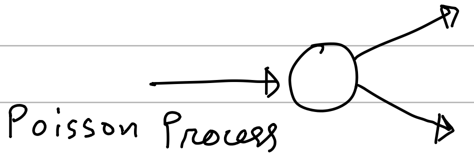
Theory Assignment 4

SWARNARUP
B21140

Q1



a) pkt lengths are Exponentially distributed.
i.e. L/Transmission time which is equivalent to service time of each output links, are also Memoryless processes (Poisson/exponential Process).

i.e. Kendall's Notation would be M/M/2/B+2

b) if pkt length is fixed, Transmission time of the output link we can assume constant,
i.e. service time of each output link is fixed.
i.e. service is Deterministic.

i.e. Kendall's notation is, $M/D/2/\infty$

c) pkt length and hence service are according to General Distribution.
System can acomodate 10 x no. of packets under service
$= 10 \times 2 = 20$
queue size = 18.

Kendall's Notation : M/G/2/20

Q2

$\lambda = \frac{1}{12}$ sec $\mu = \frac{1}{8}$ sec

$\rho = \frac{\lambda}{\mu} = \frac{8}{12} = \frac{2}{3}$

$N = \frac{\rho}{1-\rho}$

(a) avg. no. of pkts in system $= \dfrac{\frac{2}{3}}{1-2/3} = 2$

avg. Time spent in sys, $T = \frac{N}{\lambda} = \frac{2}{1/12} = 24$ sec.

avg. Waiting time $W = T -$ avg. Service time
$= 24 - 8$ sec
$= 16$ sec

(b) Little's Law For the queue as the system.

$N_q = \lambda W$

avg. no. of pkts waiting in queue $= \frac{1}{12} \times 16 = \frac{4}{3}$ pkts.

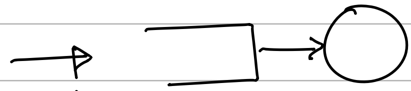
(c) Avg. time spent by a pkt in system $=$ 24 seconds.

(d) avg. no. of pkts in the system $=$ 2 pkts.

(e) Proportion of time the server is idle $= 1 -$ utilization
$= 1 - \rho$
$= 1 - \frac{2}{3}$
$= \frac{1}{3}$

i.e. 33.33% of the time.

Q3



1 call / 5 min

$\lambda = \frac{1}{5}$ call/min

service time mean = 4 min

$\mu = \frac{1}{4}$ call/min.

To find no. of servers (operators) so that wait time is 0.5 min.

Assuming m no. of operators are there

now my system is $M/M/m$
or we can simply assume $M/M/1$ system which has the server's service rate as $m\mu$.

From Earlang C Formula,

$$\frac{N_Q}{P_Q} = \frac{\rho}{1-\rho} \qquad \rho = \frac{\lambda}{c\mu}.$$

$$\Rightarrow \frac{\lambda E[W_Q]}{P_Q} = \frac{\rho}{1-\rho}$$

$$\Rightarrow E[W_q \mid W_q > 0] = \frac{1}{\lambda}\frac{\rho}{1-\rho} \qquad \text{because } P_Q = 0$$

$$\frac{\rho}{\lambda(1-\rho)} \leq 0.5$$

$$\Rightarrow \frac{\lambda / m\mu}{\lambda\left(\frac{m\mu-\lambda}{m\mu}\right)} \leq 0.5$$

$$\Rightarrow \frac{1}{m\mu - \lambda} \leq 0.5$$

$$\Rightarrow 2 \leq m\mu - \lambda$$

$$m\mu \geq 2+\lambda$$

$$m \geq \frac{2+\lambda}{\mu} \Rightarrow m \geq \frac{2+1/5}{1/4} = \frac{11/5}{1/4} = \frac{44}{5} = 8.8$$

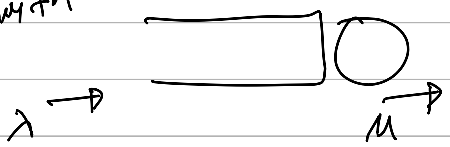
i.e. $m \geq 9$

minimum number of Operators required = 9.

