Homework 1

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1. a. Estimate Maximum Likelihood for
$f(x) = \frac{x}{\Theta^2} exp \frac{-x^2}{2\Theta^2}$
so Maximum Likelihood function (ML) $= \prod_1^N \frac{x_i}{\Theta^2} exp \frac{-x_i^2}{2\Theta^2}$
$\implies$ Maximum Log Likelihood function (MLL) =
$\sum_1^N [\log x_i - 2 \log \Theta - \frac{x_i^2}{2\Theta^2}]$

To maximize MLL we need to set $\frac{\partial MLL}{\partial \Theta} = 0 \implies \sum_1^N [0 - \frac{2}{\Theta} - \frac{x_i^2}{2} \frac{-2}{\Theta^3}] = 0$
$\implies \sum_1^N [\frac{-2}{\Theta} + \frac{x_i^2}{\Theta^3}] = 0 \implies \frac{1}{\Theta} \sum_1^N [2 + \frac{x_i^2}{\Theta^2}] = 0$
$\implies \sum_1^N [-2 + \frac{x_i^2}{\Theta^2}] = 0$
$\implies \sum_1^N -2 + \sum_1^N \frac{x_i^2}{\Theta^2} = 0$
$\implies -2N + \frac{1}{\Theta^2} \sum_1^N x_i^2 = 0$
$\implies \frac{1}{\Theta^2} \sum_1^N x_i^2 = 2N$
$\implies \frac{\sum_1^N x_i^2}{2N} = \Theta^2$
$\implies \Theta = \sqrt{\frac{\sum_1^N x_i^2}{2N}}$