STP 530: Applied Regression Analysis

Name : **Sai Swaroop Reddy Vennapusa**

Extra Credit

Instructor : **Yi Zheng**

Due Date : 9^{th} Nov 2023, 10:30AM

Step 1 and 2:

R Code:

```
# Step 1
install.packages("car")
library(car)

# Step 2
data(Duncan)
head(Duncan)
str(Duncan)
```

R Output:

```
> library(car)
Loading required package: carData
> # Step 2
> data(Duncan)
> head(Duncan)
           type income education prestige
accountant prof     62        86       82
pilot      prof     72        76       83
architect  prof     75        92       90
author     prof     55        90       76
chemist    prof     64        86       90
minister   prof     21        84       87
>
> str(Duncan)
'data.frame':   45 obs. of  4 variables:
 $ type     : Factor w/ 3 levels "bc","prof","wc": 2 2 2 2 2 2 2 2 3 2 ...
 $ income   : int  62 72 75 55 64 21 64 80 67 72 ...
 $ education: int  86 76 92 90 86 84 93 100 87 86 ...
 $ prestige : int  82 83 90 76 90 87 93 90 52 88 ...
```

Step 3:

R Code:

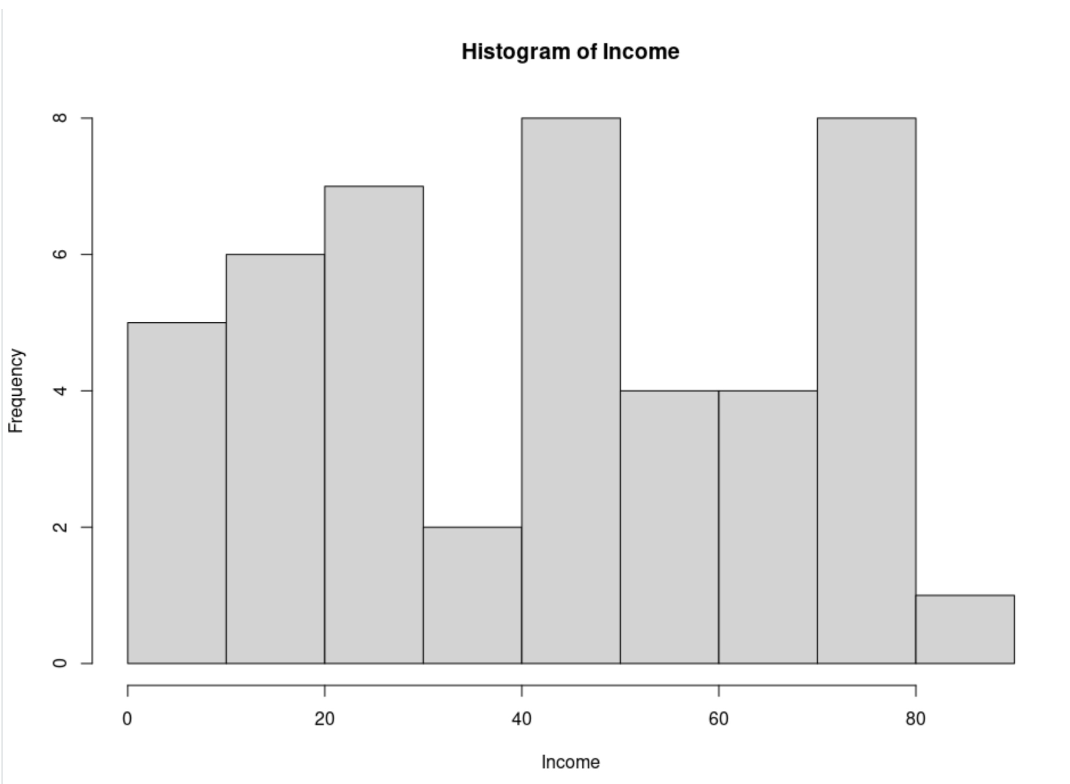
```
# Step 3
help(Duncan)
str(Duncan)
table(Duncan$type)
```

