

Assignment 8 - EE1030

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ee24btech11018 - D. Swaraj Sharma

- 1) Suppose that P_1 and P_2 are two populations with equal prior probabilities having bivariate normal distributions with mean vectors $(2, 3)$ and $(1, 1)$, respectively. The variance-covariance matrix of both the distributions is the identity matrix. Let $z_1 = (2.5, 2)$ and $z_2 = (2, 1.5)$ be two new observations. According to Fisher's linear discriminant rule,
 - a) z_1 is assigned to P_1 , and z_2 is assigned to P_2 .
 - b) z_1 is assigned to P_2 , and z_2 is assigned to P_1 .
 - c) z_1 is assigned to P_1 , and z_2 is assigned to P_1 .
 - d) z_1 is assigned to P_2 , and z_2 is assigned to P_2 .
- 2) Let X_1, \dots, X_n be a random sample from a population having probability density function $f_X(x; \theta) = \frac{2x}{\theta^2}$, $0 < x < \theta$. Then the method of moments estimator of θ is
 - a) $\frac{3 \sum_{i=1}^n X_i}{2n}$
 - b) $\frac{3 \sum_{i=1}^n X_i^2}{2n}$
 - c) $\frac{\sum_{i=1}^n X_i}{2n}$
 - d) $\frac{3 \sum_{i=1}^n X_i(X_i-1)}{2n}$
- 3) Let X be a normal random variable having mean θ and variance 1, where $1 \leq \theta \leq 10$. Then X is
 - a) sufficient but not complete.
 - b) the maximum likelihood estimator of θ .
 - c) the uniformly minimum variance unbiased estimator of θ .
 - d) complete and ancillary.
- 4) Let $\{X_n\}_{n \geq 1}$ be a sequence of independent and identically distributed random variables with mean θ and variance θ , where $\theta > 0$. Then $\frac{\sum_{i=1}^n X_i}{\sum_{i=1}^n X_i^2}$ is a consistent estimator of
 - a) $\frac{1}{1+\theta}$
 - b) $\frac{1+\theta}{\theta}$
 - c) $\frac{1}{\theta}$
 - d) $\frac{\theta}{1+\theta}$
- 5) Let X_1, \dots, X_{10} be a random sample from a population with probability density function

$$f(x; \theta) = \frac{e^{-|x-\theta|}}{2}, -\infty < x < \infty, -\infty < \theta < \infty.$$

Then the maximum likelihood estimator of θ

- a) does not exist.
- b) is not unique.
- c) is the sample mean.

d) is the smallest observation.

- 6) Consider the model $Y_i = \beta + \epsilon_i$, where ϵ_i 's are independent normal random variables with zero mean and known variance $\sigma_i^2 > 0$, for i, \dots, n . Then the best linear unbiased estimator of the unknown parameter β is

- a) $\frac{\sum_{i=1}^n (Y_i / \sigma_i^2)}{\sum_{i=1}^n (1 / \sigma_i^2)}$
- b) $\frac{\sum_{i=1}^n Y_i}{n}$
- c) $\frac{\sum_{i=1}^n Y_i / \sigma_i}{n}$
- d) $\frac{\sum_{i=1}^n (Y_i / \sigma_i)}{\sum_{i=1}^n (1 / \sigma_i)}$

- 7) Let (X, Y) be a bivariate random vector with the probability density function

$$f_{(X,Y)}(x, y) = \begin{cases} e^{-y} & 0 < x < y, \\ 0 & \text{otherwise} \end{cases}$$

Then the regression of Y on X is given by

- a) $X + 1$
- b) $\frac{X}{2}$
- c) $\frac{X}{2}$
- d) $Y + 1$

- 8) Consider a discrete time Markov chain on the state space $[1, 2]$ with one-step transition probability matrix

$$\begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{bmatrix} \end{matrix}$$

Then $\lim_{n \rightarrow \infty} P^n$ is

- a) $\begin{bmatrix} \frac{3}{11} & \frac{8}{11} \\ \frac{3}{11} & \frac{8}{11} \end{bmatrix}$
- b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- d) $\begin{bmatrix} \frac{8}{11} & \frac{3}{11} \\ \frac{8}{11} & \frac{3}{11} \end{bmatrix}$

- 9) Let (X_1, X_2) be a random vector with variance-covariance matrix $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$. The two principal components are

- a) X_1 and X_2
- b) $-X_1$ and X_2
- c) X_1 and $-X_2$
- d) $X_1 + X_2$ and X_2

10) Consider the objects $[1, 2, 3, 4]$ with the distance matrix

$$\begin{array}{c} \begin{array}{cccc} & 1 & 2 & 3 & 4 \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 11 & 5 \\ 1 & 0 & 2 & 3 \\ 11 & 2 & 0 & 4 \\ 5 & 3 & 4 & 0 \end{bmatrix} \end{array}$$

Applying the single-linkage hierarchical procedure twice, the two clusters that result are

- a) $\{2, 3\}$ and $\{1, 4\}$
 - b) $\{1, 2, 3\}$ and $\{4\}$
 - c) $\{1, 3, 4\}$ and $\{2\}$
 - d) $\{2, 3, 4\}$ and $\{1\}$
- 11) The maximum likelihood estimates of the mean vector and the variance-covariance matrix of a bivariate normal distribution based on the realization **[12, 44, 43]** of a random sample size 3 are given by

- a) $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 1 & 2/3 \end{bmatrix}$
 - b) $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 1 & 3/2 \end{bmatrix}$
 - c) $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ and $\begin{bmatrix} 3 & 3/2 \\ 3/2 & 2/3 \end{bmatrix}$
 - d) $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ and $\begin{bmatrix} 3 & 2/3 \\ 2/3 & 1 \end{bmatrix}$
- 12) Consider a fixed effects one-way analysis of variance model $Y_{ij} = \mu + \tau_i + \epsilon_{ij}$, for $i = 1, \dots, a$, $j = 1, \dots, r$, and ϵ_{ij} 's are independent and identically distributed normal random variables with mean zero and variance σ^2 . Here, r and a are positive integers. Let $\bar{Y}_i = \frac{\sum_{j=1}^r Y_{ij}}{r}$. Then \bar{Y}_i is the least squares estimator for

- a) $\mu + \frac{\tau_i}{2}$
 - b) τ_i
 - c) $\mu + \tau_i$
 - d) μ
- 13) Let A be a $n \times n$ positive semi-definite matrix with eigenvalues $\lambda_1 \geq \dots \geq \lambda_n$, and with α as the maximum diagonal entry. We can find a vector x such that $x^t x = 1$, where t denotes the transpose, and
- a) $x^t A x > \lambda_1$
 - b) $x^t A x < \lambda_n$
 - c) $\lambda_n \leq x^t A x \leq \lambda_1$
 - d) $x^t A x > n\alpha$