```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.model_selection import train_test_split, cross_val_score,
KFold, GridSearchCV
from sklearn.linear_model import LinearRegression
from sklearn.ensemble import RandomForestRegressor,
GradientBoostingRegressor, StackingRegressor
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import MinMaxScaler

# Load datasets
fed_files = ['CPIAUCSL.csv', 'RRVRUSQ156N.csv', 'MORTGAGE30US.csv',
'MSPUS.csv',
             'USAUCSFRCONDOSMSAMID.csv', 'GDPC1.csv', 'UNRATE.csv',
'PI.csv',
             'POPTOTUSA647NWDB.csv', 'CSUSHPISA.csv']
data_frames = [pd.read_csv(f, parse_dates=True, index_col=0) for f in
fed_files]

data = pd.concat(data_frames, axis=1)
data = data.ffill().dropna()

data.columns = ['CPI', 'RentalVacancyRate', 'MortgageRate',
'MedianSalePrice', 'ZillowIndex',
                 'GDP', 'UnemploymentRate', 'PersonalIncome',
'Population', 'HomePriceIndex']
data
```

```
                CPI  RentalVacancyRate  MortgageRate  MedianSalePrice  \
DATE

2004-01-08  186.300               10.4          5.87         212700.0

2004-01-15  186.300               10.4          5.66         212700.0

2004-01-22  186.300               10.4          5.64         212700.0

2004-01-29  186.300               10.4          5.68         212700.0

2004-02-01  186.700               10.4          5.68         212700.0

...             ...                ...           ...              ...

2024-03-21  312.230                6.6          6.87         426800.0

2024-03-28  312.230                6.6          6.79         426800.0

2024-04-01  313.207                6.6          6.79         412300.0
```

```
2024-05-01  313.207                 6.6          6.79        412300.0

2024-06-01  313.207                 6.6          6.79        412300.0

              ZillowIndex        GDP  UnemploymentRate  PersonalIncome  \
DATE

2004-01-08  163387.988490  15248.680               5.7          9731.8

2004-01-15  163387.988490  15248.680               5.7          9731.8

2004-01-22  163387.988490  15248.680               5.7          9731.8

2004-01-29  163387.988490  15248.680               5.7          9731.8

2004-02-01  164448.866854  15248.680               5.6          9765.4

...                   ...        ...               ...             ...

2024-03-21  359543.076444  22758.752               3.8         23746.6

2024-03-28  359543.076444  22758.752               3.8         23746.6

2024-04-01  361420.150687  22918.739               3.9         23809.6

2024-05-01  361420.150687  22918.739               4.0         23923.7

2024-06-01  361420.150687  22918.739               4.1         23923.7

             Population  HomePriceIndex
DATE
2004-01-08  292805298.0         141.647
2004-01-15  292805298.0         141.647
2004-01-22  292805298.0         141.647
2004-01-29  292805298.0         141.647
2004-02-01  292805298.0         143.192
...                 ...             ...
2024-03-21  334914895.0         318.217
2024-03-28  334914895.0         318.217
2024-04-01  334914895.0         319.048
2024-05-01  334914895.0         319.048
2024-06-01  334914895.0         319.048

[1268 rows x 10 columns]
```

```python
# Ensure the date index has frequency information
data.index = pd.to_datetime(data.index)
data = data.asfreq('MS')  # Monthly Start frequency

# Create new feature: normalized home price index
data['NormalizedHomePriceIndex'] = data['HomePriceIndex'] /
data['CPI']

# Add lag features
for lag in range(1, 13):
    data[f'HomePriceIndex_Lag_{lag}'] =
data['HomePriceIndex'].shift(lag)

# Drop rows with missing values created by lag features
data = data.dropna()

# Verify if data is correctly processed
print(f"Data after preprocessing: {data.shape}")
print(data.head())
```

```
Data after preprocessing: (233, 23)
              CPI  RentalVacancyRate  MortgageRate  MedianSalePrice  \
DATE
2005-02-01  192.4               10.1          5.66         232500.0
2005-03-01  193.1               10.1          5.69         232500.0
2005-04-01  193.7                9.8          6.04         233700.0
2005-05-01  193.6                9.8          5.78         233700.0
2005-06-01  193.7                9.8          5.65         233700.0

              ZillowIndex        GDP  UnemploymentRate  PersonalIncome
\
DATE

2005-02-01  182971.283783  15844.727               5.4         10282.7

2005-03-01  184533.914455  15844.727               5.2         10347.9

2005-04-01  186294.280193  15922.782               5.2         10412.1

2005-05-01  188164.303058  15922.782               5.1         10464.1

2005-06-01  190122.978622  15922.782               5.0         10498.0


             Population  HomePriceIndex  ...  HomePriceIndex_Lag_3  \
DATE                                     ...