R Output:

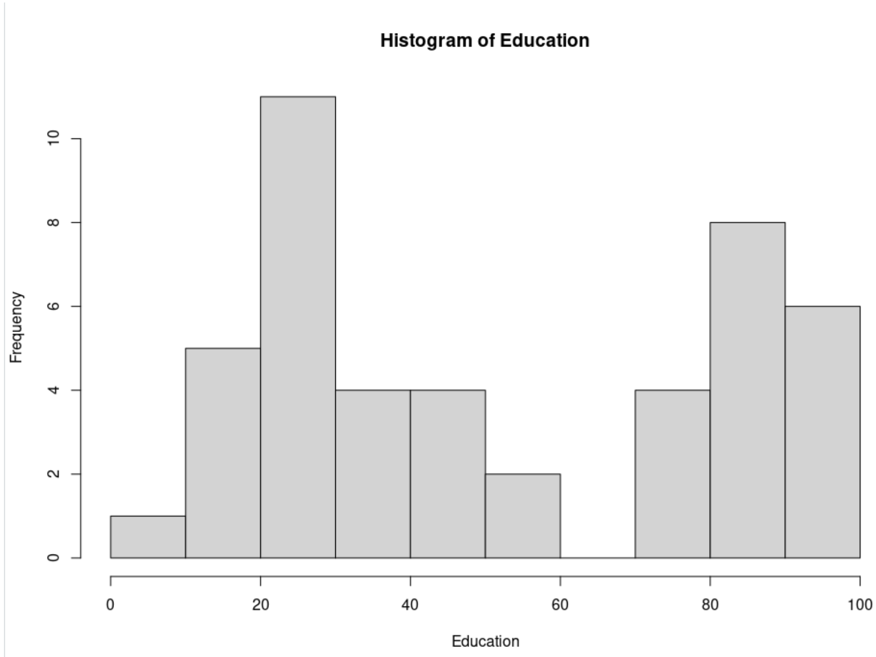
```
> str(Duncan)
'data.frame':	45 obs. of  4 variables:
 $ type     : Factor w/ 3 levels "bc","prof","wc": 2 2 2 2 2 2 2 2 3 2 ...
 $ income   : int  62 72 75 55 64 21 64 80 67 72 ...
 $ education: int  86 76 92 90 86 84 93 100 87 86 ...
 $ prestige : int  82 83 90 76 90 87 93 90 52 88 ...
> table(Duncan$type)

  bc prof   wc 
  21   18    6 
```

