Assignment 2 - EE1030

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1 E - Subjective Problems

1. A curve 'C' passes through $(2,0)$ and the slope at (x,y) as $\frac{(x+1)^2+(y-3)}{x+1}$. Find the equation of the curve. Find the area bounded by curve and x-axis in fourth quadrant. $(2004-4Marks)$
2. If length of tangent at any point on the curve $y=f(x)$ intercepted between the point and the x-axis in fourth quadrant. $(2005-4Marks)$

2 F - Match the Following

1) Match the statements/expressions in **Column I** with the open intervals in **Column II**. (2009)

Column I	**Column II**
(A) Interval contained on the domain of definition of non-zero solutions of the differential equation $(x-3)^2+y'+y=0$	(a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
(B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)\,dx$	(b) $\left(0,\frac{\pi}{2}\right)$
(C) Interval in which at least one of the points of local maximum of $\cos^2 x+\sin x$ lies	(c) $\left(\frac{\pi}{8},\frac{5\pi}{4}\right)$
(D) Interval in which $\tan^{-1}(\sin x+\cos x)$ is increasing	(d) $\left(0,\frac{\pi}{8}\right)$
	(e) $(-\pi,\pi)$

3 H - Assertion & Reason Type Questions

1) Let solution $y=y(x)$ of the differential equation $x\sqrt{x^2-1}dy-y\sqrt{y^2-1}dx=0$ satisfy $y(2)=\frac{2}{\sqrt{3}}$.

STATEMENT-1: $y(x)=\sec\left(\sec^{-1}x-\frac{\pi}{6}\right)$ and

STATEMENT-2: $y(x)$ is given by $\frac{1}{y}=\frac{2\sqrt{3}}{x}-\sqrt{1-\frac{1}{x^2}}$ (2008)

a) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
b) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
c) STATEMENT-1 is True, STATEMENT-2 is False
d) STATEMENT-2 is False, STATEMENT-2 is True

4 I - Integer Value Correct Type

1) Let $y'(x)+y(x)g'(x)= g\,(x),\ g'(x),\ y(0)= 0,\ x\epsilon\mathbb{R}$, where $f'(x)$ denotes $\frac{df(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on $\mathbb{R}$ with $g(0)= g(2)= 0$. Then the value of $y(2)$ is (2011)
2) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0)= 0$. If $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} =(2 + 5y)+(5y - 2)$, then the value of $\lim\limits_{x\to-\infty} f(x)$ is. $(JEEAdv.2018)$
3) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0)= 1$ and satisfying the differential equation $f(x + y)= f(x)f'(y)+f'(y)f(x)$ for all $x,\ y\ \epsilon\mathbb{R}$ then, the value of $\log_e\,(f\,(4))$ is. $(JEEAdv.2018)$

5 Section-B // JEE Main / AIEEE

1) The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$ are [2002]

   a) $\left(1, \frac{2}{3}\right)$
   b) $(3, 1)$
   c) $(3, 3)$
   d) $(1, 2)$

2) The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ [2002]

   a) $\frac{e^{-2x}}{4}$
   b) $\frac{e^{-2x}}{4} + cx + d$
   c) $\frac{1}{4}e^{-2x} + cx^2 + d$
   d) $\frac{1}{4}e^{-4x} + cx + d$

3) The degree and order of the differential equation of the family of all parabolas whose axis x-axis, are respectively. [2003]

   a) $2, 3$
   b) $2, 1$
   c) $1, 2$
   d) $3, 2$

4) The solution of the differential equation $\left(1 + y^2\right)+\left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$, is [2003]

   a) $xe^{2\tan^{-1} y} = e^{\tan^{-1}} + k$
   b) $(x - 2) = ke^{2\tan^{-2} y}$
   c) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$
   d) $xe^{\tan^{-1} y} = \tan^{-2} y + k$

5) The differential equation for the family of circle $x^2 + y^2 - 2ay = 0$, where a is an arbitrary contant is [2004]

   a) $\left(x^2 + y^2\right)y' = 2xy$
   b) $2\left(x^2 + y^2\right)y' = xy$
   c) $\left(x^2 - y^2\right)y' = 2xy$
   d) $2\left(x^2 - y^2\right)y' = xy$

6) Solution of the differential equation $ydx + \left(x + x^2y\right)dy = 0$ is [2004]

a) $\log y = Cx$
b) $-\frac{1}{xy} + \log y = C$
c) $\frac{1}{xy} + \log y = C$
d) $-\frac{1}{xy} = C$

7) The differential equation representing the family of curves $y^2 = 2c\left(x + \sqrt{c}\right)$, where $c > 0$, is a parameter, is of order and degree as follows: [2005]

a) order 1, degree 2
b) order 1, degree 1
c) order 1, degree 3
d) order 2, degree 2

8) If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is [2005]

a) $y\log\left(\frac{x}{y}\right) = cx$
b) $y\log\left(\frac{y}{x}\right) = cy$
c) $\log\left(\frac{y}{x}\right) = cx$
d) $\log\left(\frac{x}{y}\right) = cy$