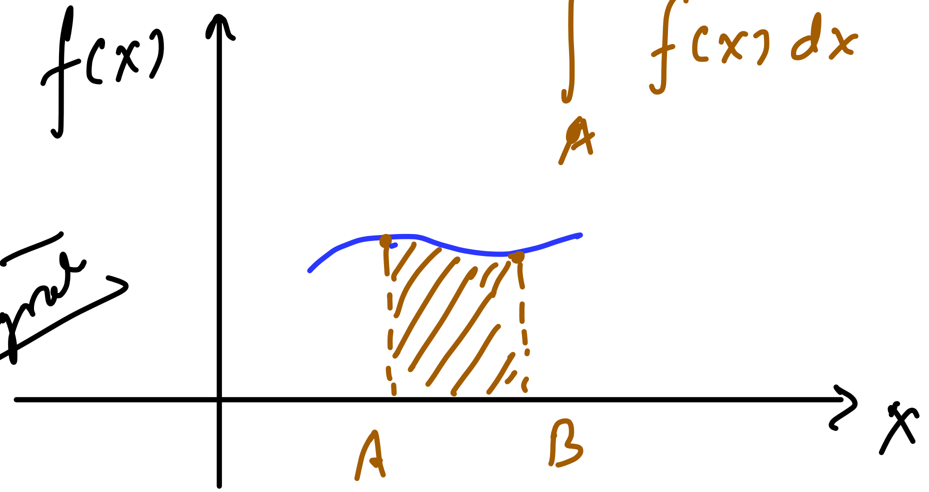
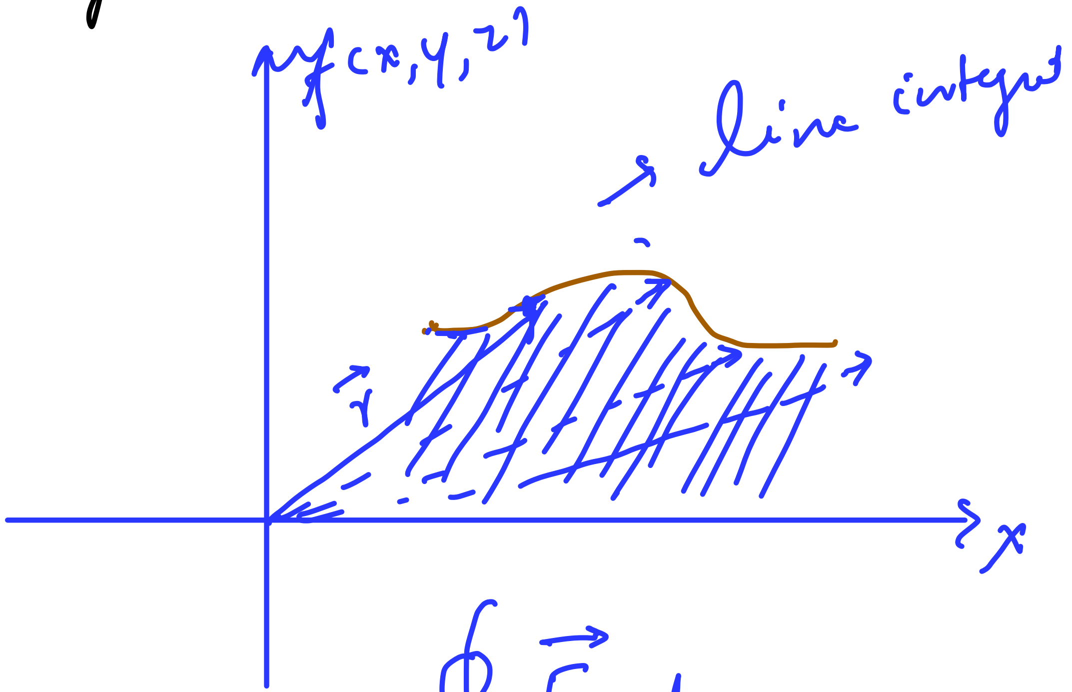
Line, Surface and Volume Integral:

