rl$

$= 3.14 \times 10 \times 30$

$= 942 \text{ cm}^2$

3.

i. From the given figure,

Area of $\Delta DEF = \frac{1}{2} \times EF \times DH$

$\therefore 300 = \frac{1}{2} \times EF \times 12$

$\therefore EF = \frac{300 \times 2}{12} = 50 \text{ cm}$

Area of $\Delta GEF = \frac{1}{2} \times EF \times GK = \frac{1}{2} \times 50 \times 20 = 500 \text{ cm}^2$

$\therefore$ Area of DFGE = Area of ΔDEF + Area of ΔGEF = $(300 + 500) \text{ cm}^2 = 800 \text{ cm}^2$

ii. From the given figure, using tangent secant property,

$BP \times PC = AP^2$

BP = 10 and AP = 18

$\therefore 10 \times PC = 18^2$

$\therefore PC = 32.4$

$\therefore BC = PC - BP$

$\therefore BC = 32.4 - 10$

$\therefore$ BC = 22.4 units

iii. Steps of construction:

1) Construct a circle with centre C and radius 3.3 cm.

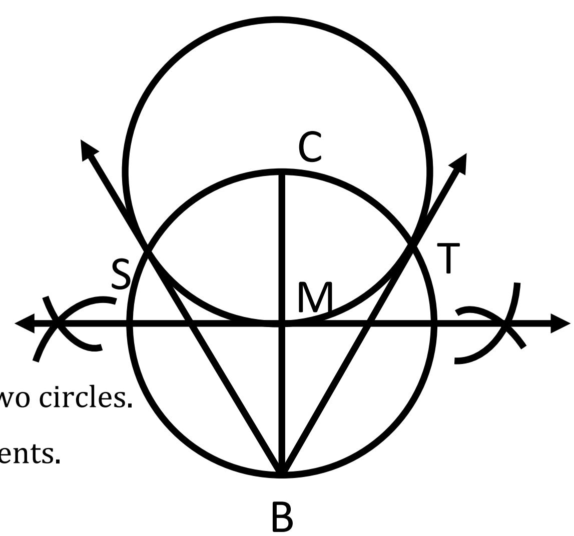
2) Take a point B such that CB = 6.6 cm.

3) Obtain midpoint M of seg CB.

4) Draw a circle with centre M and radius MB.

5) Let S and T be the points of intersection of these two circles.

6) Draw lines BS and BT which are the required tangents.



iv. $\text{L.H.S.} = \sqrt{\frac{1-\cos A}{1+\cos A}}$

$= \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}}$

$= \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}}$

$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$

$= \frac{1-\cos A}{\sin A}$

$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$

$= \operatorname{cosec} A - \cot A$

$= \text{R.H.S.}.$

v. A(-2, -3) ≡ A(x_1, y_1) and B(-4, 7) ≡ B(x_2, y_2)

Slope of line $AB = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-(-3)}{-4-(-2)} = \frac{10}{-2} = -5$

Equation of line AB is given by

$y - y_1 = m(x - x_1)$

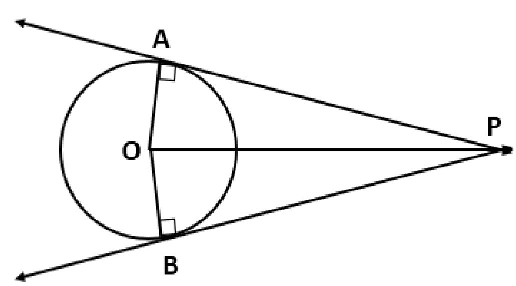
i.e. $y - (-3) = -5[x - (-2)]$

i.e. $y + 3 = -5x - 10$

i.e. $5x + y + 13 = 0$

4.

i. Given: A circle with centre O, an external point P of the circle and the two tangents through the point P are touching the circle at the points A and B.



To prove: PA = PB

Construction: Draw seg OA, seg OB and seg OP

Proof:

$\angle PAO = \angle PBO = 90^\circ$ ....(tangent perpendicular to radius)

In the right-angled triangles PAO and PBO,

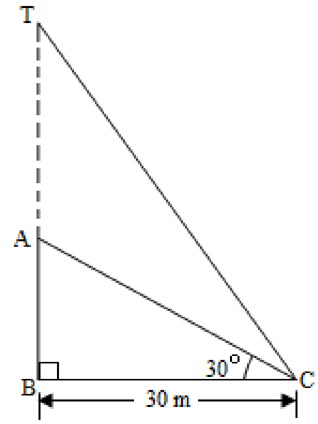
seg $OA =$ segOB ....(Radii of same circle)

Hypotenuse $PO =$ Hypotenuse PO ....(common)

$\therefore \Delta PAO \cong \Delta PBO$ ....(hypotenuse – side test)

$\Rightarrow PA = PB$ ....(c.s.c.t)

ii. Let AB represents the unbroken part and AC represents the broken part of the tree. The top of the tree (T) touches the ground at C.



