Digital Finance Calculation Questions - Comprehensive Formula Guide

This document compiles all calculative questions found in the Digital Finance course PDF slides, with verbatim examples, formula tables, sample questions, and answers.

Section 1: IPO Valuation Formulas

Formula 1.1: IPO Proceeds Calculation

Formula Name: Net IPO Proceeds

Formula:

$$\text{IPO Proceeds} = \text{New Shares} \times \text{IPO Price} \times (1 - \text{Underwriting Spread})$$

Source: Solutions_Exercise-ICOs.pdf, Slide 12/27

Verbatim Example from PDF:

"Your firm has 10 million shares outstanding and you are about to issue 5 million new shares in an IPO. The IPO price has been set at $20 per share, and the underwriting spread is 7%. The IPO is a big success with investors, and the share price rises to $50 on the first day of trading.

a) How much did your firm raise from the IPO?

The total amount of money your company raised from the IPO is calculated as follows:

$$\text{IPO Proceeds} = 5 \times 20 \times (1 - 0.07) = 93 \text{ million}$$

"

Formula 1.2: Post-IPO Market Value

Formula Name: Market Value After IPO

Formula:

$$\text{Market Value} = (\text{Original Shares} + \text{New Shares}) \times \text{First Day Price}$$

Source: Solutions_Exercise-ICOs.pdf, Slide 13/27

Verbatim Example from PDF:

"b) What is the market value of the firm after the IPO?

The market value of your company after the IPO is:

$$\text{Market Value} = (5 + 10) \times 50 = 750 \text{ million}$$

"

Formula 1.3: Fair Market Share Price (Perfect Market)

Formula Name: Perfect Market Share Price

Formula:

$$\text{Price per Share} = \frac{\text{Fair Market Value} - \text{IPO Proceeds}}{\text{Original Shares}}$$

Where Fair Market Value is derived from:

$$\text{Fair Market Value} = \text{IPO Proceeds} + \text{Original Shares} \times \text{Price per Share}$$

Source: Solutions_Exercise-ICOs.pdf, Slide 14/27

Verbatim Example from PDF:

"c) Assume that the post-IPO value of the firm is its fair market value. Suppose your firm could have issued shares directly to investors at their fair market value, in a perfect market with no underwriting fee and no underpricing. What would the share price have been in this case, if you raise the same amount as in part (a)?

The share price in a perfect market with no underwriting fee and no underpricing would be:

$$\frac{93}{1.4155} = 65.70$$

$$750 = 93 + 657$$

$$657 = 10 \times 65.70$$

$$\frac{750}{(10 + 1.4155)} = 65.70$$

"

Formula 1.4: Total IPO Cost (Market Imperfections)

Formula Name: Cost of Market Imperfections

Formula (Method 1):

$$\text{Total Cost} = \text{Underwriting Fees} + \text{Underpricing Cost}$$

$$= (\text{New Shares} \times \text{IPO Price} \times \text{Underwriting Spread}) + (\text{New Shares} \times (\text{First Day P}$$

Formula (Method 2):

$$\text{Total Cost} = (\text{Perfect Market Price} - \text{First Day Price}) \times \text{Original Shares}$$

Source: Solutions_Exercise-ICOs.pdf, Slide 15/27

Verbatim Example from PDF:

"d) Comparing part (b) and (c), what is the total cost to the firm's original investors due to market imperfections from the IPO?

The total cost that your company paid for the IPO as a result of market imperfections is:

Method 1:

$$5 \times 20 \times 0.07 = 7$$

$$5 \times (50 - 20) = 150$$

$$7 + 150 = 157$$

Method 2:

$$(65.7 - 50) \times 10 = 157$$

"

Section 2: Asset Pricing and Risk Formulas

Formula 2.1: Return Calculation

Formula Name: Asset Return

Formula:

$$r_t = \frac{P_t - P_{t-1} + C_t}{P_{t-1}}$$

Where:

- r_t = return during period t
- P_t = current price
- P_{t-1} = price in previous period
- C_t = any cash flow accruing from the investment

Source: VL6_Asset-Pricing-With-Cryptocurrencies.pdf, Slide 5/31

Verbatim Text from PDF:

"Return represents the total gain or loss on an investment. The most basic way to calculate return is as follows: Where r_t is the return during period t, P_t is the current price, P_{t-1} is the price in the previous period, C_t is any CF accruing from the investment."