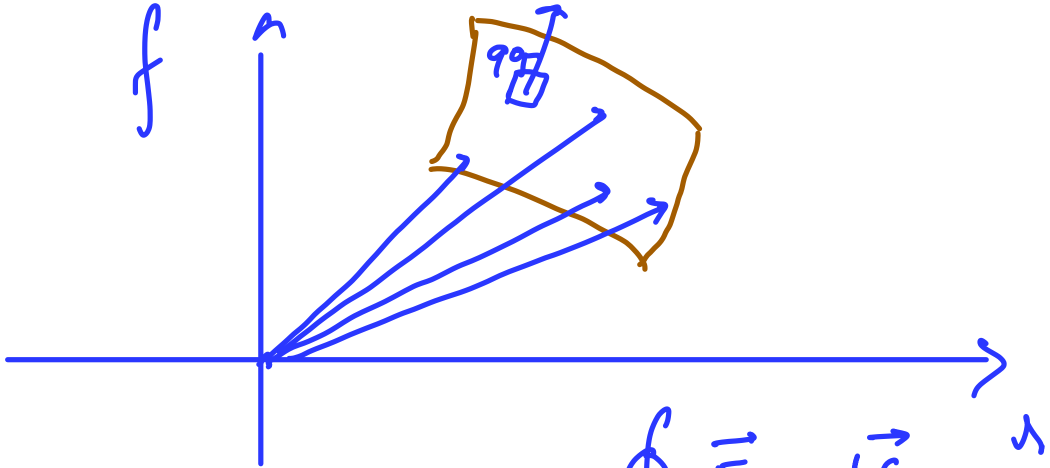
Simple integral



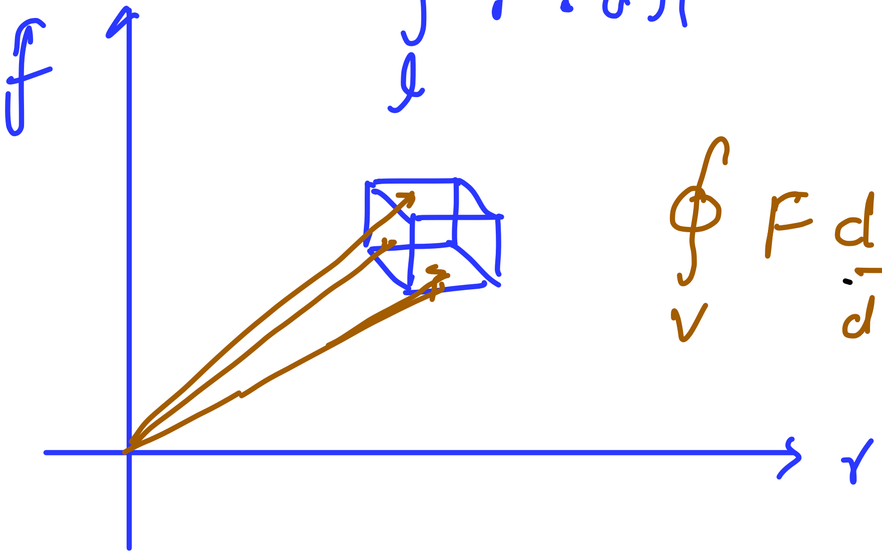
$$\int_A^B f(x)\,dx$$



$$\oint_l \vec{F}\cdot d\vec{r}$$



$$\oint_S \vec{F}\cdot d\vec{S}$$



$$\oint_V F\,\underline{dV}$$

$dx\,dy\,dz$

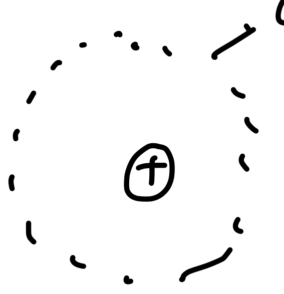
① Divergence Theorem (Gauss divergence theorem)

② Curl Theorem (Stoke's theorem)

$$\nabla \cdot \vec{F} = \lim_{V \to 0} \frac{\oint \vec{F} \cdot d\vec{S}}{dV}$$

