Assignment 8 - EE1030

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1) Suppose that P_1 and P_2 are two populations with equal prior probabilities having bivariate normal distributions with mean vectors $(2, 3)$ and $(1, 1)$, respectively. The variance-covariance matrix of both the distributions is the identity matrix. Let $z_1 = (2.5, 2)$ and $z_2 = (2, 1.5)$ be two new observations. According to Fisher's linear discriminant rule,
   a) z_1 is assigned to P_1, and z_2 is assigned to P_2.
   b) z_1 is assigned to P_2, and z_2 is assigned to P_1.
   c) z_1 is assigned to P_1, and z_2 is assigned to P_1.
   d) z_1 is assigned to P_2, and z_2 is assigned to P_2.
2) Let $X_1, \dots, X_n$ be a random sample from a population having probability density function $f_X(x; \theta) = \frac{2x}{\theta^2}$, $0 < x < \theta$. Then the method of moments estimator of θ is
   a) $\frac{3\sum_{i=1}^{n} X_i}{2n}$
   b) $\frac{3\sum_{i=1}^{n} X_i^2}{2n}$
   c) $\frac{\sum_{i=1}^{n} X_i}{2n}$
   d) $\frac{3\sum_{i=1}^{n} X_i(X_i-1)}{2n}$
3) Let X be a normal random variable having mean θ and variance 1, where $1 \le \theta \le 10$. Then X is
   a) sufficient but not complete.
   b) the maximum likelihood estimator of θ.
   c) the uniformly minimum variance unbiased estimator of θ.
   d) complete and ancillary.
4) Let $\{X_n\}_{n\ge1}$ be a sequence of independent and identically distributed random variables with mean θ and variance θ, where $\theta > 0$. Then $\frac{\sum_{i=1}^{n} X_i}{\sum_{i=1}^{n} X_i^2}$ is a consistent estimator of
   a) $\frac{1}{1+\theta}$
   b) $\frac{1+\theta}{\theta}$
   c) $\frac{1}{\theta}$
   d) $\frac{\theta}{1+\theta}$
5) Let $X_1, \dots, X_{10}$ be a random sample from a population with probability density function
$$f(x; \theta) = \frac{e^{-|x-\theta|}}{2}, -\infty < x < \infty, -\infty < \theta < \infty.$$
   Then the maximum likelihood estimator of θ
   a) does not exist.
   b) is not unique.
   c) is the sample mean.

d) is the smallest observation.

6) Consider the model $Y_i = \beta + \epsilon_i$, where ϵ_i's are independent normal random variables with zero mean and know variance $\sigma_i^2 > 0$, for $i, \cdots, n$. Then the best linear unbiased estimator of the unknown parameter β is

a) $\frac{\sum_{i=1}^{n}(Y_i/\sigma_i^2)}{\sum_{i=1}^{n}(1/\sigma_i^2)}$

b) $\frac{\sum_{i=1}^{n} Y_i}{n}$

c) $\frac{\sum_{i=1}^{n} Y_i/\sigma_i}{n}$

d) $\frac{\sum_{i=1}^{n}(Y_i/\sigma_i)}{\sum_{i=1}^{n}(1/\sigma_i)}$

7) Let (X, Y) be a bvariate random vector with the oribability density function

$$f_{(X,Y)}(x, y) = \begin{cases} e^{-y} & 0 < x < y, \\ 0 & otherwise \end{cases}$$

Then the regression of Y on X is given by

a) $X + 1$

b) $\frac{X}{2}$

c) $\frac{X}{2}$

d) $Y + 1$

8) Consider a discrete time Markov chain on the stage space $[1, 2]$ with one-step transistion probability matrix

$$\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{bmatrix} \end{array}$$

Then $\lim_{n\to\infty} P^n$ is

a) $\begin{bmatrix} \frac{3}{11} & \frac{8}{11} \\ \frac{3}{11} & \frac{8}{11} \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} \frac{8}{11} & \frac{3}{11} \\ \frac{8}{11} & \frac{3}{11} \end{bmatrix}$

9) Let (X_1, X_2) be a random vector with variance-covariance matrix $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$. The two principal components are

a) X_1 and X_2

b) $-X_1$ and X_2

c) X_1 and $-X_2$

d) $X_1 + X_2$ and X_2

10) Consider the objects $[1, 2, 3, 4]$ with the distance matrix

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 11 & 5 \\ 1 & 0 & 2 & 3 \\ 11 & 2 & 0 & 4 \\ 5 & 3 & 4 & 0 \end{bmatrix} \end{array}$$

Applying the single-linkage hierarchical procedure twice, the two clusters that result are

a) $\{2, 3\}$ and $\{1, 4\}$
b) $\{1, 2, 3\}$ and $\{4\}$
c) $\{1, 3, 4\}$ and $\{2\}$
d) $\{2, 3, 4\}$ and $\{1\}$

11) The maximum likelihood estimates of the mean vector and the variance-covariance matrix of a bivariate normal distribution based on the realization $\left\{\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}4\\4\end{pmatrix}, \begin{pmatrix}4\\3\end{pmatrix}\right\}$ of a random sample size 3 are given by

a) $\begin{pmatrix}3\\3\end{pmatrix}$ and $\begin{bmatrix}2 & 1\\1 & 2/3\end{bmatrix}$
b) $\begin{pmatrix}3\\3\end{pmatrix}$ and $\begin{bmatrix}2 & 1\\1 & 3/2\end{bmatrix}$
c) $\begin{pmatrix}3\\3\end{pmatrix}$ and $\begin{bmatrix}3 & 3/2\\3/2 & 2/3\end{bmatrix}$
d) $\begin{pmatrix}3\\3\end{pmatrix}$ and $\begin{bmatrix}3 & 2/3\\2/3 & 1\end{bmatrix}$

12) Consider a fixed effects one-way analysis of variance model $Y_{ij} = \mu + \tau_i + \epsilon_{ij}$, for $i = 1, \ldots, a$, $j = 1, \ldots, r$, and ϵ_{ij}'s are independent and identically distributed normal random variables with mean zero and variance σ^2. Here, r and a are positive integers. Let $\bar{Y}_i = \frac{\sum_{j=1}^{r} Y_{ij}}{r}$. Then $\bar{Y}_i$ is the least squares estimator for

a) $\mu + \frac{\tau_i}{2}$
b) τ_i
c) $\mu + \tau_i$
d) μ

13) Let A be a $n \times n$ positive semi-definite matrix with eigenvalues $\lambda_1 \geq \cdots \geq \lambda_n$, and with α as the maximum diagonal entry. We can find a vector x such that $x^t x = 1$, where t denotes the transpose, and

a) $x^t Ax > \lambda_1$
b) $x^t Ax < \lambda_n$
c) $\lambda_n \leq x^t Ax \leq \lambda_1$
d) $x^t Ax > n\alpha$