Q 4

join with probability $\frac{1}{q_{len}+1} = \frac{1}{\text{no. of customer in system}}$

because 1 server + q len = no. of customers.



Markov Chain for state n = no. of customers in system.

State Transition $P_{ij} = P[N_{(k+1)\delta} = j \mid N_{k\delta} = i]$

$$P_{00} = P[0 \text{ arrival}]\,P[0 \text{ service}] + P[\text{arrival but dropped}] \times P[0 \text{ service}]$$

$$= e^{-\lambda\delta} \times 1 + \underbrace{\lambda\delta e^{-\lambda\delta} \times \left(1 - \frac{1}{0+1}\right) \times 1}_{0}$$

$$= e^{-\lambda\delta}$$

$$= 1 - \lambda\delta + o(\delta)$$

when $i \geq 1$ $\quad P_{ii} = P[0 \text{ arrivals}]\,P[0 \text{ service}] + P[\text{arrival but dropped}] \times P[0 \text{ service}]$

$$= e^{-\lambda\delta} e^{-\mu\delta} + \lambda\delta e^{-\lambda\delta}\left(1 - \frac{1}{i}\right) e^{-\mu\delta} + o(\delta)$$

here i is number of pkts in system = no. of pkts in queue + 1

$$= 1 - \lambda\delta - \mu\delta + \lambda\delta\left(1 - \frac{1}{i}\right) + o(\delta)$$

$$= 1 - \left(\frac{1}{i}\right)\lambda\delta - \mu\delta + o(\delta)$$

$$P_{i,i+1} = P[1 \text{ arrival not dropped}]\,P[0 \text{ service}] + o(\delta)$$

$$= \lambda\delta e^{-\lambda\delta} \frac{1}{i} \times e^{-\mu\delta} + o(\delta)$$

$$= \left(\frac{1}{i}\right)\lambda\delta$$

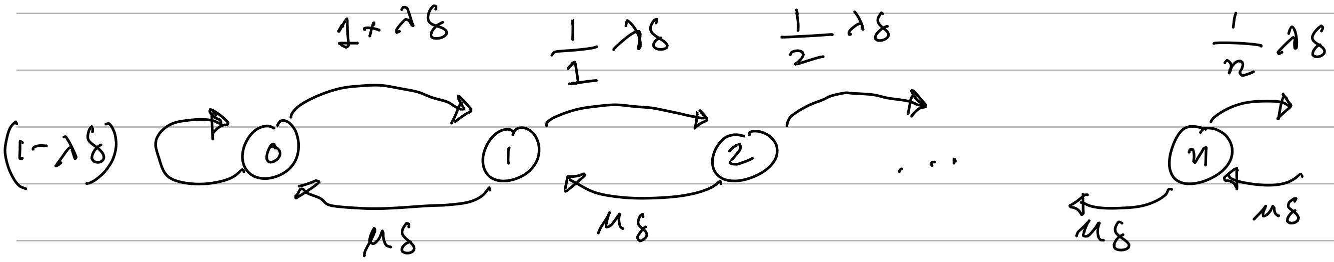
$\boxed{\text{\# For } i=0 \quad P_{01} = \lambda\delta}$

$$P_{i,i-1} = P[0 \text{ arrival}]\,P[1 \text{ service}] + P[1 \text{ arrival but dropped}] \times P[1 \text{ service}] + o(\delta)$$

$$= e^{-\lambda\delta}\mu\delta e^{-\mu\delta} + \underbrace{\lambda\delta e^{-\lambda\delta}\left(1 - \frac{1}{i+1}\right)\mu\delta e^{-\mu\delta}}_{\delta^2 \text{ term so can be ignored.}} +$$

