STP530 HW8 Solution

8.19

a.

We can use the following R codes to fit the interaction model. Note that X2 is a categorical predictor. Although the data values for X2 already reflect the dummy coding (1=small copier models, 0=large copier models), we still convert X2 to the factor type as a good practice for handling all categorical predictors.

```
setwd("~/Documents/ASU/STP530-YiZheng/HW-HaozhenXu/HW8")

library(car)

# Import the two parts of the data and merge them into a new dataset

data1 <- read.table("CH01PR20.txt")
head(data1)

data2 <- read.table("CH08PR15.txt")
head(data2)
model.factor <- factor(data2$V1, levels=c(0,1), labels=c("large","small"))

newdata <- data.frame(Call.Time=data1$V1, N.Copiers=data1$V2, Model=model.factor)
head(newdata)

# fit regression model

m <- lm(Call.Time ~ N.Copiers + Model + N.Copiers:Model, data=newdata)
summary(m)
```

```
> summary(m)

Call:
lm(formula = Call.Time ~ N.Copiers + Model + N.Copiers:Model,
    data = newdata)

Residuals:
     Min       1Q   Median       3Q      Max
-19.2072  -6.7887  -0.1708   7.1504  14.7441

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)            2.8131     3.6468   0.771   0.4449
N.Copiers             14.3394     0.6146  23.333   <2e-16 ***
Modelsmall            -8.1412     5.5801  -1.459   0.1522
N.Copiers:Modelsmall   1.7774     0.9746   1.824   0.0755 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.771 on 41 degrees of freedom
Multiple R-squared:  0.9608, Adjusted R-squared:  0.9579
F-statistic: 334.6 on 3 and 41 DF,  p-value: < 2.2e-16
```

From the output, the estimated regression function is

$$\begin{aligned}\widehat{Call.Time} =& 2.8131 + 14.3394(N.Copiers) - 8.1412(Small.Model) \\ &+ 1.7774(N.Copiers)(Small.Model)\end{aligned}$$

b.

Testing whether the interaction term X_1X_2 can be dropped from the model entails testing whether the regression coefficient for X_1X_2, β_3 equals 0. We can use the "general linear test approach" that was practiced in the Homework 5 to answer this question. Alternatively, because only one term is considered to be dropped, we can make use of the individual coefficients' t-test results that are readily available in the R output.

Recall that the individual coefficients' t-tests test the hypotheses $H_0 : \beta_k = 0$ vs. $H_1 : \beta_k \neq 0$. This is the same set of hypotheses as what you would set up with the general linear test approach. In fact, the two test-statistics are also mathematically equivalent, where $F^* = (t^*)^2$.

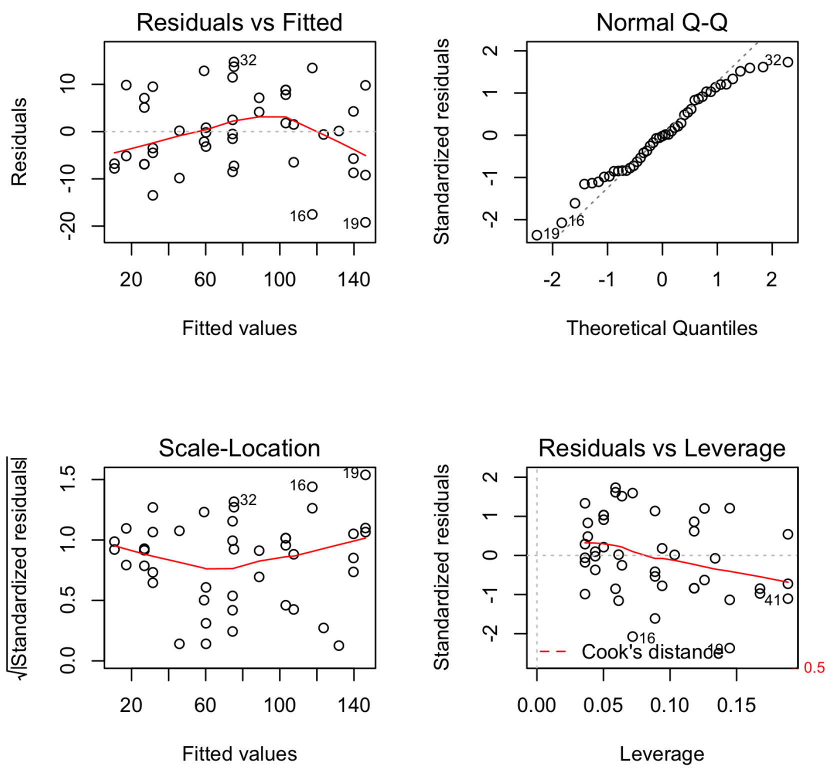
We will present the t-test approach below. For the general linear test approach, see the solution to Homework 5.