Formula 2.2: Standard Deviation (Variance)

Formula Name: Standard Deviation of Returns

Formula:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2}$$

Source: VL6_Asset-Pricing-With-Cryptocurrencies.pdf, Slide 9/31

Verbatim Text from PDF:

"The standard deviation of returns is given by the following equation:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2}$$

"

Also found in: Solutions_Exercise-3-BitcoinBlockchain.pdf, Slide 25/30

Formula 2.3: Sharpe Ratio

Formula Name: Sharpe Ratio

Formula:

$$\text{Sharpe Ratio} = \frac{r_j - r_f}{\sigma_j}$$

Where:

- r_j = return of asset j
- r_f = risk-free rate
- σ_j = standard deviation of returns

Source: VL6_Asset-Pricing-With-Cryptocurrencies.pdf, Slide 10/31

Verbatim Text from PDF:

"Introduced by Sharpe (1966). Indicates the excess return per unit of risk. Thus constitutes a reward-to-variability ratio. Formula: Where return of asset j, risk-free rate, standard deviation of returns."

Also found in: Solutions_Exercise-3-BitcoinBlockchain.pdf, Slide 29/30

Formula 2.4: Skewness

Formula Name: Skewness

Formula:

$$\text{Skewness} = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{r_t - \bar{r}}{\sigma}\right)^3$$

Source: VL6_Asset-Pricing-With-Cryptocurrencies.pdf, Slide 11/31

Verbatim Text from PDF:

"Statistical measure indicating the degree of symmetry in the data. A totally symmetric distribution will have a skewness of 0. Formula:

$$\frac{1}{n}\sum_{t=1}^{n}\left(\frac{r_t-\bar{r}}{\sigma}\right)^3$$

"

Also found in: Solutions_Exercise-3-BitcoinBlockchain.pdf, Slide 27/30

Formula 2.5: Kurtosis

Formula Name: Kurtosis

Formula:

$$\text{Kurtosis} = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{r_t-\bar{r}}{\sigma}\right)^4$$

Source: VL6_Asset-Pricing-With-Cryptocurrencies.pdf, Slide 12/31

Verbatim Text from PDF:

"Statistical measure indicating the degree of outliers (i.e., the tails) in the distribution. High kurtosis means higher amount of outliers, low kurtosis indicates lower amount of outliers. Formula:

$$\frac{1}{n}\sum_{t=1}^{n}\left(\frac{r_t-\bar{r}}{\sigma}\right)^4$$

"

Also found in: Solutions_Exercise-3-BitcoinBlockchain.pdf, Slide 28/30

Formula 2.6: Correlation Coefficient

Formula Name: Correlation Coefficient

Formula:

$$\rho_{x,y} = \frac{\text{Cov}(x,y)}{\sigma_x \times \sigma_y}$$

Where:

- $\text{Cov}(x,y)$ = Covariance between x and y
- σ_x = Standard deviation of asset x
- σ_y = Standard deviation of asset y

Source: VL6_Asset-Pricing-With-Cryptocurrencies.pdf, Slide 15/31

Verbatim Text from PDF:

"Diversification is depending upon the extent to which the returns on assets move together (co-relate). This movement is typically measured by the correlation. The correlation coefficient always lies between -1 and +1. +1 = Perfect positive correlation. -1 = Perfect negative correlation."

Section 3: CAPM and Regression Formulas

Formula 3.1: Capital Asset Pricing Model (CAPM)

Formula Name: CAPM Required Return

Formula:

$$E(r_i) = R_F + \beta_i \times [E(r_M) - R_F]$$

Or equivalently:

$$E(r_i) = R_F + \beta_i \times \text{ERP}$$

Where:

- $E(r_i)$ = Expected return of stock i
- R_F = Risk-free rate
- β_i = Systematic risk (beta) of stock i
- $E(r_M)$ = Expected return of the market
- ERP = Equity Risk Premium (Market risk premium)

Source: Solutions_Exercise-5-Robo_Intro.pdf, Slide 5/26

Verbatim Text from PDF:

"Derive the CAPM-Formula on the basis of Figure 1 (Security Market Line). Explain the following terms: $E(r_M)$, R_F, ERP, β.