2005-02-01  295516599.0         163.344  ...               157.528
2005-03-01  295516599.0         165.813  ...               159.331
2005-04-01  295516599.0         167.502  ...               161.289
2005-05-01  295516599.0         169.351  ...               163.344
2005-06-01  295516599.0         171.191  ...               165.813
```

```
            HomePriceIndex_Lag_4  HomePriceIndex_Lag_5
HomePriceIndex_Lag_6  \
DATE

2005-02-01               155.752               154.180
152.634
2005-03-01               157.528               155.752
154.180
2005-04-01               159.331               157.528
155.752
2005-05-01               161.289               159.331
157.528
2005-06-01               163.344               161.289
159.331

            HomePriceIndex_Lag_7  HomePriceIndex_Lag_8
HomePriceIndex_Lag_9  \
DATE

2005-02-01               151.338               149.851
148.186
2005-03-01               152.634               151.338
149.851
2005-04-01               154.180               152.634
151.338
2005-05-01               155.752               154.180
152.634
2005-06-01               157.528               155.752
154.180

            HomePriceIndex_Lag_10  HomePriceIndex_Lag_11  \
DATE
2005-02-01                146.592                145.058
2005-03-01                148.186                146.592
2005-04-01                149.851                148.186
2005-05-01                151.338                149.851
2005-06-01                152.634                151.338

