Assignment 2 - EE1030

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1 Section - A

1) Let M denote the median of the following frequency distributions.

Class	0-4	4-8	8-12	12-16	16-20
Frequency	3	9	10	8	6

Then $20M$ is equal to:

a) 416
b) 104
c) 52
d) 208

2) If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5}f'(0)$ is equal to

a) 0
b) 2
c) 2
d) 6

3) Let $A(2,3,5)$ and $C(-3,4,-2)$ be opposite vertices of a parallelogram $ABCD$. If the diagonal $\overrightarrow{BD} = \hat{i}+2\hat{j}+3\hat{k}$ then the area of the parallelogram is equal to

a) $\frac{1}{2}\sqrt{410}$
b) $\frac{1}{2}\sqrt{474}$
c) $\frac{1}{2}\sqrt{586}$
d) $\frac{1}{2}\sqrt{306}$

4) If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right], n \in \mathbb{N}$, then the roots of the equation $x^2 + nx + (n-3) = 0$ belong to:

a) $(0, \infty)$
b) $(-\infty, 0)$
c) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$
d) $\mathbb{Z}$

5) Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbb{R}$ be a differentiable function such that $f(0) = \frac{1}{2}$. If the $\lim_{x\to 0} \frac{x\int_o^x f(t)dt}{e^{x^2}-1} = \alpha$, then $8\alpha^2$ is equal to:

a) 16
b) 2
c) 1

d) 4

2 Section - B

1) A group of 40 students appeared in an examination of 3 subjects - Mathematics, Physics & Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, at most 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is
2) If d_1 is the shortest distance between the lines $x+1=2y=-12z$, $x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$, $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is :
3) Let the latus rectum of the hyperbola $\frac{x^2}{9}-\frac{y^2}{b^2}=1$ subtend and angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{l}{m}\left(1+\sqrt{n}\right)$, where l and m are the co-prime numbers, then $l^2+m^2+n^2$ is equal to
4) Let $A=\{1,2,3,\dots 7\}$ and let $P(A)$ denote the power set of A. If the number of functions $f : A \to P(A)$ such that $a \in f(a), \forall a \in A$ is m^n, m and $n \in \mathbb{N}$ and m is least, then $m+n$ is equal to
5) The value $9\int_0^9 \left[\sqrt{\frac{10x}{x+1}}\right] dx$, where $[t]$ denotes the greatest integer less than or equal to t, is
6) Number of integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)}+11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to
7) Let $y=y(x)$ be the solution of the differential equation $\left(1-x^2\right)dy = \left[xy+\left(x^3+2\right)\sqrt{3\left(1-x^2\right)}\right]dx$, $-1<x<1$, $y(0)=0$. If $y\left(\frac{1}{2}\right)=\frac{m}{n}$, m and n are co-prime numbers, then $m+n$ is equal to
8) Let $\alpha, \beta \in \mathbb{N}$ be the roots of the equation $x^2-70x+\lambda=0$, where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin \mathbb{N}$. if λ assumes the minimum possible value, then $\frac{\left(\sqrt{\alpha-1}+\sqrt{\beta-1}\right)(\lambda+35)}{|\alpha-\beta|}$ is equal to:
9) If the function $f(x)=\begin{cases}\frac{1}{|x|}, & |x|\geq 2\\ ax^2+2b, & |x|<2\end{cases}$ is differentiable on $\mathbb{R}$, then $48(a+b)$ is equal to
10) Let $\alpha = 1^2+4^2+8^2+13^2+19^2+26^2+\cdots$ upto 10 terms and $\beta=\sum_{n=1}^{10} n^4$. If $4\alpha-\beta=55k+40$, then k is equal to