9.5.6

EE24BTECH11018 - Durgi Swaraj Sharma

Question

Find the area of the region lying above the X axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$. $(12, 2018)$

Variables Used

Variable	Description
$\mathbf{e}$	Eccentricity of conic
$\mathbf{F}$	Focus of conic
$\mathbf{I}$	Identity matrix
$\mathbf{n}^\top \mathbf{x} = c$	Equation of directrix
$\mathbf{n}$	Slope of normal to directrix
f	$\|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2$
$\mathbf{V}$	A symmetric matrix given by eigenvalue decomposition
$\mathbf{u}$	Vertex of conic with same directrix

Equations

Equation of the circle is of form $\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$ with

$$\mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{1}$$

$$f = \|\mathbf{u}\|^2 - r^2 = 0 \tag{2}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

Equations

Equation of the parabola is of the form $vecx^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$ with

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{4}$$

$$f = 0 \tag{5}$$

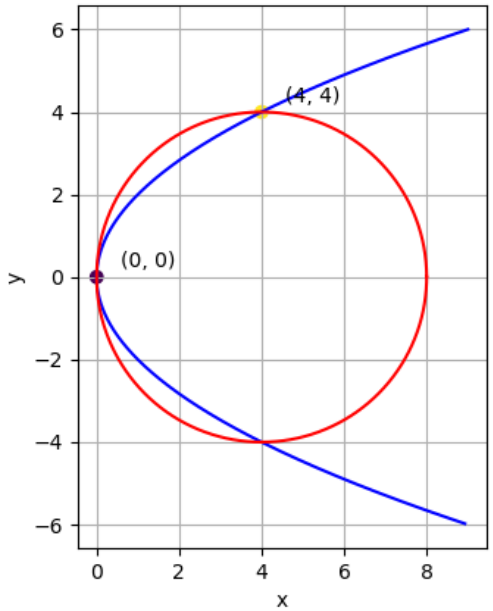
$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{6}$$

Solving

The intersection of two conics with parameters $\mathbf{V}_i, \mathbf{u}_i, f_i, i = 1, 2$ is defined as,

$$\mathbf{x}^\top (\mathbf{V}_1 - \mathbf{V}_2)\,\mathbf{x} + 2\,(\mathbf{u}_1 - \mathbf{u_2})^\top \mathbf{x} + (f_1 - f_2) = 0 \tag{7}$$

On solving using the previous equations, we get the points of intersection to be $\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.



Solving

Area between the two curves above X axis is,

$$\int_0^4 2\sqrt{x}dx - \int_0^4 \sqrt{x^2 - 8x}dx = \frac{12\pi - 32}{3} \approx 1.899. \tag{8}$$

The area between the curves $y^2 = 4x, x^2 + y^2 = 8x$ above the X axis is around 1.899 units.

Graph

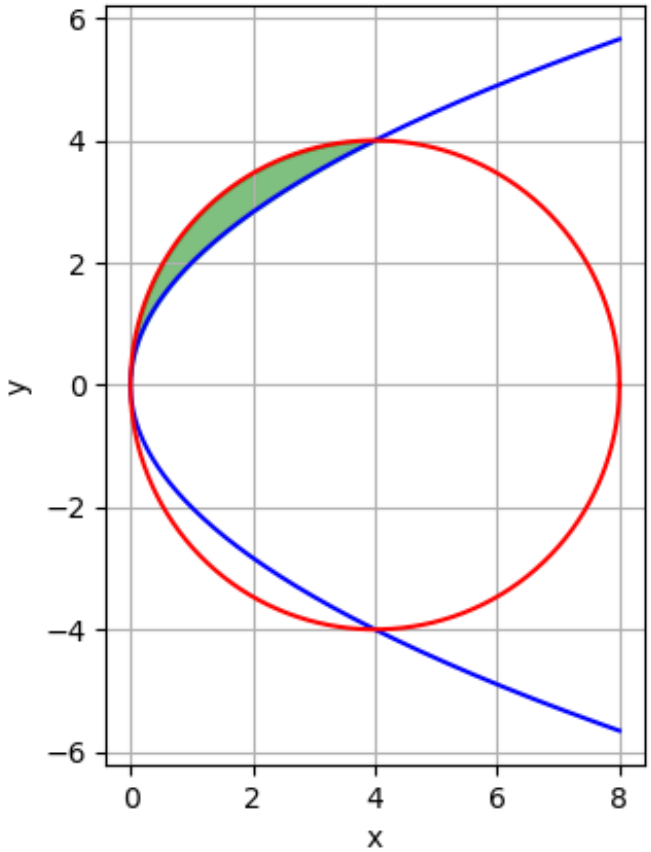


Figure: Required area

C Code I

```c
#include <math.h>
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <unistd.h>
#include <sys/socket.h>
#include <netinet/in.h>
#include "libs/matfun.h"
#include "libs/geofun.h"

int main() {
  FILE *file;
  file = fopen("values.dat", "w");
  if (file == NULL) {
    printf("Error opening file!\n");
    return 1;
  }
  double x = 0, y = 0;
  for(double i=1000;i>=0;i--){
```

C Code II

```
20      double **output=createMat(2,1);
21      output[0][0]+=i/125;
22      x = output[0][0];
23      y = sqrt(x*4);
24      output[1][0]=y;
25      fprintf(file,"%lf,%lf\n",output[0][0],output[1][0]);
26      freeMat(output,2);
27    }
28    x = 0; y = 0;
29    for(double i=0;i<=1000;i++){
30      double **output=createMat(2,1);
31      output[0][0]+=i/125;
32      x = output[0][0];
33      y = -sqrt(x*4);
34      output[1][0]=y;
35      fprintf(file,"%lf,%lf\n",output[0][0],output[1][0]);
36      freeMat(output,2);
37    }
38    x = 0; y = 0;
```

C Code III

```
39  for(double i=1000;i>=0;i--){
40    double **output=createMat(2,1);
41    output[0][0]+=i/125;
42    x = output[0][0];
43    y = sqrt(8*x-x*x);
44    output[1][0]=y;
45    fprintf(file,"%lf,%lf\n",output[0][0],output[1][0]);
46    freeMat(output,2);
47  }
48  x = 0; y = 0;
49  for(double i=0;i<=1000;i++){
50    double **output=createMat(2,1);
51    output[0][0]+=i/125;
52    x = output[0][0];
53    y = -sqrt(8*x-x*x);
54    output[1][0]=y;
55    fprintf(file,"%lf,%lf\n",output[0][0],output[1][0]);
56    freeMat(output,2);
57  }
```

C Code IV

```
58    printf("Results have been written to values.dat\n");
59    return 0;
60 }
```

Python Code I

```
import numpy as np
import matplotlib.pyplot as plt
import os
points = np.loadtxt("values.dat", delimiter=',',
    max_rows=len(list(open("./values.dat"))))
centre=np.array([points[0][0],points[0][1]])
xp = points[:2002, 0]
yp = points[:2002, 1]
xc = points[-2002:,0]
yc = points[-2002:,1]
plt.figure()
plt.plot(xp, yp, label='y^2=4x', color='blue')
plt.plot(xc, yc, label='x^2+y^2=8x', color='red')
plt.gca().set_aspect('equal', adjustable='box')
plt.fill_between(xc, yp, yc, where = (yc>0)&(yp>0)&(xc<4),
    color='green', alpha=0.5)
plt.xlabel("x")
```

Python Code II

```
16 plt.ylabel("y")
17 plt.grid(True)
18 plt.savefig('path/to/figs/fig.png')
19 plt.show()
```