$$= \mu\delta$$

Hence the markov chain is



balance Equation, $\quad p_n \frac{\lambda\delta}{n} = p_{n+1}\,\mu\delta$

$$\rho = \frac{\lambda}{\mu}$$

$$p_{n+1} = \frac{\lambda}{n\mu} p_n$$

p_0, $\quad p_1 = \frac{\lambda}{\mu} p_0$, $\quad p_2 = \frac{\lambda}{\mu} p_1 = \left(\frac{\lambda}{\mu}\right)^2 p_0$ $\quad p_3 = \frac{1}{2}\frac{\lambda}{\mu} p_2 = \frac{1}{2}\left(\frac{\lambda}{\mu}\right)^3 p_0$

$$p_n = \frac{1}{(n-1)!}\left(\frac{\lambda}{\mu}\right)^n p_0$$

$$p_0 + \sum_{n=1}^{\infty} p_n = 1$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

$$p_0\left(1 + \frac{1}{1}\rho + \frac{1}{1}\rho^2 + \frac{1}{2!}\rho^3 + \cdots\right) = 1$$

$$p_0 = \left(1 + \sum_{n=1}^{\infty} \frac{1}{(n-1)!}\rho^n\right)^{-1} = \left[1 + \rho\left(\sum_{x=0}^{\infty} \frac{1}{x!}\rho^{x!}\right)\right]^{-1}$$

$$= \left(1 + \rho \times e^{\rho}\right)^{-1}$$

$$= 1/1+\rho e^{\rho}$$

avg. no. of customers in the System

$$N = \sum_{n=0}^{\infty} n\, p_n$$

$$= p_1 + p_2 + \cdots$$

$$= \frac{1}{1}\rho^1 p_0 + \frac{1}{2!}\rho^2 p_0 + \cdots$$

$$= \sum_{n=1}^{\infty} \frac{1}{(n-1)!}\rho^n p_0 = \rho\left(\sum_{x=0}^{\infty} \frac{1}{x!}\rho^x\right) p_0 = \rho e^{\rho} p_0$$

$$N = \frac{\rho e^{\rho}}{1+\rho e^{\rho}}$$

(b) average no. of Customers in the system $= \dfrac{\rho e^{\rho}}{1+\rho e^{\rho}}$

$$= \frac{\lambda e^{\lambda/\mu}}{\mu + \lambda e^{\lambda/\mu}}.$$

(c) From little's law average time spent in the system $T = \dfrac{N}{\lambda}$

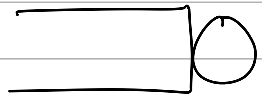
$$= \frac{1}{\lambda} \frac{\rho e^{\rho}}{1+\rho e^{\rho}}$$

$$= \frac{1}{\cancel{\lambda}} \frac{\cancel{\lambda} e^{\rho}}{\mu+\lambda e^{\rho}} = \frac{e^{\rho}}{\mu+\lambda e^{\rho}}$$

$$= \frac{e^{\lambda/\mu}}{\mu+\lambda e^{\lambda/\mu}} \text{ sec.}$$

Q5

single server G/G/1



γ distribution shape $= 5$, scale $= 2$ $\begin{cases} \bar{x} = 10 & \lambda = \frac{1}{10} \\ \sigma_a^2 = 20 \end{cases}$

Service Normal distribution $\begin{cases} \bar{x} = 8 & \mu = \frac{1}{8} \\ \sigma_b^2 = 20 \end{cases}$

(a) avg. wait time in queue $\quad \rho = \frac{8}{10}$

For G/G/1 system as $\rho \to 1$ $\quad W_q = \dfrac{\lambda(\sigma_a^2 + \sigma_b^2)}{2(1-\rho)}$

$= \frac{1}{10} \times \dfrac{20+20}{2 \times 1 - 0.8} = \dfrac{40}{20 \cdot 2/10}$

avg. wait time in queue $= 10$ min

(b) avg. no. of pkts waiting in queue N_Q

From Little's Law, $N_q = \lambda W_q = \frac{1}{10} \times 10 = 1$ pkts.

