L07 : 13.05.2022

2. Qs

- interpretation of $t(\underline{x})$

$$t(\underline{x}) = \frac{1}{\rho}\, \underline{s}^T \underline{\underline{C}}^{-1} \underline{x} \overset{\underline{\underline{C}} = \sigma^2 \underline{\underline{I}}}{\Longrightarrow} \sim \underline{s}^T \underline{x} \text{ [correlation]}$$

is a matched filter or correlator

- Signal design $\underline{s} = ?$:

Given $\underline{\underline{C}}$, choose waveform such as to max ρ

$$\max_{\underline{s}} \rho \simeq \underline{s}^T \underline{\underline{C}}^{-1} \underline{s} \quad \text{s.t. } \underline{s}^T \underline{s} = E_{max}$$

(max signal energy or max interference allowed)

We can't do an infinite maximization. It is limited by max of Energy transmitted

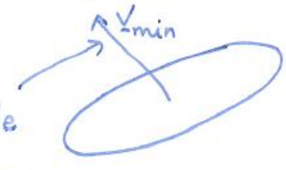
This eq.^n can be solved by using Lagrange multiplier.

$$\max_{\underline{s}} L(\underline{s}) = \underline{s}^T \underline{\underline{C}}^{-1} \underline{s} + \lambda(\underline{s}^T \underline{s} - E_{max})$$

$$\Rightarrow \underline{s}_{opt} = \sqrt{E_{max}} \cdot \underline{v}_{min}$$

$\underline{v}_{min}$ = eigen vector of $\underline{\underline{C}}$ with min eigen value.

in this dir we have min possible noise level or randomness



Practical challenge $P(\underline{x}|H)$ unknown.

(a) $p(\underline{x}|H)$ known except for some unknown parameters.

$$H_0 : \underline{x} \sim p(\underline{x} | H_0 ; \underline{\nu}_0)$$
$$H_1 : \underline{x} \sim p(\underline{x} | H_1 ; \underline{\nu}_1)$$

known functions — unknown parameters eg A and σ^2 in E 3.1 (DC in AWGN)

Is detection possible ?

Slide 3-5.

Solution :

- Estimate parameters $\hat{\nu}_0$, $\hat{\nu}_1$ from $x(n)$: SASP
- LRT $\dfrac{p(\underline{x} / H_1 ; \hat{\nu}_1)}{p(\underline{x} / H_0 ; \hat{\nu}_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \tilde{\gamma}$

(b) $p(\underline{x} \mid H)$ completely unknown

Slide 3-10

3.11.1

Solutions :

- assume or approximate the likelihood $\hat{p}(\underline{x} \mid H)$
- suppress disturbance in pre-processing
  - channel equalization.
  - interference cancellation
  - ⋮
- learning based signal processing
  - Ch 4
  - DL

4 Supervised Learning

4.1 Overview

slide 4.1

4.1.1

SUPERVISED LEARNING

Use of a set of labeled samples

training set	testing set
S	$\bar{S}$

Training set : $S = \{ (\underline{x}_1, y_1) \ldots, (x_N, y_N) \}$

$\underline{x}_n \in \mathbb{R}^d$: feature vector

$y_n \in \{ \omega_1 \ldots \omega_n \}$: correct class label for each $\underline{x}_n$

