

9.5.6

EE24BTECH11018 - Durgi Swaraj Sharma

Question

Find the area of the region lying above the X axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$. (12, 2018)

Variables Used

Variable	Description
e	Eccentricity of conic
F	Focus of conic
I	Identity matrix
$\mathbf{n}^T \mathbf{x} = c$	Equation of directrix
n	Slope of normal to directrix
f	$\ \mathbf{n}\ ^2 \ \mathbf{F}\ ^2 - c^2 e^2$
V	A symmetric matrix given by eigenvalue decomposition
u	Vertex of conic with same directrix

Equation of a circle is of form $\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$ with

$$\mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad (1)$$

$$f = \|\mathbf{u}\|^2 - r^2 \quad (2)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (3)$$

Equations

Equation of parabola with directrix $\mathbf{n}^\top \mathbf{x} = c$ is given by,

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (4)$$

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n} \mathbf{n}^\top \quad (5)$$

$$\mathbf{u} = ce^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \quad (6)$$

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \quad (7)$$

and for the parabola $y^2 = 4x$, equation of directrix is, $(-1 \ 0) \mathbf{x} = 1$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (8)$$

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (9)$$

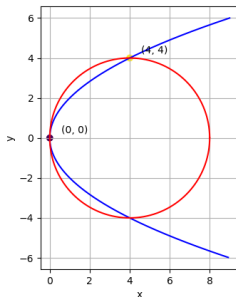
$$f = 0 \quad (10)$$

Solving

The intersection of two conics with parameters $\mathbf{V}_i, \mathbf{u}_i, f_i, i = 1, 2$ is defined as,

$$\mathbf{x}^\top (\mathbf{V}_1 - \mathbf{V}_2) \mathbf{x} + 2(\mathbf{u}_1 - \mathbf{u}_2)^\top \mathbf{x} + (f_1 - f_2) = 0 \quad (11)$$

On solving we get the points of intersection to be $\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.



Area between the two curves above X axis is,

$$\int_0^4 2\sqrt{x}dx - \int_0^4 \sqrt{x^2 - 8x}dx = \frac{12\pi - 32}{3} \approx 1.899. \quad (12)$$

The area between the curves $y^2 = 4x$, $x^2 + y^2 = 8x$ above the X axis is around 1.899 units.

Graph

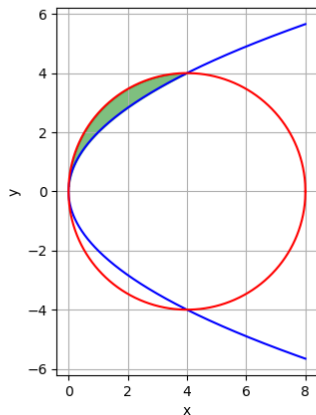


Figure: Required area

C Code I

```
1 #include <math.h>
2 #include <stdio.h>
3 #include <stdlib.h>
4 #include <string.h>
5 #include <unistd.h>
6 #include <sys/socket.h>
7 #include <netinet/in.h>
8
9 #include "libs/matfun.h"
10 #include "libs/geofun.h"
11
12 int main() {
13     FILE *file;
14     file = fopen("values.dat", "w");
15
16     if (file == NULL) {
17         printf("Error opening file!\n");
18         return 1;
19     }
```

C Code II

```
20 double x = 0, y = 0;
21 for(double i=1000;i>=0;i--){
22     double **output=createMat(2,1);
23     output[0][0]+=i/125;
24     x = output[0][0];
25     y = sqrt(x*4);
26     output[1][0]=y;
27     fprintf(file,"%lf,%lf\n",output[0][0],output[1][0]);
28     freeMat(output,2);
29 }
30 x = 0; y = 0;
31 for(double i=0;i<=1000;i++){
32     double **output=createMat(2,1);
33     output[0][0]+=i/125;
34     x = output[0][0];
35     y = -sqrt(x*4);
36     output[1][0]=y;
37     fprintf(file,"%lf,%lf\n",output[0][0],output[1][0]);
38     freeMat(output,2);
```

C Code III

```
39 }
40
41 x = 0; y = 0;
42 for(double i=1000;i>=0;i--){
43     double **output=createMat(2,1);
44     output[0][0]+=i/125;
45     x = output[0][0];
46     y = sqrt(8*x-x*x);
47     output[1][0]=y;
48     fprintf(file,"%lf,%lf\n",output[0][0],output[1][0]);
49     freeMat(output,2);
50 }
51 x = 0; y = 0;
52 for(double i=0;i<=1000;i++){
53     double **output=createMat(2,1);
54     output[0][0]+=i/125;
55     x = output[0][0];
56     y = -sqrt(8*x-x*x);
57     output[1][0]=y;
```

C Code IV

```
58     fprintf(file,"%lf,%lf\n",output[0][0],output[1][0]);  
59     freeMat(output,2);  
60 }  
61  
62 printf("Results have been written to values.dat\n");  
63  
64 return 0;  
65 }
```

Python Code I

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 import os
4
5 points = np.loadtxt("values.dat", delimiter=',',
6                     max_rows=len(list(open("./values.dat"))))
7 centre=np.array([points[0][0],points[0][1]])
8 xp = points[:2002, 0]
9 yp = points[:2002, 1]
10 xc = points[-2002:,0]
11 yc = points[-2002:,1]
12 plt.figure()
13 plt.plot(xp, yp, label='y^2=4x', color='blue')
14 plt.plot(xc, yc, label='x^2+y^2=8x', color='red')
15 plt.gca().set_aspect('equal', adjustable='box')
16 plt.fill_between(xc, yp, yc, where = (yc>0)&(yp>0)&(xc<4),
17                 color='green', alpha=0.5)
```

Python Code II

```
16 plt.xlabel("x")
17 plt.ylabel("y")
18 plt.grid(True)
19 plt.savefig('path/to/figs/fig.png')
20 plt.show()
```