(c) avg. time spent by a pkt in system T

avg. time spent in system $T = W_q + \frac{1}{\mu}$

$= 10 + 8 = 18$ min.

(d) avg. no. of pkt in system N

From Little's Law applied on the system $N = \lambda T$

avg. no. of pkts in the system, $N = (\frac{1}{10}\ \text{pkt/min}) \times (18\ \text{min})$

$= 1.8$ pkts.

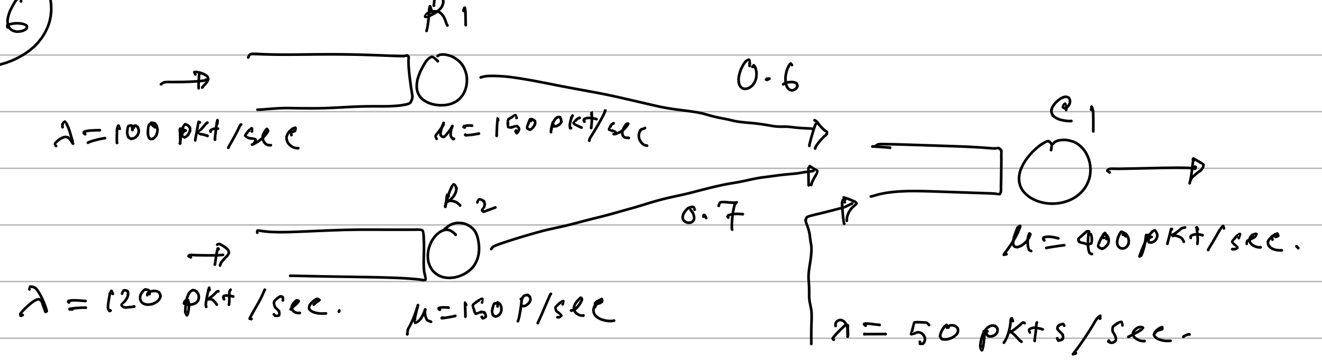
(e) Proportion of time server is idle.

proportion of time server is idle $= 1 -$ utilization

$= 1 - \frac{8}{10} = 2/10$

i.e. 20% of the time.

Q6



$$\begin{bmatrix} \bar\lambda_1 \\ \bar\lambda_2 \\ \bar\lambda_3 \end{bmatrix} = \begin{bmatrix} FAR_1 \\ FAR_2 \\ FAC_1 \end{bmatrix} + R.M \times \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

$\boxed{\begin{array}{l} R_1 \to 1 \\ R_2 \to 2 \\ C_1 \to 3 \end{array}}$

Routing Matrix = $\begin{bmatrix} P_{11} & P_{21} & P_{31} \\ P_{21} & P_{22} & P_{32} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$

only $R_1 \to C_1$ i.e. $1 \to 3$ or $P_{31} = 0.6$
and $R_2 \to C_1$ i.e. $2 \to 3$ or $P_{32} = 0.7$

(a) Then Routing Matrix becomes, = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.6 & 0.7 & 0 \end{bmatrix}$

(b) total arrival rate at C_1 $= \bar\lambda_3 =$ Fresh Arrival at C_1 $+ P_{31}\lambda_1 + P_{32}\lambda_2$

$= 50 + 100 \times 0.6 + 120 \times 0.7$

$= 194$ pkts/sec.

(c) Avg. no. of Packets in the Queuing system according to assuming C_1 as the M/M/1 system having arrival rate 194 pkts/sec and service rate $\mu = 400$ pkts/sec.

$$\rho = \frac{194}{400}$$

avg. no. of pkts in the system become

$$= \frac{\rho}{1-\rho} = \frac{194}{400-194} = 0.941 \text{ packets}.$$