Market Portfolio expected one-period return of stock i, Systematic risk of stock i, Risk-free rate, Market risk premium, i.e., return of the market over risk-free rate, ERP = Equity Risk Premium."

Formula 3.2: Alpha Calculation

Formula Name: Jensen's Alpha

Formula:

$$\alpha = E(r_i) - [R_F + \beta_i \times \text{ERP}]$$

Source: Solutions_Exercise-5-Robo_Intro.pdf, Slide 6/26

Verbatim Example from PDF:

"b) An analyst makes the following predictions about the 1-year expected return of two stocks A and B. A has an expected return of 11% and a beta of 1.5. B has an expected return of 7% and a beta of 0.5. Which stock is more attractive?

Alpha is the unexplained part of the model. Active managers look for stocks with high alpha.

Stock A:

$$\alpha_A = 11\% - [2\% + 1.5 \times (8\% - 2\%)] = 0\%$$

Stock B:

$$\alpha_B = 7\% - [2\% + 0.5 \times (8\% - 2\%)] = 2\%$$

"

Formula 3.3: Regression Beta Estimation

Formula Name: Beta Coefficient (from regression)

Estimated via linear regression:

$$r_{i,t} - r_{f,t} = \alpha + \beta(r_{M,t} - r_{f,t}) + \epsilon_t$$

Where:

- β = Slope coefficient from regression
- α = Intercept (Jensen's alpha)

Source: Solutions_Exercise-5-Robo_Intro.pdf, Slides 10-12/26

Verbatim Text from PDF:

"e) Please estimate the CAPM betas for the two Companies as of Dec 31, 2016. On the basis of your estimate, calculate the required returns for the two companies using the CAPM. You have a plausible assumption about the market risk premium and the risk-free rate.

Assumptions:

- Risk-free rate: 0.02
- Market risk premium: 0.05

Required return of BASF with regard to:

- MSCI World: 7.86%
- DAX: 7.91%"

Section 4: AI and Classification Metrics

Formula 4.1: Accuracy

Formula Name: Classification Accuracy

Formula:

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

Where:

- TP = True Positives

- TN = True Negatives
- FP = False Positives
- FN = False Negatives

Source: Solutions_Exercise-AI_in_Finance.pdf, Slide 8/29

Verbatim Example from PDF:

"How well does the test measure the reality?

$$\text{Accuracy} = \frac{308 + 21}{308 + 21 + 105 + 66} = \frac{329}{500} = 65.8\%$$

"

Also found in Slide 21/29:

"

$$\text{Accuracy} = \frac{436 + 90}{436 + 90 + 76 + 198} = 65.75\%$$

"

Formula 4.2: Precision

Formula Name: Precision (Positive Predictive Value)

Formula:

$$\text{Precision} = \frac{TP}{TP + FP}$$

Source: Solutions_Exercise-AI_in_Finance.pdf, Slide 8/29

Verbatim Example from PDF:

"When an entity is classified as Positive, how often are they actually Positive?

$$\text{Precision} = \frac{308}{308 + 105} = \frac{308}{413} = 74.58\%$$

"

Also found in Slide 21/29:

"

$$\text{Precision} = \frac{436}{436 + 76} = 85.16\%$$

"

Formula 4.3: Recall (Sensitivity)

Formula Name: Recall/Sensitivity

Formula:

$$\text{Recall} = \frac{TP}{TP + FN}$$

Source: Solutions_Exercise-AI_in_Finance.pdf, Slide 8/29

Verbatim Example from PDF:

"When an entity is actually Positive, how often are they classified as such?

$$\text{Recall} = \frac{308}{308 + 66} = \frac{308}{374} = 82.35\%$$

"

Also found in Slide 21/29:

"

$$\text{Recall} = \frac{436}{436 + 198} = 68.77\%$$

"

Formula 4.4: F1 Score

Formula Name: F1 Score

Formula:

$$F_1 = 2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$

Or equivalently:

$$F_1 = \frac{2 \times TP}{2 \times TP + FP + FN}$$

Source: Solutions_Exercise-AI_in_Finance.pdf, Slide 8/29

Verbatim Example from PDF:

"Harmonic mean of Precision and Recall, giving both similar importance.

$$F_1 = 2 \times \frac{0.7458 \times 0.8235}{0.7458 + 0.8235} = 78.27\%$$

"

