Assignment 2 - EE1030

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1 Subjective Problems

1. A curve 'C' passes through $(2, 0)$ and the slope at (x, y) as $\frac{(x+1)^2+(y-3)}{x+1}$. Find the equation of the curve. Find the area bounded by curve and x-axis in fourth quadrant.
(2004 – 4*Marks*)
2. If length of tangent at any point on the curve $y = f(x)$ intercepted between the point and the x-axis in fourth quadrant.
(2005 – 4*Marks*)

2 Match the Following

1) Match the statements/expressions in **Column I** with the open intervals in **Column II**.
(2009)

Column I	**Column II**
(A) Interval contained on the domain of definition of non-zero solutions of the differential equation $(x-3)^2 + y' + y = 0$	(p) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)\,dx$	(q) $\left(0, \frac{\pi}{2}\right)$
(C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	(r) $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
(D) Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing	(s) $\left(0, \frac{\pi}{8}\right)$
	(t) $(-\pi, \pi)$

3 Assertion & Reason Type Questions

1) Let solution $y = y(x)$ of the differential equation $x\sqrt{x^2-1}dy - y\sqrt{y^2-1}dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$. **STATEMENT-1**: $y(x) = \sec\left(\sec^{-1} x - \frac{\pi}{6}\right)$ and **STATEMENT-2**: $y(x)$ is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$
(2008)
   a) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
   b) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
   c) STATEMENT-1 is True, STATEMENT-2 is False
   d) STATEMENT-2 is False, STATEMENT-2 is True

4 Integer Value Correct Type

1) Let $y'(x)+y(x)g'(x)= g(x)$, $g'(x)$, $y(0)= 0$, $x \in \mathbb{R}$, where $f'(x)$ denotes $\frac{df(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on $\mathbb{R}$ with $g(0)= g(2)= 0$. Then the value of $y(2)$ is

(2011)

2) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0)= 0$. If $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} =(2 + 5y)+(5y - 2)$, then the value of $\lim_{x \to -\infty} f(x)$ is.

$(JEEAdv.2018)$

3) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0)= 1$ and satisfying the differential equation $f(x + y)= f(x)f'(y)+f'(y)f(x)$ for all $x, y \in \mathbb{R}$ then, the value of $\log_e (f(4))$ is.

$(JEEAdv.2018)$

5 Section-B // JEE Main / AIEEE

1) The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$ are

[2002]

a) $\left(1, \frac{2}{3}\right)$
b) $(3, 1)$
c) $(3, 3)$
d) $(1, 2)$

2) The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$

[2002]

a) $\frac{e^{-2x}}{4}$
b) $\frac{e^{-2x}}{4} + cx + d$
c) $\frac{1}{4}e^{-2x} + cx^2 + d$
d) $\frac{1}{4}e^{-4x} + cx + d$

3) The degree and order of the differential equation of the family of all parabolas whose axis x-axis, are respectively.

[2003]

a) $2, 3$
b) $2, 1$
c) $1, 2$
d) $3, 2$

4) The solution of the differential equation $\left(1 + y^2\right)+\left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$, is

[2003]

a) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$
b) $(x - 2) = ke^{2\tan^{-1} y}$
c) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$
d) $xe^{\tan^{-1} y} = \tan^{-1} y + k$

5) The differential equation for the family of circle $x^2 + y^2 - 2ay = 0$, where a is an arbitrary contant is

[2004]

a) $\left(x^2+y^2\right)y' = 2xy$
b) $2\left(x^2+y^2\right)y' = xy$
c) $\left(x^2-y^2\right)y' = 2xy$
d) $2\left(x^2-y^2\right)y' = xy$

6) Solution of the differential equation $ydx + \left(x + x^2y\right)dy = 0$ is

[2004]

a) $\log y = Cx$
b) $-\frac{1}{xy} + \log y = C$
c) $\frac{1}{xy} + \log y = C$
d) $-\frac{1}{xy} = C$

7) The differential equation representing the family of curves $y^2 = 2c\left(x + \sqrt{c}\right)$, where $c > 0$, is a parameter, is of order and degree as follows:

[2005]

a) order 1, degree 2
b) order 1, degree 1
c) order 1, degree 3
d) order 2, degree 2

8) If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is

[2005]

a) $y\log\left(\frac{x}{y}\right) = cx$
b) $y\log\left(\frac{y}{x}\right) = cy$
c) $\log\left(\frac{y}{x}\right) = cx$
d) $\log\left(\frac{x}{y}\right) = cy$