Homework 1

Swaraj Khadanga

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1. a. Estimate Maximum Likelihood for
$f(x) = \frac{x}{\theta^2} exp \frac{-x^2}{2\theta^2}$
so Maximum Likelihood function (ML) $= \prod_1^N \frac{x_i}{\theta^2} exp \frac{-x_i^2}{2\theta^2}$
$\implies$ Maximum Log Likelihood function (MLL) =
$\sum_1^N [\log x_i - 2 \log \theta - \frac{x_i^2}{2\theta^2}]$

To maximize MLL we need to set $\frac{\partial MLL}{\partial \theta} = 0 \implies \sum_1^N [0 - \frac{2}{\theta} - \frac{x_i^2}{2} \frac{-2}{\theta^3}] = 0$
$\implies \sum_1^N [\frac{-2}{\theta} + \frac{x_i^2}{\theta^3}] = 0 \implies \frac{1}{\theta} \sum_1^N [2 + \frac{x_i^2}{\theta^2}] = 0$
$\implies \sum_1^N [-2 + \frac{x_i^2}{\theta^2}] = 0$
$\implies \sum_1^N -2 + \sum_1^N \frac{x_i^2}{\theta^2} = 0$
$\implies -2N + \frac{1}{\theta^2} \sum_1^N x_i^2 = 0$
$\implies \frac{1}{\theta^2} \sum_1^N x_i^2 = 2N$
$\implies \frac{\sum_1^N x_i^2}{2N} = \theta^2$
$\implies \hat{\theta} = \sqrt{\frac{\sum_1^N x_i^2}{2N}}$

c. $f(x) = \frac{1}{\theta} \forall 0 <= x <= \theta$
so Maximum Likelihood (ML) $= \prod_1^N \frac{1}{\theta}$
$\implies ML = \frac{1}{\theta^N}$
To maximize the above likelihood we need to minimize θ
But we also have a constraint that $0 <= x <= \theta$
so $\hat{\theta} = max[x_i]$