Also found in Slide 21/29:

"

$$F_1 = 2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} = 76.09\%$$

"

Part 2: Formula Table Summary

Formula Name	Formula	Key Variables	Application
Net IPO Proceeds	$\text{Proceeds} = N \times P \times (1 - s)$	N = new shares, P = IPO price, s = spread	IPO valuation
Post-IPO Market Value	$\text{MV} = (O + N) \times P_1$	O = original shares, P_1 = first day price	IPO valuation
Perfect Market Price	$\text{Price} = \frac{FMV - \text{Proceeds}}{O}$	FMV = fair market value	IPO cost analysis
Total IPO Cost	$\text{Cost} = (P_{perfect} - P_1) \times O$	$P_{perfect}$ = perfect price	IPO imperfections
Asset Return	$r_t = \frac{P_t - P_{t-1} + C_t}{P_{t-1}}$	P = price, C = cash flow	Performance
Standard Deviation	$\sigma = \sqrt{\frac{1}{n-1} \sum (r_t - \bar{r})^2}$	r_t = returns, $\bar{r}$ = mean	Risk measurement
Sharpe Ratio	$SR = \frac{r - r_f}{\sigma}$	r_f = risk-free rate	Risk-adjusted return
Skewness	$S = \frac{1}{n} \sum \left(\frac{r_t - \bar{r}}{\sigma} \right)^3$	-	Distribution shape

Formula Name	Formula	Key Variables	Application
Kurtosis	$K = \frac{1}{n} \sum \left(\frac{r_t - \bar{r}}{\sigma} \right)^4$	-	Tail risk
Correlation	$\rho = \frac{\text{Cov}(x,y)}{\sigma_x \sigma_y}$	Cov = covariance	Diversification
CAPM	$E(r_i) = R_F + \beta_i \times \text{ERP}$	β = systematic risk, ERP = market premium	Required return
Alpha	$\alpha = E(r_i) - [R_F + \beta_i \times \text{ERP}]$	-	Abnormal return
Accuracy	$A = \frac{TP+TN}{TP+TN+FP+FN}$	TP, TN, FP, FN = confusion matrix	Classification
Precision	$P = \frac{TP}{TP+FP}$	-	Classification
Recall	$R = \frac{TP}{TP+FN}$	-	Classification
F1 Score	$F_1 = 2 \times \frac{P \times R}{P+R}$	P = precision, R = recall	Classification

Part 3: Sample Examination Questions

Question Set 1: IPO Valuation (Based on Formula 1.1-1.4)

Question 1.1:

TechStart GmbH has 8 million shares outstanding and plans to issue 3 million new shares in an IPO. The IPO price has been set at €25 per share, and the underwriting spread is 6%. On the first day of trading, the share price closes at €42.

a) Calculate the net proceeds TechStart raised from the IPO.

b) What is the market value of TechStart after the IPO?

c) Assuming the post-IPO value represents the firm's fair market value, what would the share price have been in a perfect market with no underwriting fees and no underpricing if TechStart raised the same amount as in part (a)?

d) What is the total cost to the firm's original investors due to market imperfections?

Question 1.2:

BioInnovate AG plans an IPO with the following details:

- Existing shares: 15 million
- New shares to be issued: 6 million
- IPO price: €18 per share
- Underwriting spread: 5.5%
- First-day closing price: €28 per share

a) How much capital did BioInnovate raise from the IPO?

b) Calculate the market capitalization after the IPO.

c) In a perfect market scenario (no fees, no underpricing), what would the equilibrium share price have been if the firm raised the same amount?

d) Compute the total wealth transfer from original shareholders due to underpricing and underwriting fees using both methods.

