

# 11.16.1.9

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**Problem:** A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment. [Additional: Find the probability mass function too.]

**Solution:**

The sample space  $\sigma$  is given by:

$$\sigma = \{WW, RW, WR\} \quad (0.1)$$

We shall define random variable  $X$ , which denotes the position at which the Red ball is drawn.

Then  $X$  takes the values

$$X = \begin{cases} 0, & \text{red ball never drawn} \\ 1, & \text{red ball drawn first} \\ 2, & \text{red ball drawn second} \end{cases} \quad (0.2)$$

There is a total of  $4 \cdot 3$  outcomes that can happen, with 4 in first draw and 3 in second. These outcomes relate with  $X$  as:

- $X = 0$  when both balls drawn are white, and there's a total of  $3 \cdot 2$  these outcomes.
- $X = 1$  when the first ball is red, the second ball has to be white. Giving a total of  $1 \cdot 3$ .
- $X = 2$  when the second ball is red, the first has to be white. Giving a total of  $3 \cdot 1$

This translates to the Probability Mass Function as

$$p_X(k) = \begin{cases} \frac{6}{12} = 0.50, & k = 0 \\ \frac{3}{12} = 0.25, & k = 1 \\ \frac{3}{12} = 0.25, & k = 2 \end{cases} \quad (0.3)$$

## Computing the Probabilities

We simulate the experiment using random variables and boolean logic.

- Let  $X_1$  be 1 if the first draw is red, otherwise 0.
- Let  $X_2$  be 1 if the second draw is red (only if  $X_1 = 0$ ), otherwise 0.
- Define outcomes:

$$RW : X_1 = 1, X_2 = 0$$

$$WR : X_1 = 0, X_2 = 1$$

$$WW : X_1 = 0, X_2 = 0$$

We estimate probabilities by running a large number of trials and counting occurrences.