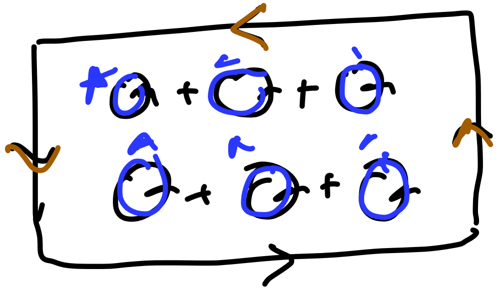
$$\boxed{\int_V (\nabla \cdot \vec{F})\, dV = \oint_S \vec{F} \cdot d\vec{S}}$$

$$\int_V (\nabla \cdot \vec{E})\, dV = \oint \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$



$$\nabla \times \vec{F} = \underset{\Delta s \to 0}{\mathrm{Lt.}} \frac{\oint \vec{F} \cdot d\vec{l}}{ds}$$

$$\underbrace{\int_s (\nabla \times \vec{F}) \cdot d\vec{s}}_{\text{Surface integral}} = \underbrace{\oint_l \vec{F} \cdot d\vec{l}}_{\text{line integral}}$$



$$\int_V (\nabla \cdot \vec{F})\, dV = \oint_S \vec{F} \cdot \vec{dS}$$ Gauss Theorem

$$\int_S (\nabla \times \vec{F}) \cdot \vec{dS} = \oint_l \vec{F} \cdot \vec{dl}$$ Stoke's theorem

$\iint \equiv$ Surface integral

$\iiint \equiv$ volume integral

Scalar and Vector

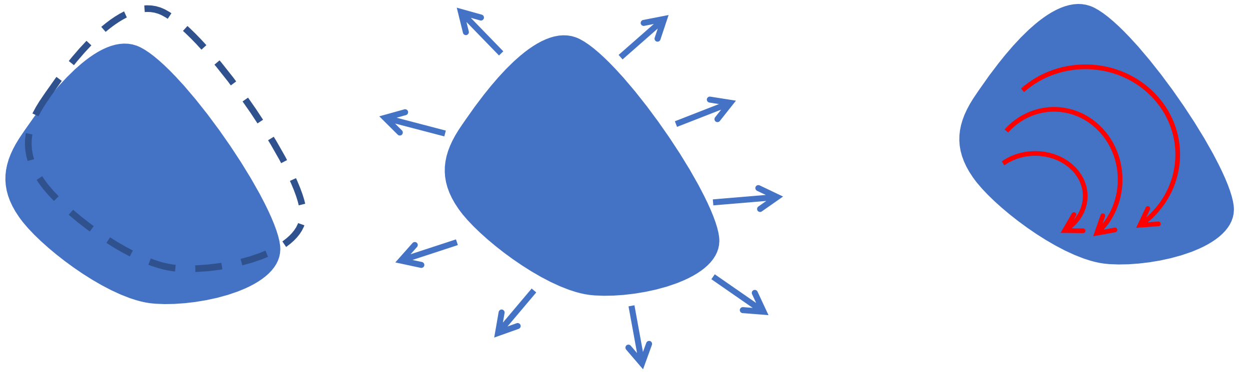
- **Scalars** are quantities that are fully described by a magnitude (or numerical value) alone. For example, length, speed, work, mass, density, etc.

- **Vectors** are quantities that are fully described by both a magnitude and a direction. For example, force, torque, momentum, acceleration, velocity, etc.

$$\vec{r} = a\hat{x} + b\hat{y} + c\hat{z}$$
$$= a\hat{\imath} + b\hat{\jmath} + c\hat{k} = \langle a, b, c \rangle$$

- Vector possess both magnitudes as well as direction.
- They do not obey the ordinary laws of Algebra.
- Vector change if either the magnitude or direction change or both changes.

Gradient, Divergence and Curl



A. In a smalltime *t* the center of the body will move.
B. The body can expand or contract, changing its volume.
C. The body can rotate.

Gradient: the derivative of a function with respect to distance along the direction in which the function changes most rapidly.