            HomePriceIndex_Lag_12
DATE
2005-02-01                143.192
2005-03-01                145.058
2005-04-01                146.592
2005-05-01                148.186
2005-06-01                149.851

[5 rows x 23 columns]
```

```python
# Features and target variable
X = data.drop(columns=['HomePriceIndex'])
y = data['HomePriceIndex']

# Train-test split
X_train, X_test, y_train, y_test = train_test_split(X, y,
test_size=0.2, random_state=42)

# Initialize models
lr = LinearRegression()
rf = RandomForestRegressor(n_estimators=100, random_state=42)
gbr = GradientBoostingRegressor(random_state=42)

# Initialize k-fold cross-validation
kf = KFold(n_splits=5, shuffle=True, random_state=42)

# Cross-validation for Linear Regression
cv_scores_lr = cross_val_score(lr, X, y, cv=kf,
scoring='neg_mean_squared_error')
mse_cv_lr = -cv_scores_lr.mean()

# Cross-validation for Random Forest
cv_scores_rf = cross_val_score(rf, X, y, cv=kf,
scoring='neg_mean_squared_error')
mse_cv_rf = -cv_scores_rf.mean()

# Fit models on the entire training set
lr.fit(X_train, y_train)
rf.fit(X_train, y_train)
gbr.fit(X_train, y_train)

# Predict on the test set
y_pred_lr = lr.predict(X_test)
y_pred_rf = rf.predict(X_test)
y_pred_gbr = gbr.predict(X_test)

# Ensemble predictions by averaging
y_pred_ensemble = (y_pred_lr + y_pred_rf) / 2

# Stacking Regressor
param_grid_rf = {
    'n_estimators': [50, 100, 200],
    'max_depth': [None, 10, 20, 30],
    'min_samples_split': [2, 5, 10],
