

**why sieve big o n log log n**

The reason why the Sieve of Eratosthenes has a time complexity of **O(n log log n)** is based on how many times numbers get "crossed out" during the sieve process. Here's a clear explanation:

* For each prime $ p $, you mark its multiples as not prime. The number of multiples of $ p $ up to $ n $ is approximately $ \frac{n}{p} $.
* Summing the cost of marking multiples for all primes $ p $ less than or equal to $ n $ gives:
* Taking $ n $ common:
* The sum of reciprocals of primes up to $ n $ asymptotically behaves like $ \log \log n $ (this is a known result in number theory about the harmonic series of primes).
* Therefore, total complexity is:

In essence, the sieve’s efficiency comes from the fact that the sum of the reciprocals of primes grows very slowly — roughly as $ \log \log n $ — which is much slower growth than $ \log n $ or $ n $.

This result relies on advanced math about the behavior of prime numbers and harmonic series but is well established.

**Summary:**

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| Reason for O(n log log n) Complexity |
| Marking multiples for each prime takes $ n/p $ time |
| Sum over all primes of $ n/p $ leads to $ n \times \sum 1/p $ |
| Sum of reciprocals of primes approximates $ \log \log n $ |
| Hence, overall complexity is $ O(n \log \log n) $ |

This explanation is from detailed algorithm analysis and number theory, and can be found on platforms like GeeksforGeeks and Wikipedia about the Sieve of Eratosthenes time complexity.[[1]](#fn1)[[2]](#fn2)[[3]](#fn3)

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1. <https://www.geeksforgeeks.org/dsa/how-is-the-time-complexity-of-sieve-of-eratosthenes-is-nloglogn/>

1. <https://en.wikipedia.org/wiki/Sieve_of_Eratosthenes>

1. <https://www.baeldung.com/cs/sieve-of-eratosthenes>

1. <https://stackoverflow.com/questions/2582732/time-complexity-of-sieve-of-eratosthenes-algorithm>

1. <https://cp-algorithms.com/algebra/sieve-of-eratosthenes.html>

1. <https://www.codementor.io/@svenkatadileepkumar/find-all-the-prime-numbers-less-than-n-in-o-n-time-complexity-1amti1lm2p>

1. <https://www.geeksforgeeks.org/dsa/sieve-eratosthenes-0n-time-complexity/>

1. <https://codility.com/media/train/9-Sieve.pdf>