Question Set 2: Asset Pricing and Risk (Based on Formulas 2.1-2.6)

Question 2.1:

An investor purchased a cryptocurrency at €5,000 on January 1. On December 31, the price was €6,500, and the investor received no dividends or cash flows during the year.

a) Calculate the annual return.

b) If the investor held the asset for 5 years with the following year-end prices: €5,000, €6,500, €5,800, €7,200, €8,100, €9,000, calculate:

- Average annual return
- Standard deviation of returns
- If the risk-free rate is 2%, calculate the Sharpe ratio

Question 2.2:

A portfolio manager tracks monthly returns of three cryptocurrencies over 36 months with the following statistics:

Crypto	Mean Return	Std Dev	Min Return	Max Return
Bitcoin	2.5%	15%	-35%	48%
Ethereum	3.1%	18%	-28%	52%
Cardano	1.8%	22%	-45%	67%

Given risk-free rate = 0.5% per month:

a) Calculate the Sharpe ratio for each cryptocurrency.

b) If you could only invest in one, which would you choose based on risk-adjusted returns?

c) The manager notes that Bitcoin returns have skewness of 0.8 and kurtosis of 4.2. What do these values indicate about the return distribution?

Question Set 3: CAPM and Alpha (Based on Formulas 3.1-3.3)

Question 3.1:

An analyst provides the following predictions for two German stocks:

- **Stock X (SAP):** Expected return = 13%, Beta = 1.8
- **Stock Y (Siemens):** Expected return = 9%, Beta = 1.2

Assume: Risk-free rate = 2.5%, Market risk premium = 6%

a) Using CAPM, calculate the required return for each stock.

b) Calculate the alpha for each stock.

c) Based on alpha, which stock appears more attractive? Explain.

d) Name two potential issues with this stock-picking approach (measurement errors, joint hypothesis problem, etc.).

Question 3.2:

A portfolio manager wants to estimate the beta of Adidas stock. Using 60 months of historical data, they run a regression of Adidas excess returns against DAX excess returns and obtain:

- Beta coefficient: 0.95
- Alpha (intercept): 0.3% per month
- R-squared: 0.72

Given: Risk-free rate = 2% annually, Market risk premium = 7% annually

a) Calculate the required annual return for Adidas using CAPM.

b) Convert the monthly alpha to an annual figure. Is it statistically significant?

c) What does the R-squared of 0.72 indicate?

Question Set 4: Classification Metrics (Based on Formulas 4.1-4.4)

Question 4.1:

A machine learning model predicts credit defaults for 1,000 loan applications. The confusion matrix shows:

	Actual Default	Actual No Default
Predicted Default	180	45
Predicted No Default	70	705

a) Draw the complete confusion matrix with TP, TN, FP, FN labels.

b) Calculate:

- Accuracy
- Precision
- Recall
- F1 Score

c) The bank's current rule-based system has 85% accuracy. Should they switch to the ML model based on accuracy alone? Explain.

d) If the bank is more concerned about missing actual defaults (false negatives) than false alarms (false positives), which metric should they prioritize?

Question 4.2:

A robo-advisor develops a model to predict "winning" stocks (outperforming the market). After backtesting on 500 stocks over one year:

- The model predicts 280 stocks as "winners"
- In reality, 320 stocks actually outperformed
- Of the 280 predicted winners, 215 were correct

a) Construct the confusion matrix.

b) Calculate Accuracy, Precision, Recall, and F1 score.

c) Compare this to a naive strategy of predicting all stocks as winners. Which has higher accuracy?

d) Despite potentially lower accuracy, explain a scenario where the ML model still provides value (hint: think about Precision vs the naive strategy).

Part 4: Answers to Sample Questions

Answers to Question Set 1: IPO Valuation

Answer 1.1:

a) **Net IPO Proceeds:**

$$\text{Proceeds} = 3 \text{ million} \times € \ 25 \times (1 - 0.06) = 3 \times 25 \times 0.94 = € \ 70.5 \text{ million}$$

b) **Market Value After IPO:**

$$\text{Market Value} = (8 + 3) \text{ million} \times € \ 42 = 11 \times 42 = € \ 462 \text{ million}$$

c) **Perfect Market Share Price:**

Let P = perfect market price per share

Fair Market Value = Proceeds + Value of original shares:

$$462 = 70.5 + 8P$$

$$8P = 391.5$$

$$P = € \ 48.9375 \text{ per share}$$

Alternatively, if we need to raise €70.5 million:

$$\text{Shares needed} = \frac{70.5}{48.9375} = 1.441 \text{ million}$$

Total shares = $8 + 1.441 = 9.441$ million

$$\text{Price} = \frac{462}{9.441} = € \ 48.94 \text{ per share}$$

d) **Total Cost of Market Imperfections:**

Method 1 (Component approach):