    'min_samples_leaf': [1, 2, 4],
    'bootstrap': [True, False]
}

grid_search_rf = GridSearchCV(estimator=rf, param_grid=param_grid_rf,
cv=kf, scoring='neg_mean_squared_error', n_jobs=-1, verbose=2)
grid_search_rf.fit(X_train, y_train)
```

```
best_rf = grid_search_rf.best_estimator_

param_grid_gbr = {
    'n_estimators': [50, 100, 200],
    'learning_rate': [0.01, 0.1, 0.2],
    'max_depth': [3, 5, 7]
}

grid_search_gbr = GridSearchCV(estimator=gbr,
param_grid=param_grid_gbr, cv=kf, scoring='neg_mean_squared_error',
n_jobs=-1, verbose=2)
grid_search_gbr.fit(X_train, y_train)
best_gbr = grid_search_gbr.best_estimator_

estimators = [
    ('lr', lr),
    ('rf', best_rf),
    ('gbr', best_gbr)
]
stacking_regressor = StackingRegressor(estimators=estimators,
final_estimator=LinearRegression())
stacking_regressor.fit(X_train, y_train)

# Predictions
y_pred_stacking = stacking_regressor.predict(X_test)
```

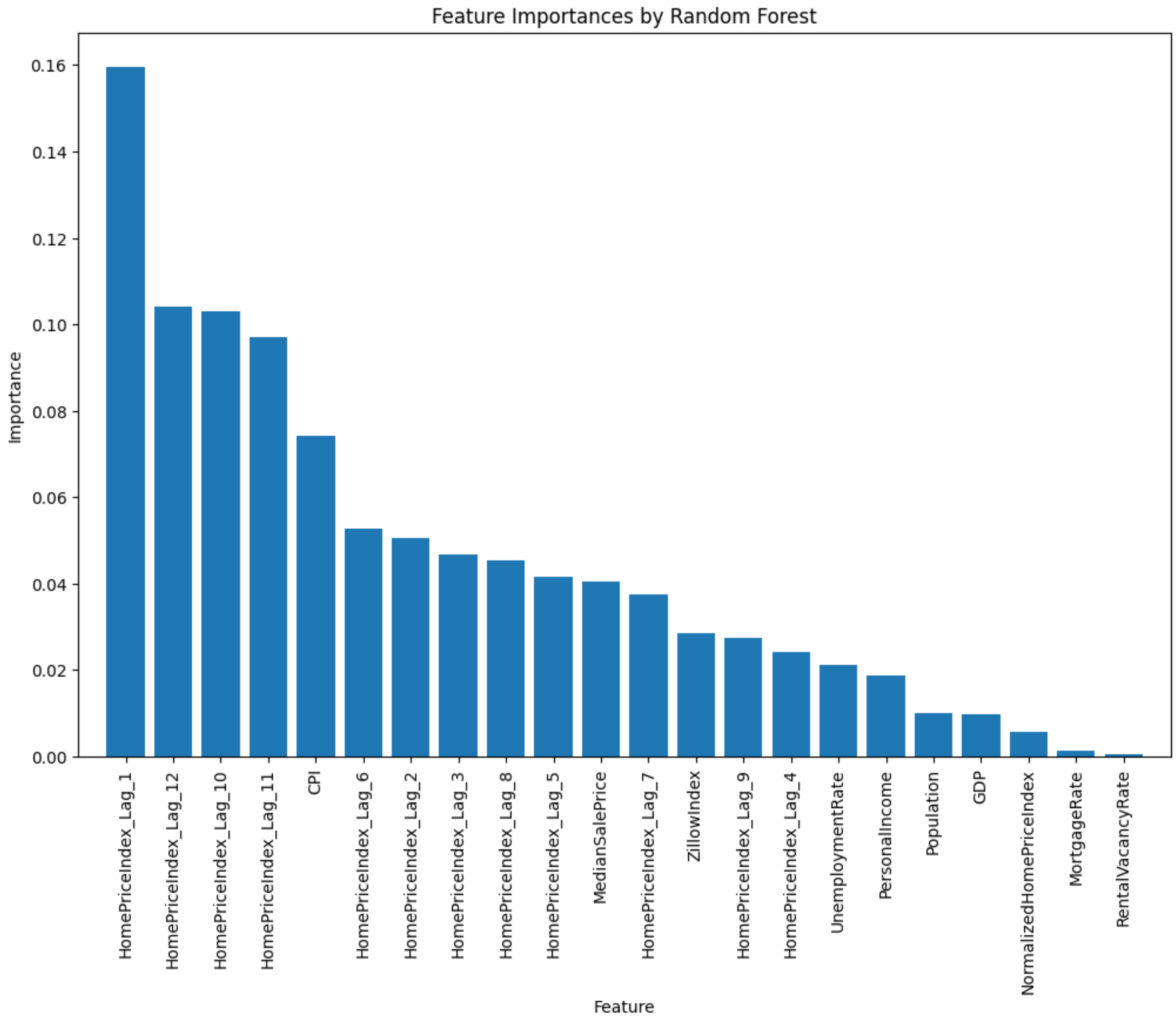
```
Fitting 5 folds for each of 216 candidates, totalling 1080 fits
Fitting 5 folds for each of 27 candidates, totalling 135 fits
```

```
# Evaluation
def evaluate_model(y_true, y_pred, model_name):
    mse = mean_squared_error(y_true, y_pred)
    r2 = r2_score(y_true, y_pred)
    mae = mean_absolute_error(y_true, y_pred)
    print(f'{model_name} - MSE: {mse:.4f}')
    print(f'{model_name} - R2: {r2:.4f}')
    print(f'{model_name} - MAE: {mae:.4f}')

# Evaluate each model
evaluate_model(y_test, y_pred_lr, 'Linear Regression')