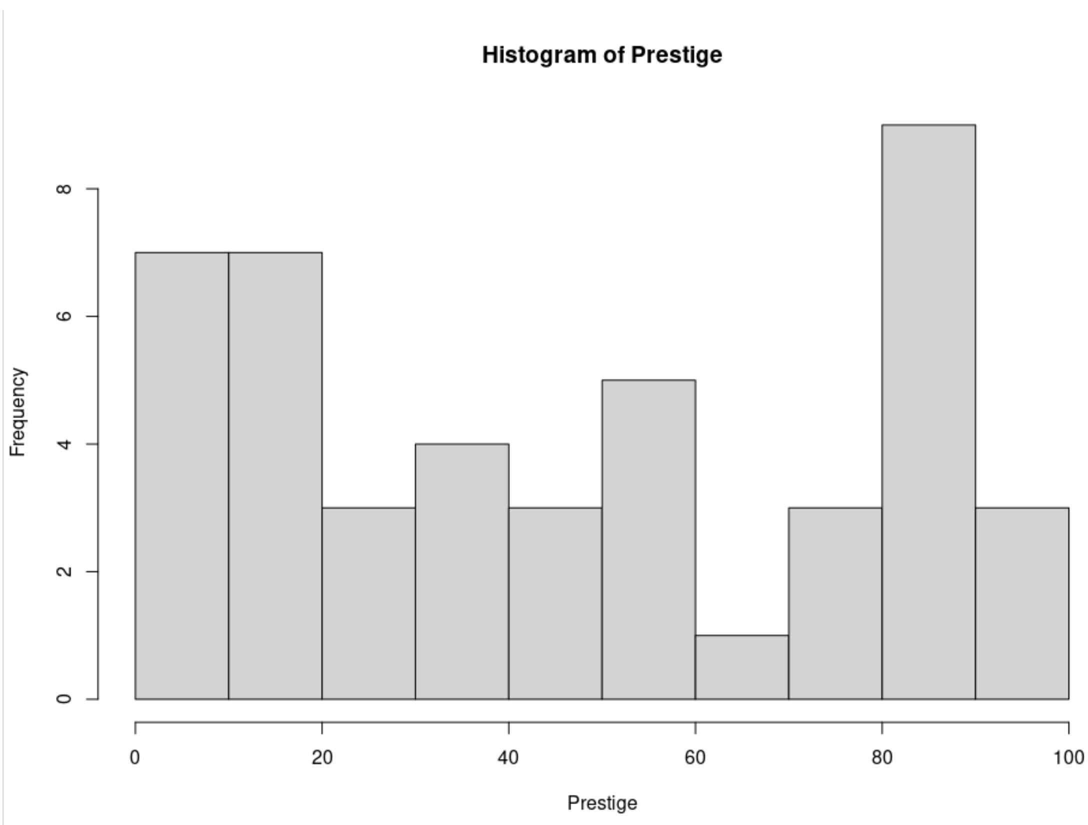
Histogram for numeric variable:
hist(Duncan$income, main="Histogram of Income", xlab="Income")



hist(Duncan$education, main="Histogram of Education", xlab="Education")




```
hist(Duncan$prestige, main="Histogram of Prestige", xlab="Prestige")
```



e. For each variable, discussion whether the data distribution look reasonable. Are there any signs of possible data errors?

Answer:

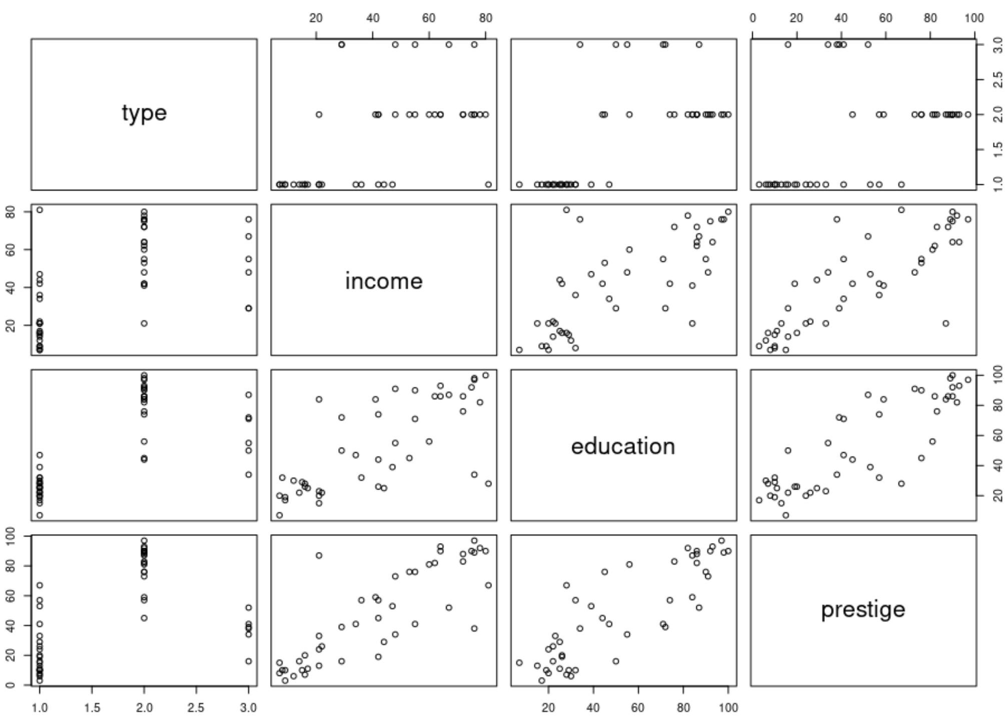
Type:
The frequency table for the categorical variable `type` shows that the majority of observations are classified as either 'bc' or 'prof', with 'wc' being less frequent. This distribution seems reasonable given that 'bc' (blue-collar) and 'prof' (professional) occupations tend to be more common than 'wc' (white-collar) occupations in general datasets.

