Maxwell’s Equation

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Qualitative

1.Gauss's law

1. There are two types of charge, positive and negative, just as there are two types of real numbers, positive and negative.
2. Electric field lines diverge from positive charge and converge on negative charge

2.No one's law (Gauss’s Law for magnetism)

- There is no magnetic monopole
- All magnetic field lines form loops

3.Faraday's law

- A changing magnetic field induces an electromotive force (emf) and, hence, an electric field. The direction of the emf opposes the change.

4.Ampère's law

- Magnetic fields are generated by moving charges or by changing electric fields.

① Divergence Theorem (Gauss div. theo.)

$$\int_V (\nabla \cdot \vec{F})\, dV = \oint_S \vec{F} \cdot \vec{dS}$$

② Curl Theorem (Stokes's theo.)

$$\int_S (\nabla \times \vec{F}) \cdot \vec{dS} = \oint_l \vec{F} \cdot \vec{dl}$$

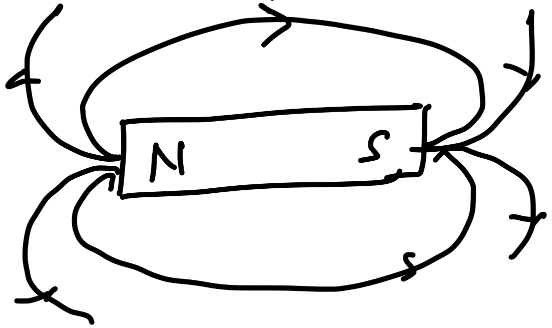
Related with divergence

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon} \quad \text{(Gauss law)}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{(Magnetic monopole can not exsist)}$$

$\rho \rightarrow$ volume charge density

$\epsilon \rightarrow$ permittivity



Related to curl :-

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's law)}$$

$$\nabla \times \vec{B} = \mu \vec{J} + \underbrace{\mu \epsilon \frac{\partial \vec{E}}{\partial t}}_{\text{Maxwell's correction}} \quad \text{(Ampere's law)}$$

N or S

$$\oint B \cdot dl = \mu_0 I$$

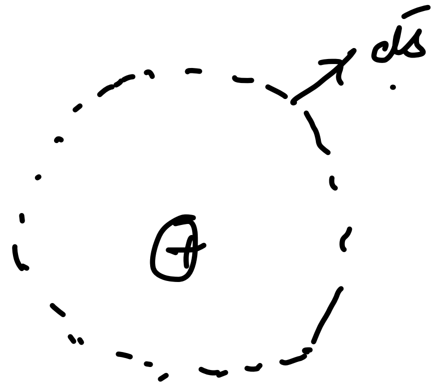
$J \rightarrow$ current charge density

$\mu \rightarrow$ permability ; $\vec{E}$ - Electric field , $\vec{B}$ - magnetic field

* For static case $\frac{\partial \vec{B}}{\partial t} = 0$ and $\frac{\partial \vec{E}}{\partial t} = 0$

$$\int_V (\nabla\cdot\vec{F})\,dV = \oint_S \vec{F}\cdot d\vec{S}$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon}$$



Voulme integral both the sides

$$\int_V (\nabla\cdot\vec{E})\,dV = \int_V \frac{\rho}{\epsilon}\,dV$$

Using div. theo.

$$\boxed{\oint_S \vec{E}\cdot d\vec{S} = \frac{1}{\epsilon}\int_V \rho\,dV = \frac{Q_{enc}}{\epsilon}}$$

Integral form.

$$\nabla \cdot \vec{B} = 0$$

volume integral both side

$$\int_V (\nabla \cdot \vec{B})\, dV = 0$$

apply divergence theorem.

$$\boxed{\oint_S \vec{B} \cdot d\vec{S} = 0}$$

$$\oint_S \bar{E} \cdot d\bar{S} = \frac{Q_{enc}}{\epsilon}$$

$$\oint_S \bar{B} \cdot d\bar{S} = 0$$

$$\int_S (\nabla \times \vec{F}) \cdot d\vec{S} = \oint_l \vec{F} \cdot d\vec{l}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Taking surface integral both sides

$$\int_S (\nabla \times \vec{E}) \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

Apply Stoke's theo.