evaluate_model(y_test, y_pred_rf, 'Random Forest')
evaluate_model(y_test, y_pred_gbr, 'Gradient Boosting')
evaluate_model(y_test, y_pred_ensemble, 'Ensemble Model')
evaluate_model(y_test, y_pred_stacking, 'Stacking Ensemble Model')
```

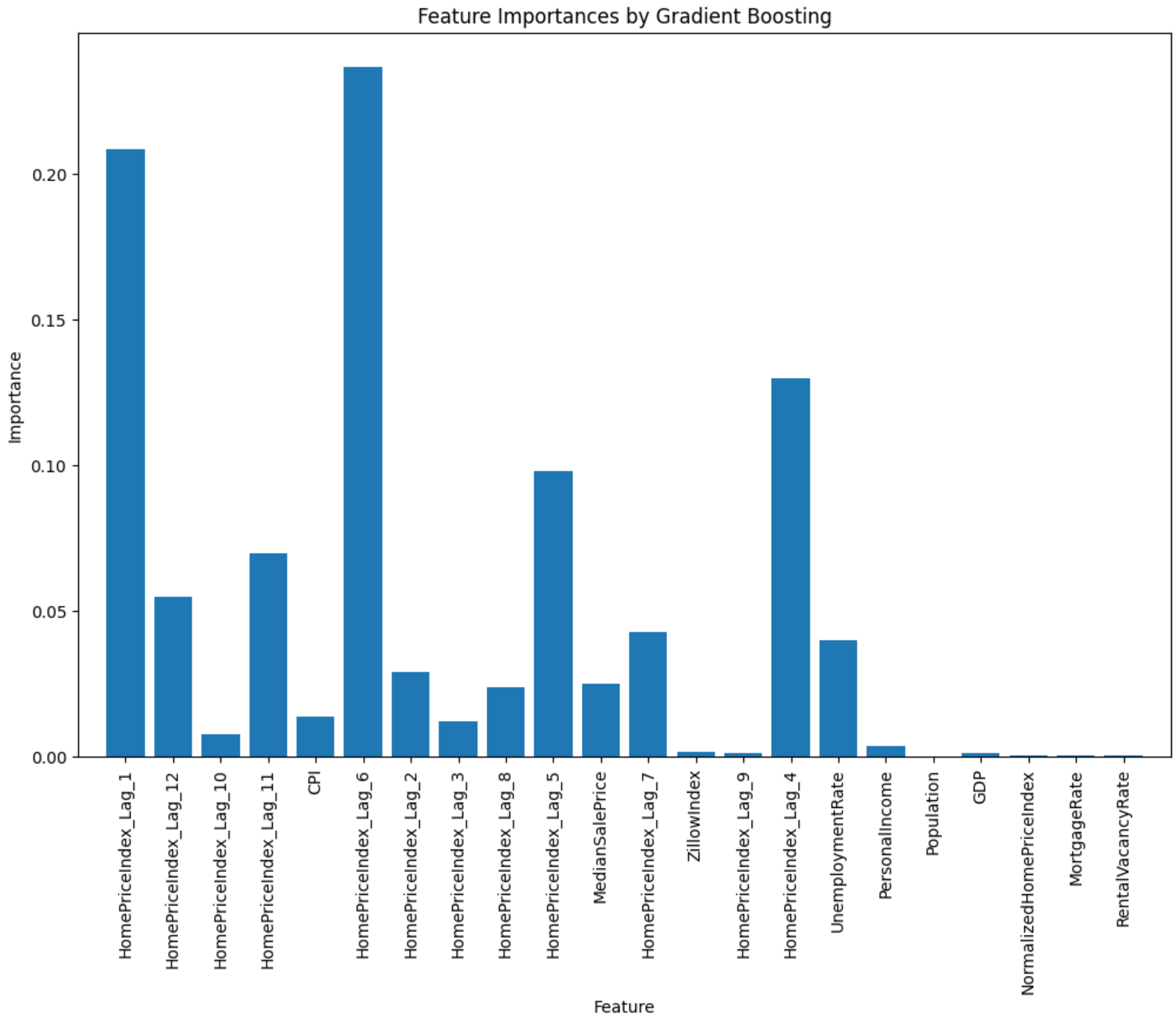
```
Linear Regression - MSE: 0.1508
Linear Regression - R2: 0.9999
Linear Regression - MAE: 0.2783
Random Forest - MSE: 1.8930
Random Forest - R2: 0.9993
```

```
Random Forest - MAE: 0.8921
Gradient Boosting - MSE: 2.3872
Gradient Boosting - R2: 0.9991
Gradient Boosting - MAE: 1.1100
Ensemble Model - MSE: 0.4940
Ensemble Model - R2: 0.9998
Ensemble Model - MAE: 0.4716
Stacking Ensemble Model - MSE: 0.1342
Stacking Ensemble Model - R2: 0.9999
Stacking Ensemble Model - MAE: 0.2581
```

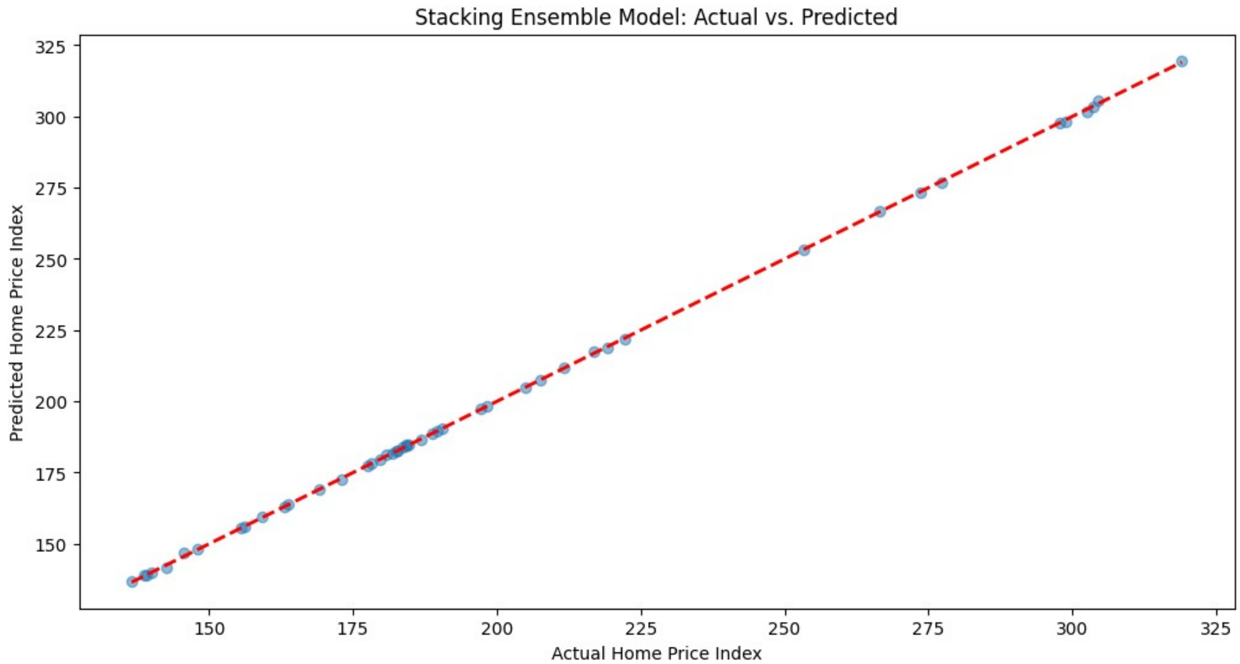
```python
# Plot feature importances for Random Forest
plt.figure(figsize=(12, 8))
importances = best_rf.feature_importances_
features = X.columns
indices = np.argsort(importances)[::-1]

plt.title('Feature Importances by Random Forest')
plt.bar(range(X.shape[1]), importances[indices], align='center')
plt.xticks(range(X.shape[1]), features[indices], rotation=90)
plt.xlim([-1, X.shape[1]])
plt.xlabel('Feature')
plt.ylabel('Importance')
plt.show()
```



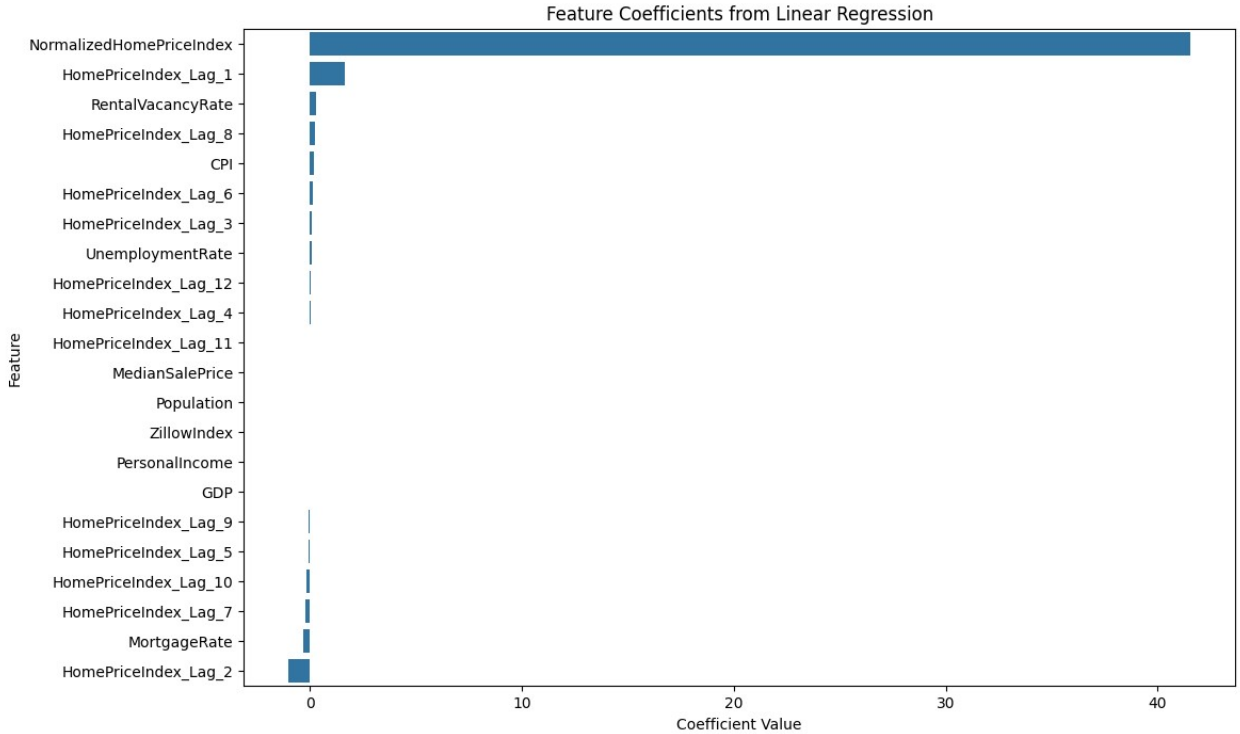

```
# Plot feature importances for Gradient Boosting
plt.figure(figsize=(12, 8))
importances = best_gbr.feature_importances_
plt.title('Feature Importances by Gradient Boosting')
plt.bar(range(X.shape[1]), importances[indices], align='center')