Income:
The histogram of income shows a somewhat uniform distribution with two peaks, which could suggest that there are two groups within this variable. There are no signs of extreme outliers, and the distribution does not appear to be heavily skewed. However, the presence of two peaks might warrant a closer inspection to understand if this represents two different subgroups within the data.

Education:
The education histogram shows a distribution that is slightly left-skewed, with fewer observations at the lower end of the education scale. This could be expected as the dataset possibly represents occupations that require a certain level of education. There are no obvious signs of data errors or extreme outliers.

Prestige:
The prestige histogram appears to be somewhat bimodal, with two peaks around the middle of the scale and fewer observations at both the high and low ends. This could suggest distinct groups in terms of occupational prestige, which could be expected given the nature of the variable. No clear signs of data errors are evident from this plot.

Step 4:



a. Relationship between Prestige and Other Variables:

- Prestige and Income: There appears to be a positive relationship between income and prestige. As income increases, the prestige of the occupation also seems to increase, which is intuitive as higher-paying jobs often come with greater prestige.

- Prestige and Education: A similar positive relationship is observed between education and prestige. Higher levels of education tend to be associated with more prestigious occupations. This reflects the societal value placed on education and its role in accessing higher-prestige positions.

b. Relationships Among Predictors (Implications of Multicollinearity):

- Income and Education: There is a noticeable positive correlation between income and education. This suggests that as education increases, so does income, which is expected in many societal contexts where higher education often leads to higher-paying jobs. However, this positive correlation could also imply multicollinearity when both are used as predictors in a regression model. Multicollinearity could inflate the variances of the parameter estimates and make the estimates less reliable. It's essential to assess the variance inflation factors (VIF) during model diagnostics to check for multicollinearity issues.

In summary, the scatterplot matrix reveals expected relationships between prestige and both income and education, suggesting that these variables could be good predictors for prestige in a regression model. However, the positive correlation between income and education warrants further investigation for potential multicollinearity before finalizing the model.

Step 5:

```
> m <- lm(prestige ~ education + income + type, data=Duncan)
> summary(m)


Call:
lm(formula = prestige ~ education + income + type, data = Duncan)


Residuals:
    Min      1Q  Median      3Q     Max
-14.890  -5.740  -1.754   5.442  28.972


Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.18503    3.71377  -0.050  0.96051
education     0.34532    0.11361   3.040  0.00416
income        0.59755    0.08936   6.687 5.12e-08 *
typeprof     16.65751    6.99301   2.382  0.02206 *
typewc      -14.66113    6.10877  -2.400  0.02114 *
---
Signif. codes:  0 ‘*’ 0.001 ‘’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 9.744 on 40 degrees of freedom
```

Multiple R-squared: 0.9131,   Adjusted R-squared: 0.9044
F-statistic: 105 on 4 and 40 DF, p-value: < 2.2e-16

Step 6:

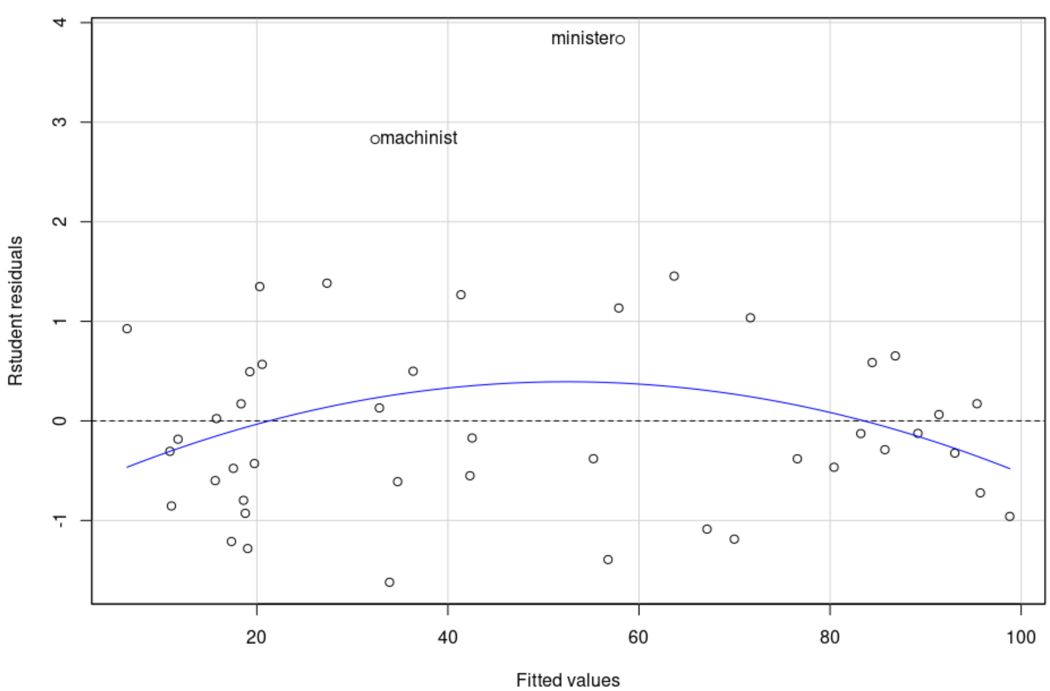
```
> vif(m)
              GVIF Df GVIF^(1/(2*Df))
education 5.297584  1        2.301648
income    2.209178  1        1.486330
type      5.098592  2        1.502666
```

- Education has a VIF of approximately 2.30, which is below the cutoff of 3.16. This suggests that `education` does not have concerning multicollinearity with the other variables.
- Income has a VIF of approximately 1.49, which is well below the cutoff of 3.16. This suggests that `income` does not have concerning multicollinearity with the other variables.
- Type has a combined VIF for the dummy variables at about 1.50, also below the cutoff. This indicates that `type` does not have concerning multicollinearity with the other variables.

In summary, none of the variables exceed the common threshold of concern for multicollinearity, and therefore, there is no evidence of high multicollinearity affecting the regression estimates in this model.

Step 7:

```
> residualPlots(m, ~1, type="rstudent", id=list(labels=row.names(Duncan)))
           Test stat Pr(>|Test stat|)
Tukey test   -1.6035           0.1088
```