- Underwriting fees: $3 \times 25 \times 0.06 = € \ 4.5$ million
- Underpricing cost: $3 \times (42 - 25) = € \ 51$ million
- Total cost: $4.5 + 51 = € \ 55.5$ million

Method 2 (Wealth transfer approach):

$$\text{Cost} = (48.94 - 42) \times 8 = 6.94 \times 8 = € \ 55.52 \text{ million}$$

Answer 1.2:

a) **Capital Raised:**

$$\text{Proceeds} = 6 \times € \ 18 \times (1 - 0.055) = 6 \times 18 \times 0.945 = € \ 102.06 \text{ million}$$

b) **Market Capitalization:**

$$\text{Market Cap} = (15+6)\times \text{€ } 28 = 21 \times 28 = \text{€ } 588 \text{ million}$$

c) **Perfect Market Price:**

Fair Market Value equation:

$$588 = 102.06 + 15P$$

$$P = \frac{485.94}{15} = \text{€ } 32.396 \text{ per share}$$

Or solving for shares needed at this price:

$$\text{New shares} = \frac{102.06}{32.396} = 3.15 \text{ million}$$

$$\text{Price} = \frac{588}{15+3.15} = \text{€ } 32.40 \text{ per share}$$

d) **Total Cost:**

Method 1:

- Underwriting: $6 \times 18 \times 0.055 = \text{€ } 5.94$ million
- Underpricing: $6 \times (28-18) = \text{€ } 60$ million
- Total: $5.94 + 60 = \text{€ } 65.94$ million

Method 2:

$$(32.40 - 28) \times 15 = 4.40 \times 15 = \text{€ } 66 \text{ million}$$

Answers to Question Set 2: Asset Pricing and Risk

Answer 2.1:

a) **Annual Return:**

$$r = \frac{6,500 - 5,000 + 0}{5,000} = \frac{1,500}{5,000} = 0.30 = 30\%$$

b) **5-Year Analysis:**

Year-by-year returns:

- Year 1: $(6,500 - 5,000)/5,000 = 30.0\%$
- Year 2: $(5,800 - 6,500)/6,500 = -10.77\%$
- Year 3: $(7,200 - 5,800)/5,800 = 24.14\%$
- Year 4: $(8,100 - 7,200)/7,200 = 12.50\%$
- Year 5: $(9,000 - 8,100)/8,100 = 11.11\%$

Average Return:

$$\bar{r} = \frac{30.0 - 10.77 + 24.14 + 12.50 + 11.11}{5} = \frac{66.98}{5} = 13.40\%$$

Standard Deviation:

$$\sigma = \sqrt{\frac{1}{4}[(30.0 - 13.4)^2 + (-10.77 - 13.4)^2 + (24.14 - 13.4)^2 + (12.50 - 13.4)^2 + (\text{:}$$

$$= \sqrt{\frac{1}{4}[275.56 + 584.02 + 115.40 + 0.81 + 5.24]}$$

$$= \sqrt{\frac{981.03}{4}} = \sqrt{245.26} = 15.66\%$$

Sharpe Ratio:

$$\text{Sharpe} = \frac{13.40\% - 2\%}{15.66\%} = \frac{11.40\%}{15.66\%} = 0.728$$

Answer 2.2:

a) **Sharpe Ratios:**

Bitcoin:

$$\text{Sharpe} = \frac{2.5\% - 0.5\%}{15\%} = \frac{2.0\%}{15\%} = 0.133$$

Ethereum:

$$\text{Sharpe} = \frac{3.1\% - 0.5\%}{18\%} = \frac{2.6\%}{18\%} = 0.144$$

Cardano:

$$\text{Sharpe} = \frac{1.8\% - 0.5\%}{22\%} = \frac{1.3\%}{22\%} = 0.059$$

b) **Investment Choice:**

Based on Sharpe ratios, **Ethereum offers the best risk-adjusted return** (0.144), followed closely by Bitcoin (0.133). Cardano has the lowest risk-adjusted performance (0.059).

c) **Skewness and Kurtosis Interpretation:**

- **Skewness = 0.8 (positive):** The distribution is right-skewed, meaning there are more extreme positive returns than extreme negative returns. This is generally favorable for investors.
- **Kurtosis = 4.2 (high):** The distribution has fat tails, indicating a higher probability of extreme outcomes (both positive and negative) compared to a normal distribution. This suggests higher tail risk and potential for large price swings.