plt.xticks(range(X.shape[1]), features[indices], rotation=90)
plt.xlim([-1, X.shape[1]])
plt.xlabel('Feature')
plt.ylabel('Importance')
plt.show()
```



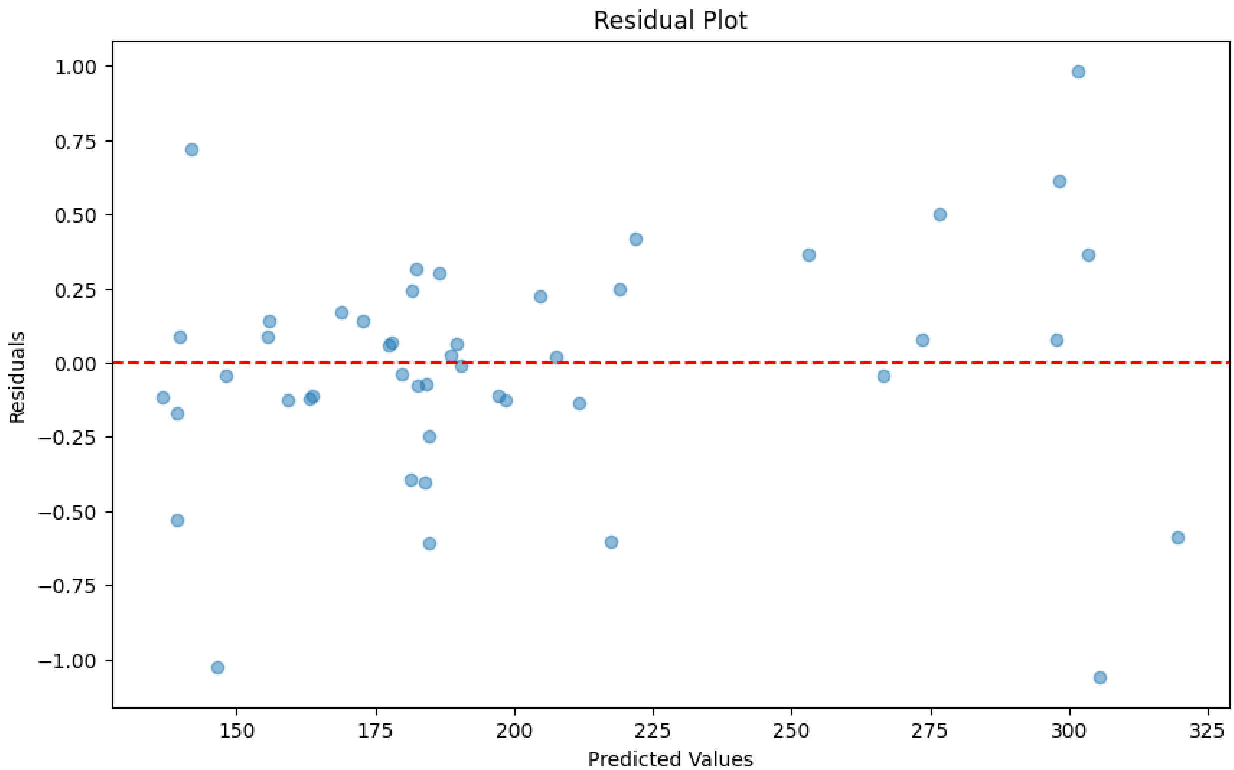

```
The above plot represents the feature importance of Gradient Boosting
model

# Plot predictions vs. actual values for Stacking Ensemble Model
plt.figure(figsize=(12, 6))
plt.scatter(y_test, y_pred_stacking, alpha=0.5)
plt.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()],
'r--', lw=2)
plt.xlabel('Actual Home Price Index')
plt.ylabel('Predicted Home Price Index')
plt.title('Stacking Ensemble Model: Actual vs. Predicted')
plt.show()
```



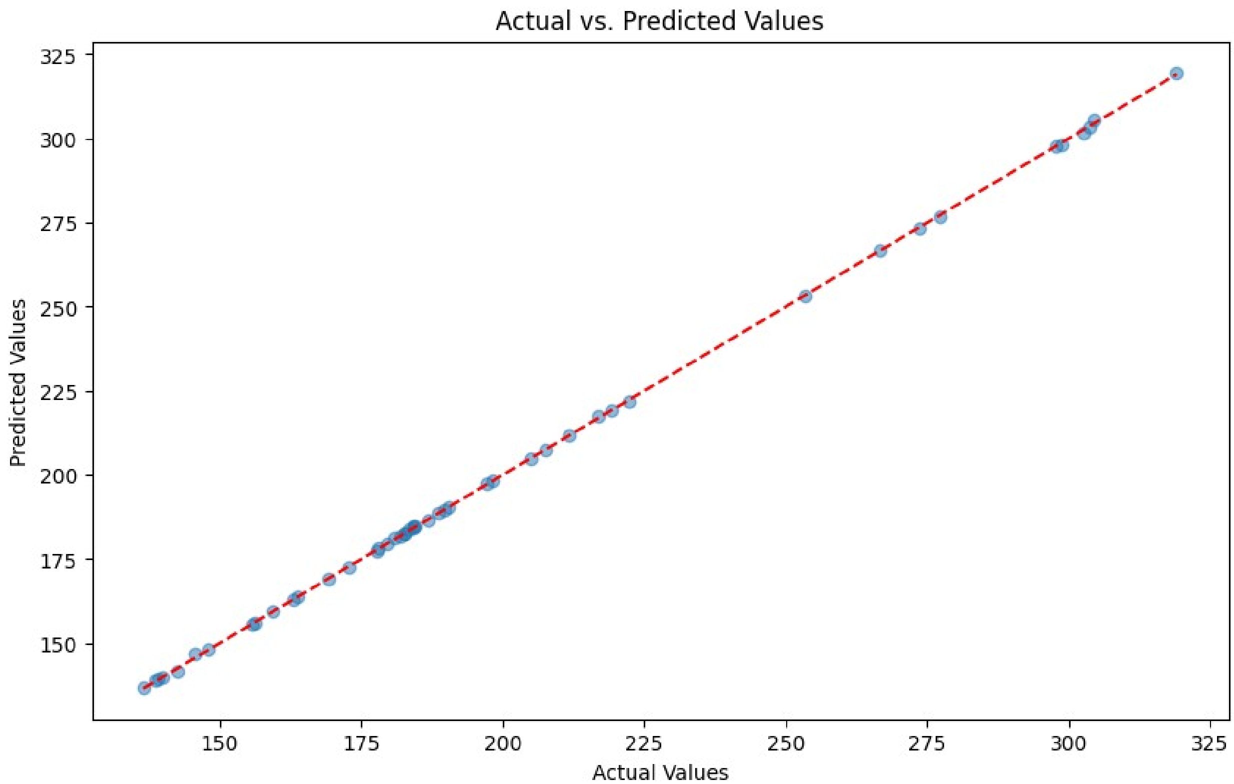

```python
import matplotlib.pyplot as plt
import seaborn as sns

# Fit the Linear Regression model
lr = LinearRegression()
lr.fit(X_train, y_train)

# Get feature coefficients
coefficients = lr.coef_
features = X.columns

# Create a DataFrame for better visualization
coef_df = pd.DataFrame({
    'Feature': features,
    'Coefficient': coefficients
}).sort_values(by='Coefficient', ascending=False)

# Plot feature coefficients
plt.figure(figsize=(12, 8))
sns.barplot(x='Coefficient', y='Feature', data=coef_df)
plt.title('Feature Coefficients from Linear Regression')
plt.xlabel('Coefficient Value')
plt.ylabel('Feature')
plt.show()
```



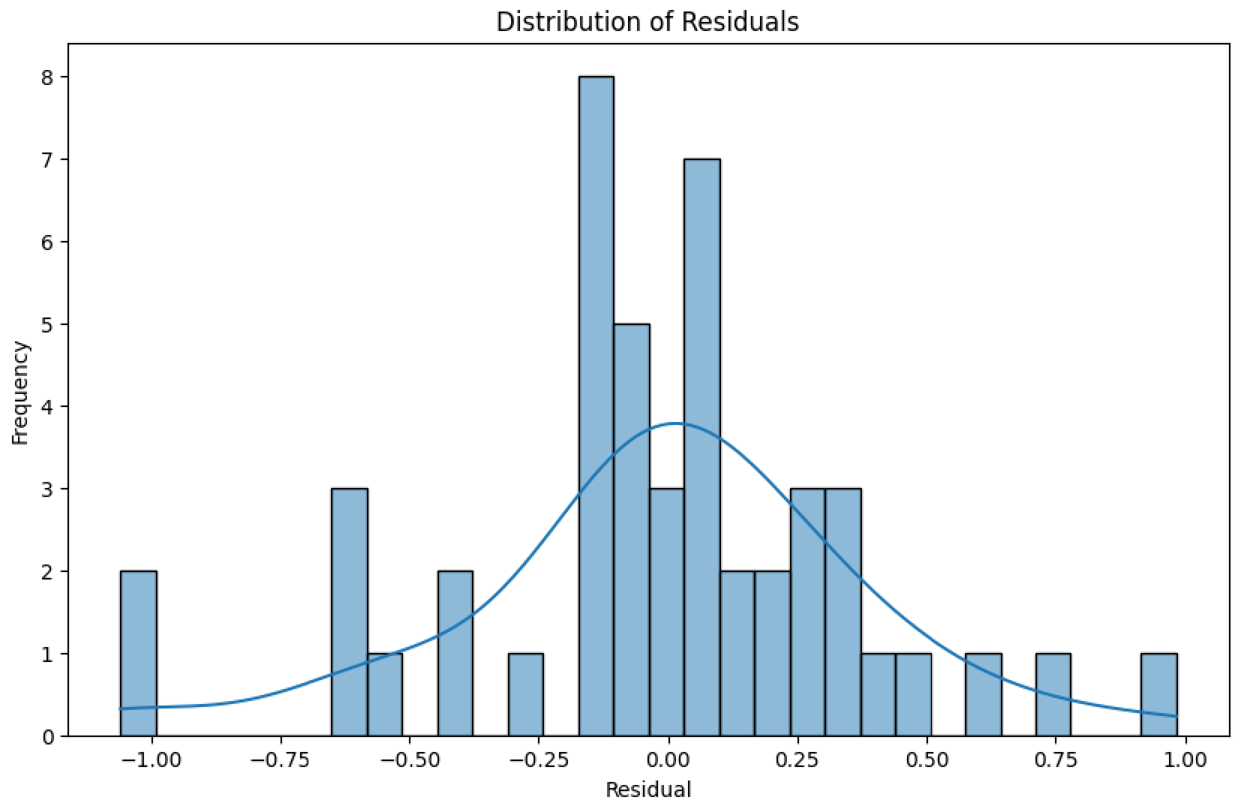

```python
# Predict on the test set
y_pred_lr = lr.predict(X_test)