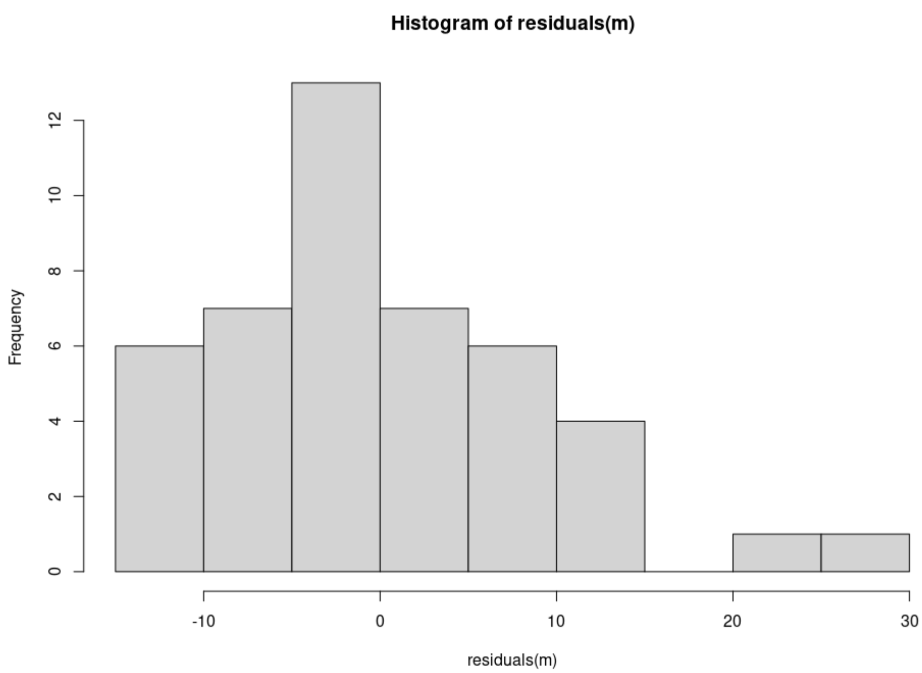


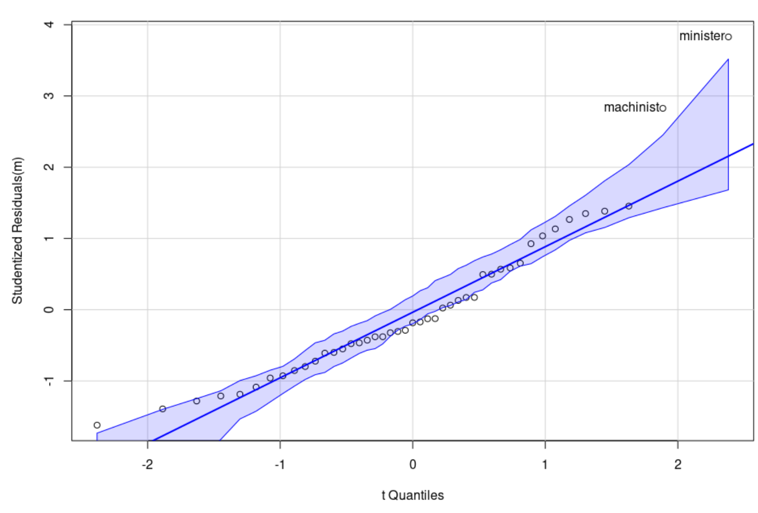
The residual plot does suggest the possibility of a nonlinear relationship, as indicated by the curve in the plotted line. The residuals should ideally be scattered randomly around the horizontal axis (zero line). Here, the curve suggests that the residuals have a systematic pattern, implying that the model may not be capturing some of the non-linear patterns in the data.

As for the assumption of homoscedasticity, the plot shows that the spread of residuals is relatively uniform across the range of fitted values. While there are a couple of outliers (such as "ministero"), there is not a clear pattern of increasing or decreasing spread. This suggests that the homoscedasticity assumption is roughly met, although the presence of outliers might warrant further investigation.

In summary, while the assumption of constant variance seems to be met, the residual plot indicates that the model may benefit from including non-linear terms to better capture the relationship between the predictors and the response variable.

Step 8:





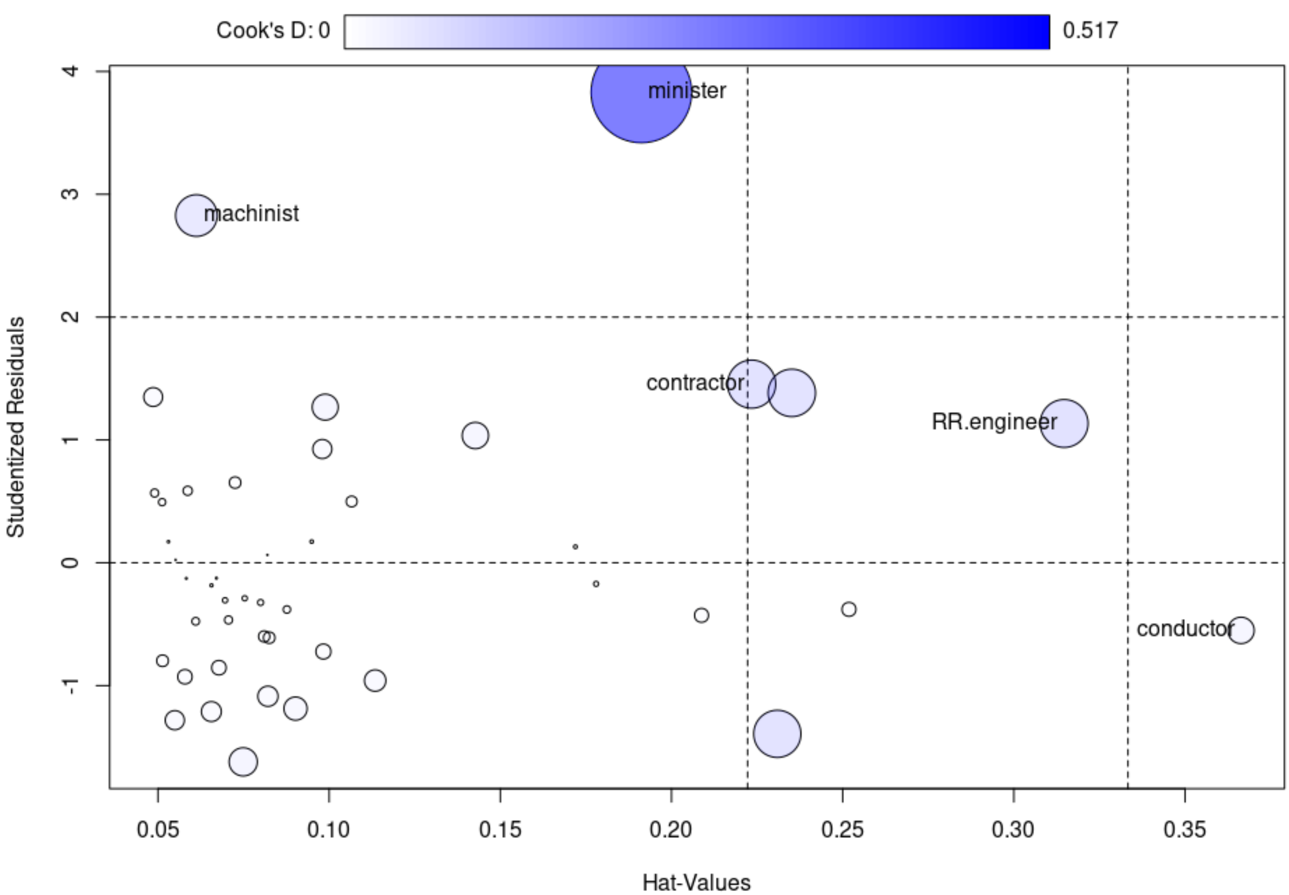
The histogram of residuals displays a fairly symmetrical distribution, but there is a noticeable skewness towards the right, indicating some larger positive residuals. However, there aren't any extreme outliers, and the bulk of the data clusters around the center, which is consistent with a normal distribution's characteristics.

In the Q-Q plot, most of the points follow the expected line, but there's a clear deviation at both ends of the distribution, especially in the upper tail where we can see a couple of points labeled, such as "ministero" and "machinisto," standing out significantly from the line. This suggests that the residuals may have heavier tails than a normal distribution, indicating potential issues with normality.

In conclusion, while the assumption of normality is somewhat supported by these plots, the deviations in the Q-Q plot, especially, suggest that the normality assumption does not hold perfectly for this dataset.

Step 9:

a.



The influence plot displays standardized residuals against the leverage (hat-values) for each observation, with the size of the circles proportional to the Cook's D values of the observations. Observations with larger Cook's D values are more influential to the regression model.