Answers to Question Set 3: CAPM and Alpha

Answer 3.1:

a) **Required Returns using CAPM:**

Stock X (SAP):

$$E(r_X) = 2.5\% + 1.8 \times 6\% = 2.5\% + 10.8\% = 13.3\%$$

Stock Y (Siemens):

$$E(r_Y) = 2.5\% + 1.2 \times 6\% = 2.5\% + 7.2\% = 9.7\%$$

b) **Alpha Calculations:**

Stock X:

$$\alpha_X = 13\% - 13.3\% = -0.3\%$$

Stock Y:

$$\alpha_Y = 9\% - 9.7\% = -0.7\%$$

c) **Stock Attractiveness:**

Stock X is more attractive because it has a smaller negative alpha (-0.3% vs -0.7%). Both stocks are slightly overvalued according to CAPM, but Stock X is closer to fair value. In practice, Stock X offers expected returns closer to what its risk level demands.

d) **Two Issues with Stock-Picking Approach:**

1. **Measurement Errors:** Expected returns and betas must be estimated from historical data or forecasts, which are subject to significant estimation errors. Small changes in estimates can dramatically alter the alpha calculation and investment decision.
2. **Joint Hypothesis Problem:** When testing for mispricing (non-zero alpha), we simultaneously test whether (a) the stock is mispriced AND (b) CAPM is the correct model. If we find significant alpha, we cannot determine whether it reflects true mispricing or a missing risk factor that CAPM doesn't capture (e.g., size, value, momentum factors from Fama-French models).

Answer 3.2:

a) **Required Annual Return for Adidas:**

Given beta = 0.95:

$$E(r) = 2\% + 0.95 \times 7\% = 2\% + 6.65\% = 8.65\%$$

b) **Annual Alpha:**

Monthly alpha = 0.3% per month

Simple annualization (approximation):

$$\alpha_{annual} = 0.3\% \times 12 = 3.6\% \text{ per year}$$

Compound annualization (more accurate):

$$\alpha_{annual} = (1.003)^{12} - 1 = 1.0366 - 1 = 3.66\% \text{ per year}$$

Statistical Significance: A positive alpha of 3.6-3.66% annually suggests potential outperformance. However, to determine statistical significance, we would need the standard error of the alpha estimate and conduct a t-test. Generally, for 60 observations, a t-statistic > 2.0 indicates significance at the 5% level.

c) **R-squared Interpretation:**

R^2 = 0.72 means that 72% of Adidas's return variation is explained by market (DAX) movements, with the remaining 28% attributed to company-specific factors (idiosyncratic risk). This is a relatively high R^2, indicating that Adidas stock moves quite closely with the overall market, though it still retains meaningful independent variation.

Answers to Question Set 4: Classification Metrics

Answer 4.1:

a) **Complete Confusion Matrix:**

	Actual Positive (Default)	Actual Negative (No Default)	Total
Predicted Positive	TP = 180	FP = 45	225
Predicted Negative	FN = 70	TN = 705	775
Total	250	750	1,000

b) **Metric Calculations:**

Accuracy:

$$A = \frac{TP + TN}{TP + TN + FP + FN} = \frac{180 + 705}{1,000} = \frac{885}{1,000} = 88.5\%$$

Precision:

$$P = \frac{TP}{TP + FP} = \frac{180}{180 + 45} = \frac{180}{225} = 80.0\%$$

Recall (Sensitivity):

$$R = \frac{TP}{TP + FN} = \frac{180}{180 + 70} = \frac{180}{250} = 72.0\%$$

F1 Score:

$$F_1 = 2 \times \frac{P \times R}{P + R} = 2 \times \frac{0.80 \times 0.72}{0.80 + 0.72} = 2 \times \frac{0.576}{1.52} = 2 \times 0.3789 = 75.8\%$$

c) **Comparison with Rule-Based System:**

The ML model has **88.5% accuracy vs. 85% for the current system**, so it performs better on overall accuracy. **However, accuracy alone is insufficient** for this decision because:

- The dataset is imbalanced (25% defaults vs. 75% non-defaults)
- A naive model predicting "no default" for everyone would achieve 75% accuracy
- The cost of false negatives (missing defaults) is likely much higher than false positives (rejected good loans)

Recommendation: Evaluate additional metrics (especially Recall and F1) and consider the business costs of different error types before switching systems.

d) **Priority Metric:**

If the bank is most concerned about **missing actual defaults (false negatives)**, they should prioritize **Recall (72.0%)**.