# Calculate residuals
residuals = y_test - y_pred_lr

# Plot Residuals
plt.figure(figsize=(10, 6))
plt.scatter(y_pred_lr, residuals, alpha=0.5)
plt.axhline(y=0, color='r', linestyle='--')
plt.title('Residual Plot')
plt.xlabel('Predicted Values')
plt.ylabel('Residuals')
plt.show()
```



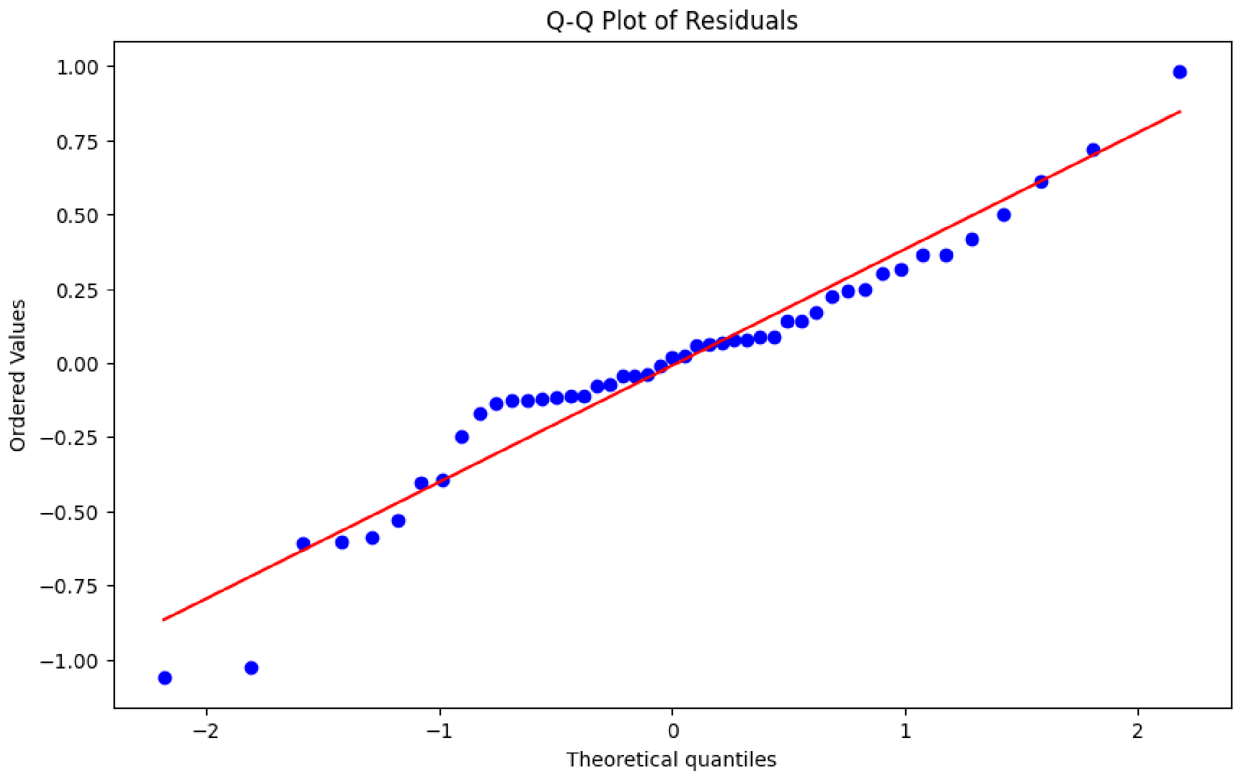

```
# Plot Actual vs. Predicted Values
plt.figure(figsize=(10, 6))
plt.scatter(y_test, y_pred_lr, alpha=0.5)
plt.plot([min(y_test), max(y_test)], [min(y_test), max(y_test)],
color='r', linestyle='--')
plt.title('Actual vs. Predicted Values')
plt.xlabel('Actual Values')
plt.ylabel('Predicted Values')
plt.show()
```



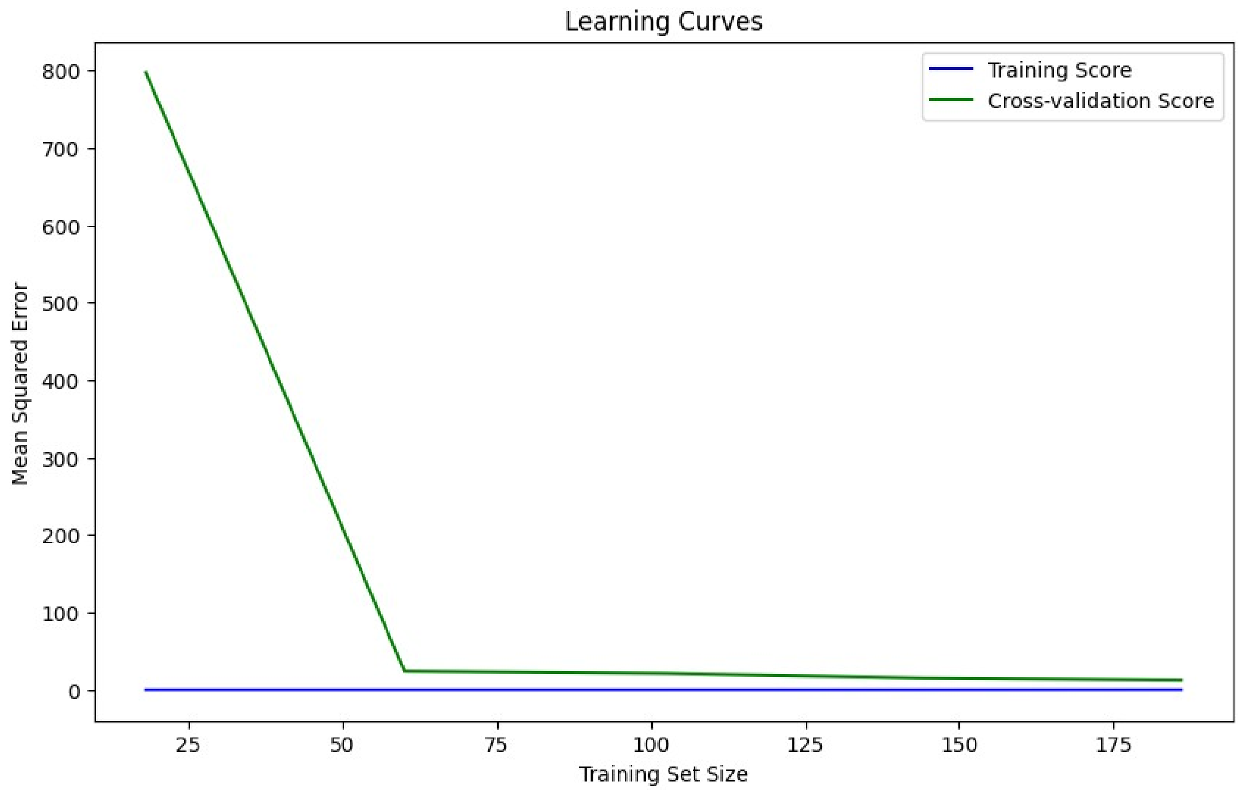

```python
# Plot Distribution of Residuals
plt.figure(figsize=(10, 6))
sns.histplot(residuals, kde=True, bins=30)
plt.title('Distribution of Residuals')
plt.xlabel('Residual')
plt.ylabel('Frequency')
plt.show()
```



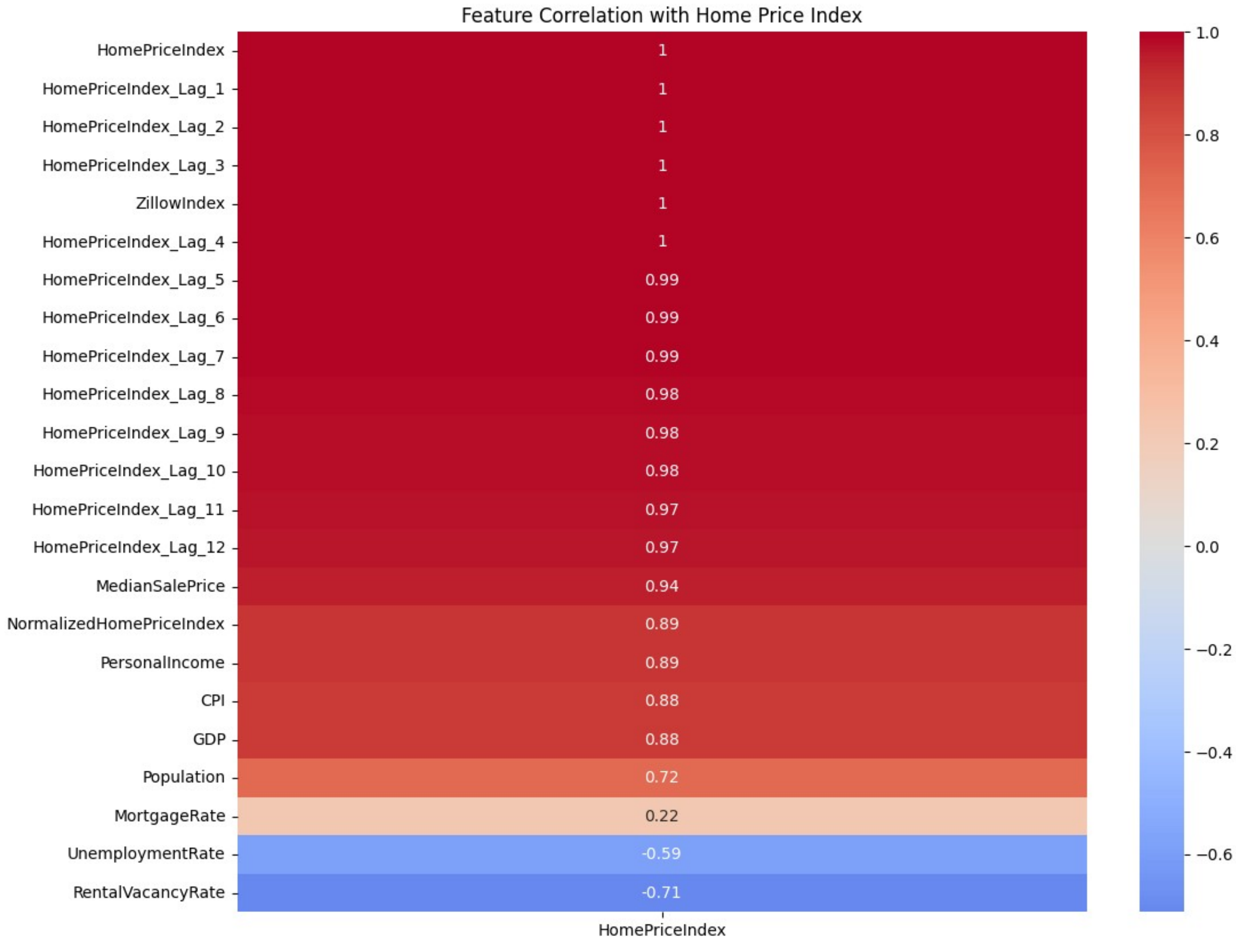

```python
import scipy.stats as stats

# Q-Q Plot
plt.figure(figsize=(10, 6))
stats.probplot(residuals, dist="norm", plot=plt)
plt.title('Q-Q Plot of Residuals')
plt.show()
```



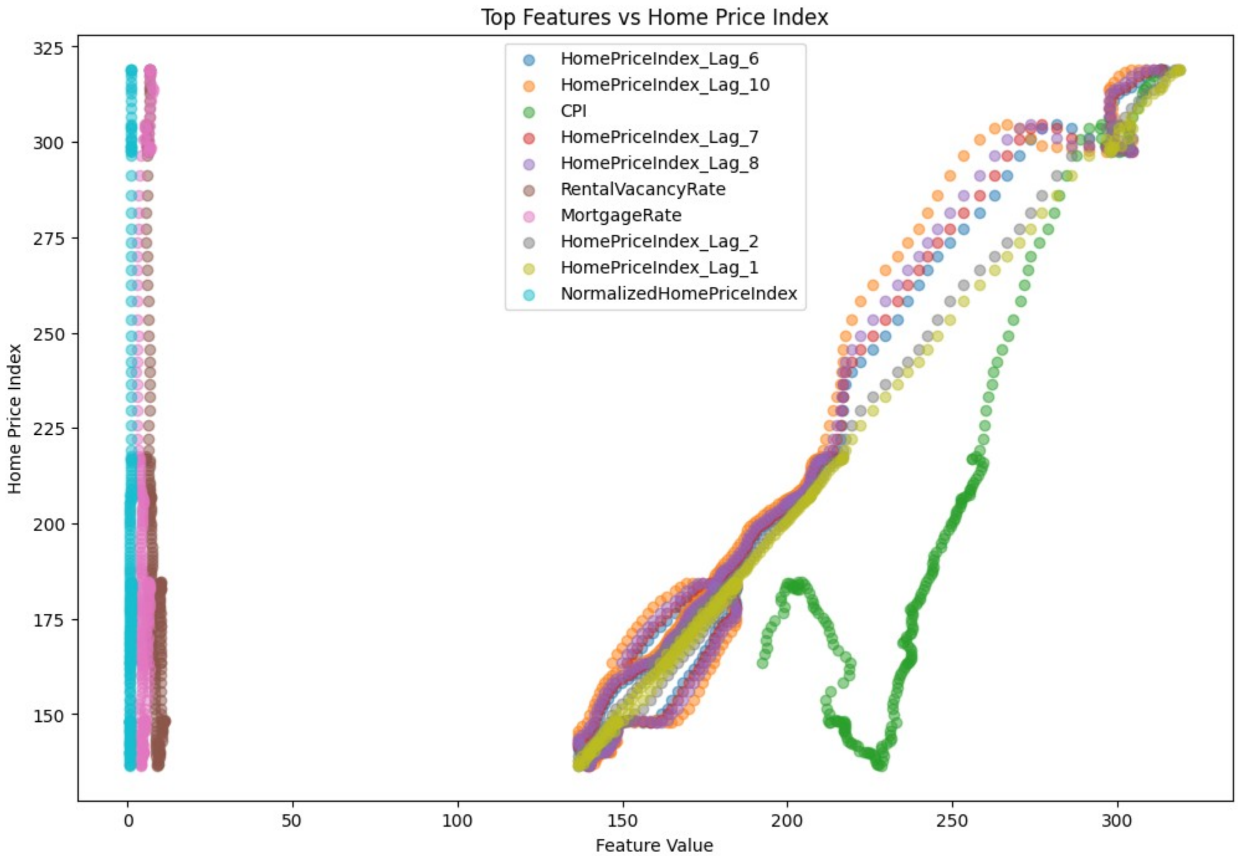

```
from sklearn.model_selection import learning_curve

# Calculate learning curves
train_sizes, train_scores, test_scores = learning_curve(lr, X, y,
cv=5, scoring='neg_mean_squared_error')

# Plot Learning Curves
plt.figure(figsize=(10, 6))
plt.plot(train_sizes, -train_scores.mean(axis=1), label='Training
Score', color='blue')
