10.4.ex.13.1

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Problem: A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succe ssion without replacement. Write the sample space for this experiment. [Additional: Find the probability mass function too.]

Solution:

The sample space σ is given by:

$$\sigma = \{WW, RW, WR\} \tag{0.1}$$

We shall define random variable X, which denotes the position at which the Red ball is drawn.

Then X takes the values

$$X = \begin{cases} 0, & \text{red ball never drawn} \\ 1, & \text{red ball drawn first} \\ 2, & \text{red ball drawn second} \end{cases} \tag{0.2}$$

There is a total of $4 \cdot 3$ outcomes that can happen, with 4 in first draw and 3 in second. These outcomes relate with X as:

- $X = 0$ when both balls drawn are white, and there's a total of $3 \cdot 2$ these outcomes.
- $X = 1$ when the first ball is red, the second ball has to white. Giving a total of $1 \cdot 3$.
- $X = 2$ when the second ball is red, the first has to be white. Giving a total of $3 \cdot 1$

This translates to the Probability Mass Function as

$$p_X(k) = \begin{cases} \frac{6}{12} = 0.50, & k = 0 \\ \frac{3}{12} = 0.25, & k = 1 \\ \frac{3}{12} = 0.25, & k = 2 \end{cases} \tag{0.3}$$

Computing the Probabilities

We simulate the experiment using random variables and boolean logic.

- Let X_1 be 1 if the first draw is red, otherwise 0.
- Let X_2 be 1 if the second draw is red (only if $X_1 = 0$), otherwise 0.
- Define outcomes:

$$\begin{aligned} RW: &\quad X_1 = 1, X_2 = 0 \\ WR: &\quad X_1 = 0, X_2 = 1 \\ WW: &\quad X_1 = 0, X_2 = 0 \end{aligned}$$

We estimate probabilities by running a large number of trials and counting occurrences.