N_j training examples for ω_j

$$\sum_{j=1}^{c} N_j = N$$

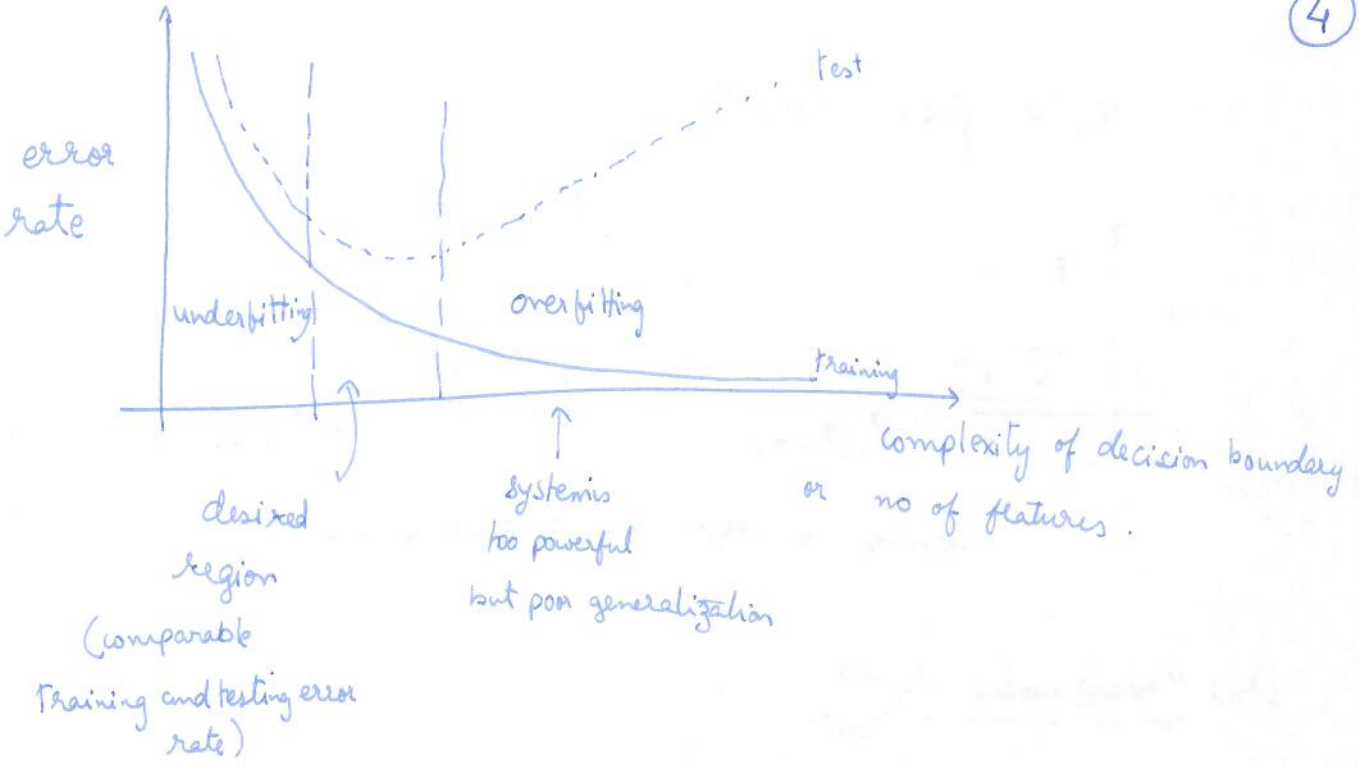
Training: learn classifier $\hat{y}(\underline{x}) \in \{ \omega_1 \ldots \omega_c \}$ from $\underline{S}$

Training error rate : $\dfrac{\text{no of } \hat{y}(\underline{x}_n) \neq y_n \text{ from } S}{N}$

Test set: $\bar{S}$ of $\bar{N} = |\bar{S}|$ test sample not used in training

Test error rate : $\dfrac{\text{no of } \hat{y}(\underline{x}_n) \neq y_n \text{ from } \bar{S}}{\bar{N}}$

$$= \frac{\text{no of wrong classifications from } \bar{S}}{\bar{N}}$$



4.2 Template matching

*) intuitive method

*) based on a distance metric $D(\underline{x}_1, \underline{x}_2)$: how similar are both vectors $\underline{x}_1, \underline{x}_2$

4.2.1 Nearest ~~neighbours~~ mean

old and simple

Idea:

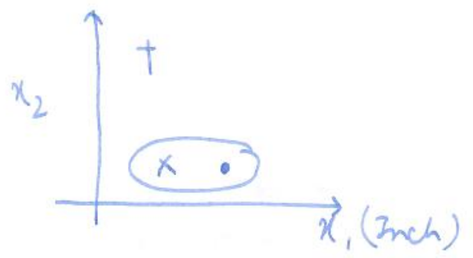
- estimate class centers $\hat{\underline{\mu}}_j$ from the training set ~~and the~~ S
- nearest class center wins.

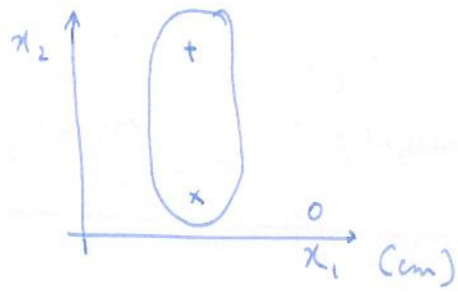
Q Which distance metric to use?

(a) Euclidian dist^n. - $D_{Eucl}(\underline{x}, \underline{\mu}) = \sqrt{(\underline{x}-\underline{\mu})^T (\underline{x}-\underline{\mu})}$

- simple metric
- linear decision boundary see Ch 2.2.6
- Sensitive to feature scaling, transform etc.

x_1 = fish length





change in class due to feature scaling.

(b) Mahalanobis distn

Slide 4.9

4.9 /

- Estimate $\hat{\mu}_j$ and $\hat{\underline{\underline{C}}}_j$ from S
- $D_{maha}(\underline{x}, \hat{\underline{\mu}}_i, \hat{\underline{\underline{C}}}_i) \underset{\omega_j}{\overset{\omega_i}{\gtrless}} D_{maha}(\underline{x}, \hat{\underline{\mu}}_j, \hat{\underline{\underline{C}}}_j)$

- quadratic decision boundary see ch 2.2.6

Slide 4.10