plt.plot(train_sizes, -test_scores.mean(axis=1), label='Cross-
validation Score', color='green')
plt.title('Learning Curves')
plt.xlabel('Training Set Size')
plt.ylabel('Mean Squared Error')
plt.legend()
plt.show()
```




```
import seaborn as sns

# Calculate correlation matrix
correlation_matrix = X.copy()
correlation_matrix['HomePriceIndex'] = y
correlation_matrix = correlation_matrix.corr()

# Plot Heatmap
plt.figure(figsize=(12, 10))
sns.heatmap(correlation_matrix[['HomePriceIndex']].sort_values(by='Hom
ePriceIndex', ascending=False),
            annot=True, cmap='coolwarm', center=0)
plt.title('Feature Correlation with Home Price Index')
plt.show()
```




```
# Sort features by coefficient magnitude
feature_names = X.columns
sorted_indices = np.argsort(np.abs(coefficients))
top_features = feature_names[sorted_indices][-10:]  # Get top 10
features
top_coefficients = coefficients[sorted_indices][-10:]

# Plot Top Features vs Target
plt.figure(figsize=(12, 8))
for i, feature in enumerate(top_features):
    plt.scatter(X[feature], y, label=feature, alpha=0.5)
plt.xlabel('Feature Value')
plt.ylabel('Home Price Index')
plt.title('Top Features vs Home Price Index')
plt.legend()
plt.show()
```


Top Features vs Home Price Index
HomePriceIndex_Lag_6
HomePriceIndex_Lag_10
CPI
HomePriceIndex_Lag_7
HomePriceIndex_Lag_8
RentalVacancyRate
MortgageRate
HomePriceIndex_Lag_2
HomePriceIndex_Lag_1
NormalizedHomePriceIndex
Home Price Index
Feature Value
325
300
275
250
225
200
175
150
0
50
100
150
200
250
300