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$$

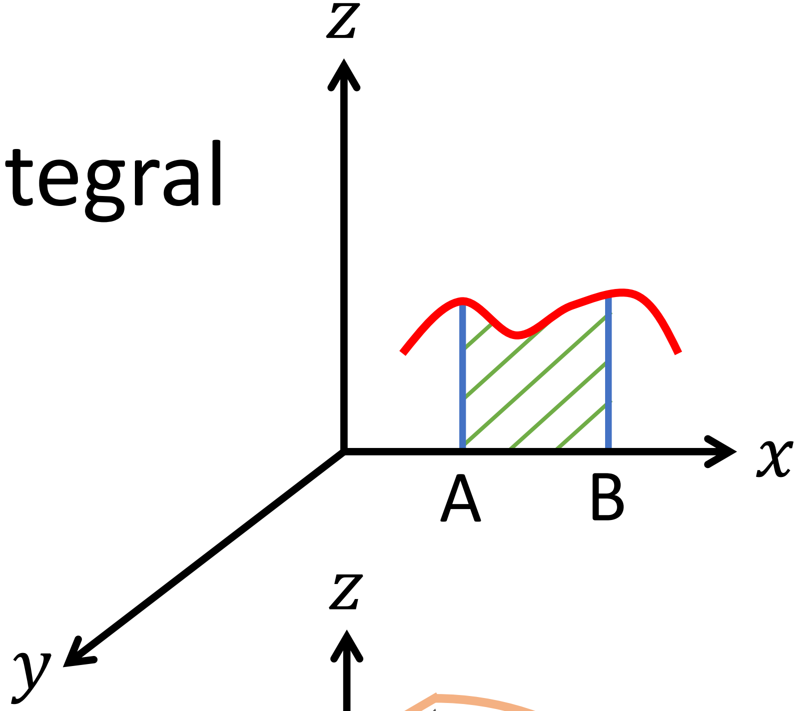
Divergence: $\nabla . f(x,y,z) = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) . f(x,y,z)$

Curl: $\nabla \times f(x,y,z) = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) \times f(x,y,z)$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f & f & f \end{vmatrix} = \left(\frac{\partial f}{\partial y} - \frac{\partial f}{\partial z}\right)\hat{x} + \left(\frac{\partial f}{\partial z} - \frac{\partial f}{\partial x}\right)\hat{y} + \left(\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}\right)\hat{z}$$