Although the t-test results are already available in the model output, we should run model diagnostics before reporting and interpreting the statistical test results.

```
> vif(m)
      N.Copiers           Model N.Copiers:Model
       1.671186        4.280903        4.698387
```

The VIF values are fairly low, indicating mild multicollinearity (lower than the common rule-of-thumb thresholds of 5 or 10). So we will not center X1 for this problem. (Of course you can center X1 to obtain more truthful standard errors for the main effects, but note that the standard error and p-value of the interaction term are unchanged with centering.)

```
# More diagnostics
par(mfrow=c(2, 2))
plot(m)
```



The diagnostic plots show:

- No compelling curvilinear trend (top left plot)
- Roughly normal distribution of residuals (top right plot)
- No excessive outliers with large studentized residual values (bottom left plot)
- No heavily influential outliers (bottom right plot)

Additionally, based on the data collection process given in the problem description, we can reasonably assume that the data points are independent from each other. Hence we consider the model diagnostics are passed and we can confidently proceed with reporting and interpreting the following t-test for the interaction term.

- Step 1. Assumptions:
$$\varepsilon \ i.i.d. \sim N(0, \sigma^2)$$
- Step 2. Hypotheses:
$$H_0 : \beta_3 = 0 \quad H_a : \beta_3 \neq 0$$
- Step 3. Test-statistic:
$$t^* = \frac{b_3}{s\{b_3\}} = \frac{1.7774}{0.9746} = 1.824$$
- Step 4. P-value: Here the P-value is $P_{H_0}\{|t^*| > 1.824\} = 0.0755 < 0.1$
- Step 5. Conclusion: Because the P-value = $.0755 < \alpha = 0.1$, we can reject H_0 at the significance level of 0.1, which means that the interaction term contributes significantly to the model, and we cannot drop the interaction term from the regression model.

Describe the nature of this interaction effect:

In the fitted regression function

$$\begin{aligned}\widehat{Call.Time} =& 2.8131 + 14.3394(N.Copiers) - 8.1412(Small.Model) \\ &+ 1.7774(N.Copiers)(Small.Model)\end{aligned}$$

the interaction coefficient 1.7774 means that the slope for the fitted regression line of "service call time" on "number of copiers serviced" for small copier models is 1.7774 larger than the slope for large copier models. In other words, the marginal effect of additional copiers on the service call time is larger for the small copier models than the large copier models.

In terms of reinforcement vs. interference interaction, I would not call it either, because X2 (Small.Model) is a categorical predictor, so the positive or negative sign of the interaction coefficient merely reflects the arbitrary dummy coding system of X2 (i.e., 1=small copier models, 0=large copier models).

The above interpretation is only about point estimates. We should also look at the statistical significance and practical significance to evaluate the nature of the interaction. Additional R codes are outputs are given on the next page.

```
> m.additive <- lm(Call.Time ~ N.Copiers + Model, data=newdata)
> summary(m.additive)

Call:
lm(formula = Call.Time ~ N.Copiers + Model, data = newdata)

Residuals:
     Min       1Q   Median       3Q      Max 
-22.5390  -4.2515   0.5995   6.5995  14.9330 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -0.9225     3.0997  -0.298    0.767    
N.Copiers    15.0461     0.4900  30.706   <2e-16 ***
Modelsmall    0.7587     2.7799   0.273    0.786    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 9.011 on 42 degrees of freedom
Multiple R-squared:  0.9576, Adjusted R-squared:  0.9556 
F-statistic: 473.9 on 2 and 42 DF,  p-value: < 2.2e-16

> 
> anova(m)
Analysis of Variance Table

Response: Call.Time
                Df Sum Sq Mean Sq   F value  Pr(>F)    
N.Copiers        1  76960   76960 1000.2987 < 2e-16 ***
Model            1      6       6    0.0786 0.78059    
N.Copiers:Model  1    256     256    3.3260 0.07549 .  
Residuals       41   3154      77                      
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

From the outputs above as well as the output of the interaction model earlier, we can observe the following:

- Statistical significance: The interaction term is only marginally significant, with a p-value of 0.0755. We have weak evidence to support the claim that such a differential effect caused by copier models is real as opposed to random noise.

- Practical significance:

  – The interaction term raised the R-squared value from .9576 to .9608. The increase of .0032 is meager.

  – Compared to the Mean Square of X1 (number of copiers serviced), which is 76960, the Mean Square of the interaction term is only 256, which is only .3% of the former.

  Both aspects indicate that the interaction term has ignorable practical significance in accounting for time spent on the service calls.