- High Recall ensures the model identifies most of the actual defaulters
- This minimizes the risk of approving loans that will default
- The tradeoff is accepting more false positives (some good borrowers get rejected)

In practice, the bank should optimize for a high Recall while maintaining acceptable Precision, perhaps using the F1 score as a balanced metric or adjusting the classification threshold to increase Recall at the expense of Precision.

Answer 4.2:

a) **Confusion Matrix Construction:**

Given information:

- Model predicts 280 stocks as winners (Predicted Positive)
- Reality: 320 stocks actually outperform (Actual Positive)
- Of 280 predicted winners, 215 were correct (TP = 215)

Calculations:

- TP = 215
- FP = 280 - 215 = 65
- FN = 320 - 215 = 105
- TN = 500 - (215 + 65 + 105) = 115

	Actual Winner	Actual Loser	Total
Predicted Winner	TP = 215	FP = 65	280
Predicted Loser	FN = 105	TN = 115	220
Total	320	180	500

b) **Metric Calculations:**

Accuracy:

$$A = \frac{215 + 115}{500} = \frac{330}{500} = 66.0\%$$

Precision:

$$P = \frac{215}{280} = 76.79\%$$

Recall:

$$R = \frac{215}{320} = 67.19\%$$

F1 Score:

$$F_1 = 2 \times \frac{0.7679 \times 0.6719}{0.7679 + 0.6719} = 2 \times \frac{0.5159}{1.4398} = 71.66\%$$

c) **Naive Strategy Comparison:**

Naive "All Winners" Strategy:

- Predicts all 500 stocks as winners
- Correct predictions: 320 (all actual winners)
- Incorrect predictions: 180 (all actual losers)
- **Accuracy: 320/500 = 64.0%**

Comparison: The ML model has **66.0% accuracy vs. 64.0% for the naive strategy**, so the ML model is slightly better but not dramatically so.

d) **ML Model Value Despite Lower/Similar Accuracy:**

Scenario where ML model provides value:

Consider a portfolio manager with limited capital who can only invest in **50 stocks**. The key question is not "which strategy has higher overall accuracy?" but rather **"among stocks predicted as winners, how accurate is the prediction?"**

ML Model Precision: 76.79%

- If the manager selects 50 stocks the model predicts as winners, approximately 38-39 of them will actually be winners (76.79% × 50 ≈ 38)

Naive Strategy Precision: 64.0%

- If the manager randomly selects 50 stocks (or uses the "all winners" heuristic proportionally), only about 32 of them will actually be winners (64.0% × 50 ≈ 32)

Value Proposition:
The ML model delivers **6-7 more winning stocks** in a 50-stock portfolio compared to random selection, representing a **19% improvement in portfolio quality** (38/32 - 1 ≈ 0.19).

Conclusion: When the use case involves **selective investment** (not investing in all stocks), **Precision becomes more important than Accuracy**. The ML model's superior Precision (76.79% vs 64.0%) provides substantial value by helping investors construct better-performing portfolios, even though its overall accuracy advantage is modest.

References

All formulas and examples were extracted from the following Digital Finance course materials:

1. Solutions_Exercise-ICOs.pdf (Prof. Dr. Sebastian Müller, TUM)
2. VL6_Asset-Pricing-With-Cryptocurrencies.pdf (Prof. Dr. Sebastian Müller, TUM)
3. Solutions_Exercise-3-BitcoinBlockchain.pdf (Prof. Dr. Sebastian Müller, TUM)
4. Solutions_Exercise-5-Robo_Intro.pdf (Prof. Dr. Sebastian Müller, TUM)
5. Solutions_Exercise-AI_in_Finance.pdf (Prof. Dr. Sebastian Müller, TUM)
6. Solutions_Exercise-7-Robo_Customization_Trends.pdf (Prof. Dr. Sebastian Müller, TUM)

Course: Digital Finance, Winter Term 2025/2026
Institution: TUM Center for Digital Transformation, TUM School of Management, TUM Campus Heilbronn