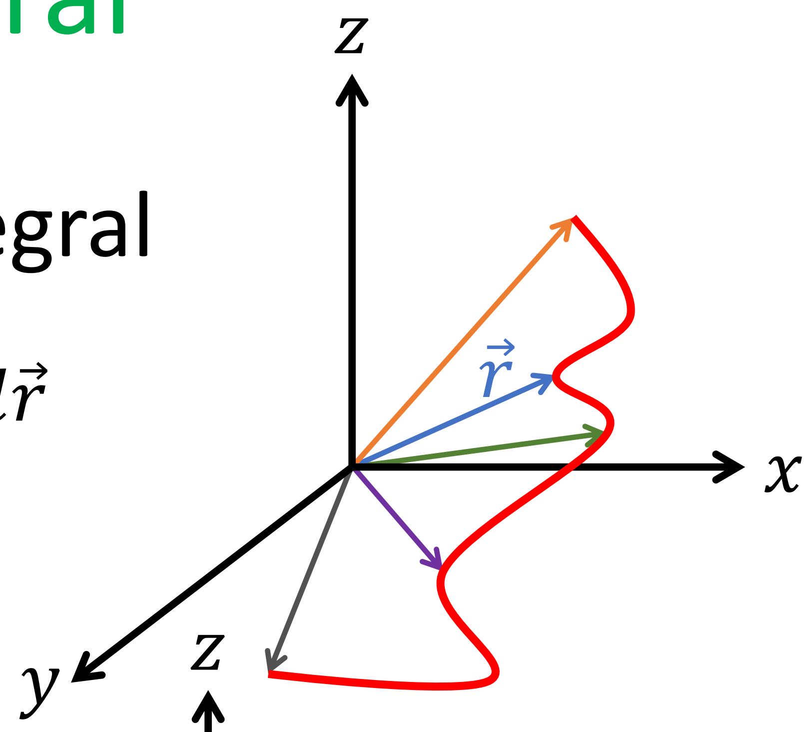
Line, Surface and Volume Integral

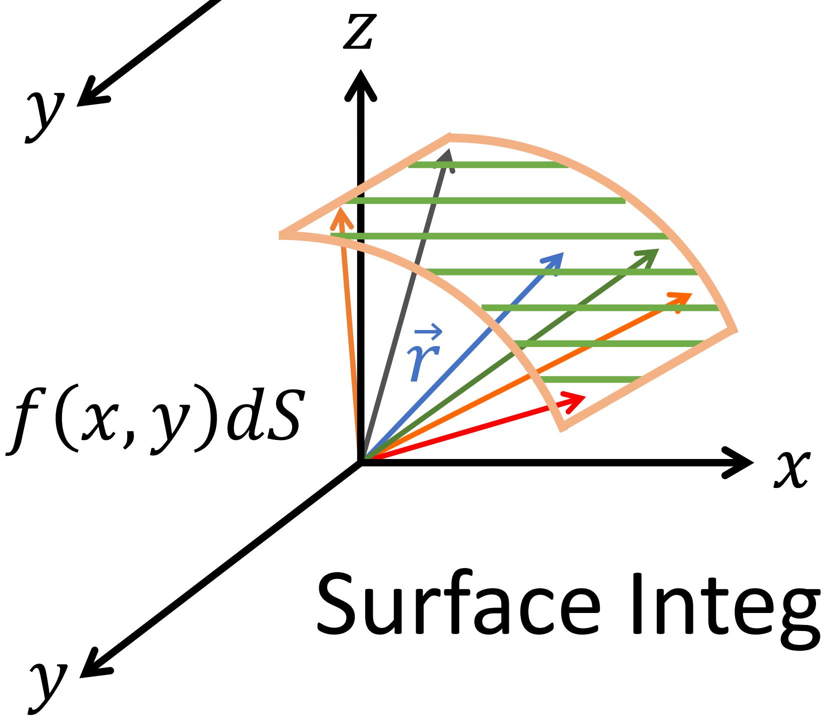
Integral



Line Integral

$$\int \vec{F}.d\vec{r}$$



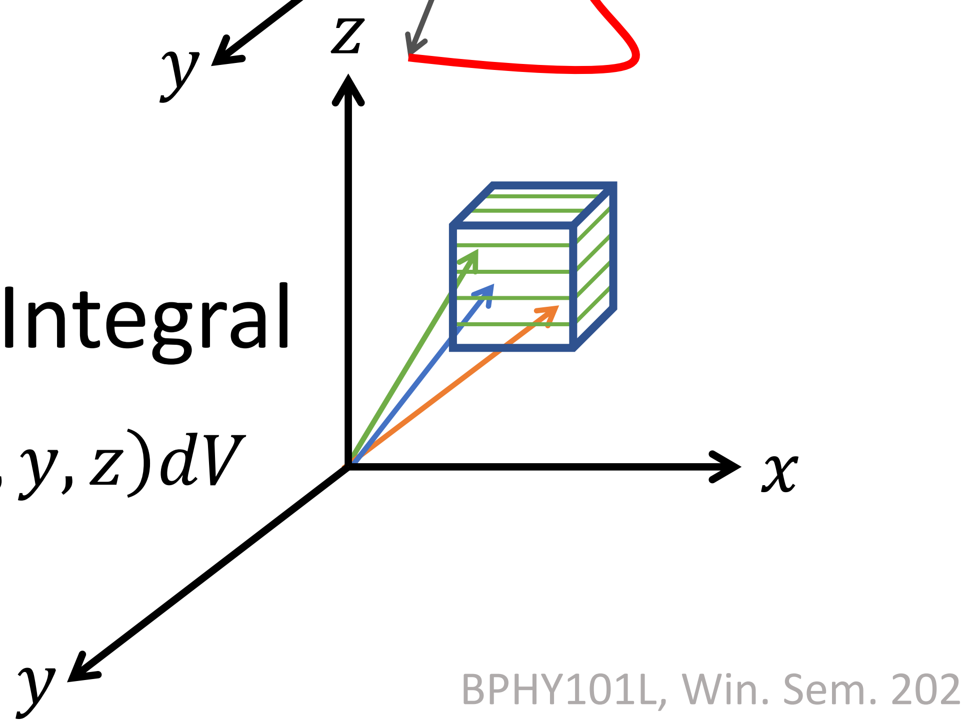


$$\iint f(x,y)dS$$

Surface Integral

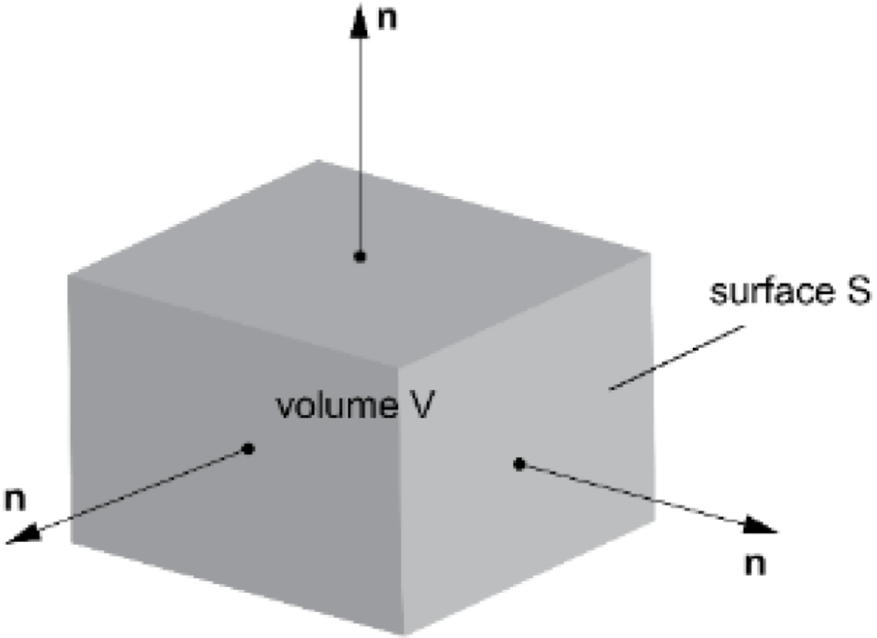
Volume Integral

$$\iiint f(x,y,z)dV$$



Divergence (Gauss') Theorem

$$\overbrace{\iiint\limits_V (\nabla \cdot \vec{F})}^{\text{div}}\, dV = \oiint\limits_S (\vec{F} \cdot \hat{n}) dS = \oiint\limits_S \vec{F} \cdot d\vec{S}$$



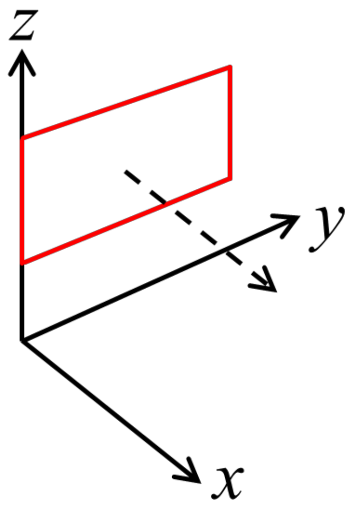
Integral of divergence of vector field ($\vec{F}$) over volume V inside closed boundary S **equals** outward flux of vector field ($\vec{F}$) through closed surface S

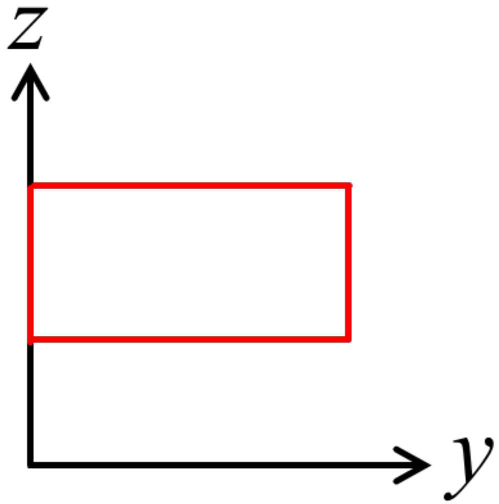
$$\nabla \cdot \vec{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (F_x, F_y, F_z) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Curl (Stokes') Theorem

$$\iint_S (\overbrace{\nabla \times \vec{F}}^{\text{curl}}) \cdot d\vec{S} = \oiint_S ((\nabla \times \vec{F}) \cdot \hat{n}) dS = \oint_{\partial S} \vec{F} \cdot d\vec{l}$$

Integral of curl of vector field ($\vec{F}$) over surface *S* **equals**
line integral of vector field ($\vec{F}$) over closed boundary d*S* defined by surface *S*





Problem:1 $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$,

Find $div\vec{F}, and\ curl\vec{F}$

Problem:2 $\vec{F} = (2x + yz)\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$,

Find $div\vec{F}, and\ curl\vec{F}$

Problem:3 $F = xyz$,

Find ∇F