$$\oint_l \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{S} = -\frac{\partial \phi_B}{\partial t} \quad ; \quad \phi_B = \text{magnetic flux.}$$

$$\boxed{\oint_l \vec{E} \cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t}}$$

$$\nabla \times \vec{B} = \mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\int_s (\nabla \times \vec{B}) \cdot d\vec{S} = \mu \underbrace{\int_s \vec{J} \cdot d\vec{S}}_{I_{enc}} + \mu \epsilon \int_s \frac{\partial \vec{E}}{\partial t} \cdot d\vec{S}$$

$$\oint_l \vec{B} \cdot d\vec{l} = \mu I_{enc} + \mu \epsilon \frac{\partial}{\partial t} \int_s \vec{E} \cdot d\vec{S}$$

$$\boxed{\oint_l \vec{B} \cdot dl = \mu I_{enc} + \mu \epsilon \frac{\partial \phi_E}{\partial t}}$$

$\phi_E \rightarrow$ electric flux

$I_{enc} \rightarrow$ current enclosed.

Integral form

$$\oint_s \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\epsilon}$$

$$\oint_s \vec{B} \cdot d\vec{S} = 0$$

$$\oint_l \vec{E} \cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t}$$

$$\oint_l \vec{B} \cdot d\vec{l} = \mu I_{enc} + \mu \epsilon \frac{\partial \phi_E}{\partial t}$$

$$\nabla \cdot \bar{E} = \frac{\rho}{\epsilon}$$

$$\nabla \cdot \bar{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{B} = \mu \bar{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$$

Integral form to differential form

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\epsilon} \quad ; \quad Q = \int_V \rho \, dV$$

from div. theo.

$$\oint \vec{F}\cdot d\vec{S} = \int_V (\nabla\cdot\vec{F})\, dV$$

$$\int_V \underbrace{(\nabla\cdot\vec{E})}\, dV = \frac{1}{\epsilon}\int_V \rho \, dV = \int_V \underbrace{\left(\frac{\rho}{\epsilon}\right)}\, dV$$

$$\boxed{\nabla\cdot\vec{E} = \frac{\rho}{\epsilon}}$$

Differential Form

$$\nabla.\vec{E} = \frac{\rho}{\epsilon}, \text{Gauss Law} \qquad \vec{D} = \epsilon\vec{E}$$

$$\nabla.\vec{B} = 0, \text{No One's Law} \qquad \vec{B} = \mu\vec{H}$$

$$\nabla \times \vec{E} = -\frac{\partial\vec{B}}{\partial t}, \text{Faraday's Law}$$

$$\nabla \times \vec{B} = \mu\epsilon\frac{\partial\vec{E}}{\partial t} + \mu\vec{J}, \text{modified Ampere's Law}$$

ρ = electric charge density (Unit: As/m^3)
ϵ = permittivity
D = electric flux density/displacement field (Unit: As/m^2)
E = electric field intensity (Unit: V/m)
μ = permeability
H = magnetic field intensity (Unit: A/m)
B = magnetic field (Unit: Tesla=Vs/m^2)

Differential Form

$$\nabla.\vec{E} = \frac{\rho}{\epsilon}$$

$$\nabla.\vec{B} = 0$$

$$\iiint_V (\nabla.\vec{F})dV = \oiint_S \vec{F}.d\vec{S}$$

Gauss' Theorem

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu\epsilon\frac{\partial \vec{E}}{\partial t} + \mu\vec{J}$$

$$\iint_S (\nabla \times \vec{F}).d\vec{S} = \oint_l \vec{F}.d\vec{l}$$

Stokes' Theorem

Integral Form

$$\oiint_S \vec{E}.d\vec{S} = \iiint_V \frac{\rho}{\epsilon}\, dV = \frac{Q_{en}}{\epsilon}$$

Gauss's law

$$\oiint_S \vec{B}.d\vec{S} = 0$$

Gauss's law for magnetism

$$\oint_l \vec{E}.d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t}.d\vec{S} = -\frac{\partial \phi_B}{\partial t}$$

Faraday's law for induction

$$\oint_l \vec{B}.d\vec{l} = \mu\epsilon\iint_S \frac{\partial \vec{E}}{\partial t}.d\vec{S} + \mu\iint_S \vec{J}.d\vec{S}$$

$$= \mu\epsilon\frac{\partial \phi_E}{\partial t} + \mu I_{enc}$$

Ampere's law