BC = 30 m, $\angle ACB = 30°$

Total height of the tree = AB+AT

= AB + AC ...(1)

In right angled $\triangle ABC$,

$$\tan\angle ACB = \frac{AB}{BC}$$

$$\therefore \tan 30^\circ = \frac{AB}{BC}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$\therefore AB = \frac{30}{\sqrt{3}}\,\text{m} \quad ...(2)$$

$$\text{Also, } \cos\angle ACB = \frac{BC}{AC}$$

$$\therefore \cos 30^\circ = \frac{BC}{AC}$$

$$\therefore \frac{\sqrt{3}}{2} = \frac{30}{AC}$$

$$\therefore AC = 30 \times \frac{2}{\sqrt{3}}$$

$$\therefore AC = \frac{60}{\sqrt{3}}$$

$$\therefore AT = \frac{60}{\sqrt{3}} \quad ...(3)$$

$$\text{Height of the tree} = AB + AT \quad ...[\text{From}(1)]$$

$$= \frac{30}{\sqrt{3}} + \frac{60}{\sqrt{3}} \quad ...[\text{From (2) and (3)}]$$

$$= \frac{30+60}{\sqrt{3}}$$

$$= \frac{90}{\sqrt{3}}$$

$$= \frac{90}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{90\sqrt{3}}{3}$$

$$= 30\sqrt{3}$$

$$= 30 \times 1.73$$

$$= 51.90 \text{ m}$$

iii. A ≡ (5, 4), B ≡ (-3, -2) and C ≡ (1, -8) are the vertices of a triangle ABC.

AD is the median.

Now, D is the mid-point of BC.

$\therefore$ x-coordinate of point $D = \frac{-3+1}{2} = -\frac{2}{2} = -1$

y-coordinate of point $D = \frac{-2+(-8)}{2} = -\frac{10}{2} = -5$

$\therefore$ Point $D \equiv (-1, -5)$

Slope of $AD = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-4}{-1-5} = \frac{-9}{-6} = \frac{3}{2}$

Thus, equation of median AD is given by $y - y_1 = m(x - x_1)$

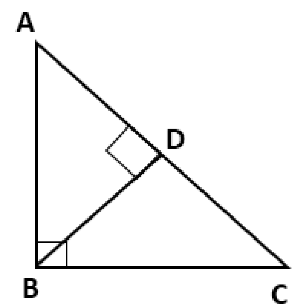
i.e. $y - 4 = \frac{3}{2}(x-5)$

i.e. $2y - 8 = 3x - 15$

i.e. $2y - 3x + 7 = 0$

5.

i. Consider the following figure:



Given: In ΔABC, $\angle ABC = 90°$

To prove: $AC^2 = AB^2 + BC^2$

Construction: Draw seg $BD \perp$ hypotenuse AC

and $A - D - C$

Proof:

In ΔABC, seg $BD \perp$ hypotenuse AC ....(construction)

$\therefore \Delta ABC \sim \Delta ADB$ ....(Similarity in right angled triangles)

$\therefore \frac{AB}{AD} = \frac{AC}{AB}$ ....(Corresponding sides of similar triangles)

$\therefore AB^2 = AC \times AD$ ....(i)

Similarly, $\Delta ABC \sim \Delta BDC$ ....(Similarity in right angled triangles)

$\therefore \frac{BC}{DC} = \frac{AC}{BC}$ ....(Corresponding sides of similar triangles)

$\therefore BC^2 = AC \times DC$ ....(ii)

$AB^2 + BC^2 = AC \times AD + AC \times DC$ ....[Adding equations (i) and (ii)]

$= AC(AD + DC)$

$= AC \times AC$

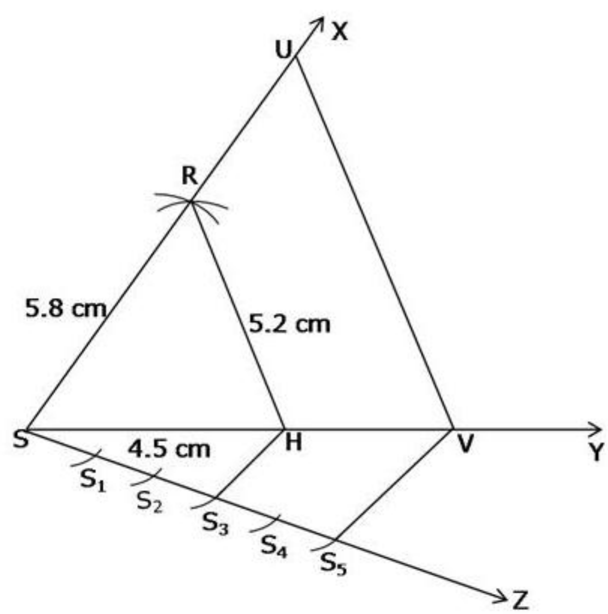
$= AC^2$

$\therefore AC^2 = AB^2 + BC^2$

ii. Steps of construction:

1) Draw ΔSHR with $SH = 4.5$ cm, $HR = 5.2$ cm and $SR = 5.8$ cm.
2) Construct an acute $\angle HSZ$ at S on opposite side of vertex R of ΔSHR.
3) Mark-off five $\left(\text{greater of 3 and 5 in } \frac{3}{5}\right)$ points S_1, S_2, S_3, S_4, S_5 on SZ, such that $HS_1 = S_1S_2 = S_2S_3 = S_3S_4 = S_4S_5$.
4) Join S_3 to H and draw a line S_5V parallel to S_3H, intersecting the extended SH at V.
5) Draw a line through V parallel to HR intersecting the extended line segment SR at U.

Thus, ΔSVU is the required triangle.



iii. Given,

Volume of cuboid = V = a × b × c

Surface area of a cuboid = S = 2(ab + bc +ca)

Now,

$$\frac{2}{S}\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right] = \frac{2}{2(ab+bc+ca)}\left[\frac{bc+ca+ab}{abc}\right]$$

$$= \frac{1}{abc}$$

$$= \frac{1}{V}$$

$$\therefore \frac{1}{V} = \frac{2}{S}\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right]$$