From the plot, it appears that several points stand out:

- "minister" has high leverage and a large residual, making it a point of concern as it is quite influential on the model fit.
- "machinist" also has a relatively high Cook's D value, suggesting it is an influential observation.
- "RR.engineer" and "conductor" have high leverage but their residuals are not as large, which implies they have potential influence but perhaps not as strong as "minister".
- "contractor" has a moderate Cook's D value and leverage, indicating some influence but not as pronounced as "minister" or "machinist".

The presence of these influential points suggests that the model's estimates might be unduly affected by a few data points.

b.

```
> Cooks.d <- cooks.distance(m)
> p <- 5
> n <- nrow(Duncan)
> percentile <- 100 * pf(q=Cooks.d, df1=p, df2=n-p)
> data.frame(Duncan, Cooks.d=round(Cooks.d, 3), percentile=round(percentile, 1))
                   type income education prestige Cooks.d percentile
accountant         prof     62        86       82   0.000        0.0
pilot              prof     72        76       83   0.001        0.0
architect          prof     75        92       90   0.002        0.0
author             prof     55        90       76   0.003        0.0
chemist            prof     64        86       90   0.004        0.0
minister           prof     21        84       87   0.517       23.8
professor          prof     64        93       93   0.007        0.0
dentist            prof     80       100       90   0.024        0.0
reporter             wc     67        87       52   0.010        0.0
engineer           prof     72        86       88   0.000        0.0
undertaker         prof     42        74       57   0.021        0.0
lawyer             prof     76        98       89   0.012        0.0
physician          prof     76        97       97   0.001        0.0
welfare.worker     prof     41        84       59   0.028        0.0
teacher            prof     48        91       73   0.003        0.0
conductor            wc     76        34       38   0.036        0.1
contractor         prof     53        45       76   0.118        1.2
factory.owner      prof     60        56       81   0.036        0.1
store.manager      prof     42        44       45   0.114        1.1
banker             prof     78        82       92   0.000        0.0
bookkeeper           wc     29        72       39   0.115        1.2
```

mail.carrier	wc	48	55	34	0.001	0.0
insurance.agent	wc	55	71	41	0.001	0.0
store.clerk	wc	29	50	16	0.010	0.0
carpenter	bc	21	23	33	0.018	0.0
electrician	bc	47	39	53	0.035	0.1
RR.engineer	bc	81	28	67	0.117	1.2
machinist	bc	36	32	57	0.089	0.6
auto.repairman	bc	22	22	26	0.003	0.0
plumber	bc	44	25	29	0.007	0.0
gas.stn.attendant	bc	15	29	10	0.011	0.0
coal.miner	bc	7	7	15	0.019	0.0
streetcar.motorman	bc	42	26	19	0.041	0.1
taxi.driver	bc	9	19	10	0.000	0.0
truck.driver	bc	21	15	13	0.003	0.0
machine.operator	bc	21	20	24	0.003	0.0
barber	bc	16	26	20	0.000	0.0
bartender	bc	16	28	7	0.019	0.0
shoe.shiner	bc	9	17	3	0.011	0.0
cook	bc	14	22	16	0.000	0.0
soda.clerk	bc	12	30	6	0.020	0.0
watchman	bc	17	25	11	0.007	0.0
janitor	bc	7	20	8	0.001	0.0
policeman	bc	34	47	41	0.006	0.0
waiter	bc	8	32	10	0.006	0.0

Based on the calculated Cook's D values and their corresponding percentile scores, it appears that none of the cases exceed the 50th percentile on the reference distribution F(p, n – p). The highest percentile score reported is 23.8 for the "minister" observation, which is below the 50th percentile. This suggests that there are no cases with disproportionately large influence on the regression model, as all Cook's D values fall below the median of the F distribution, which is a common benchmark for identifying influential points.

Step 10:

> dfbetas(m)

	(Intercept)	education	income	typeprof	typewc
accountant	5.166618e-03	-0.0064582754	-0.0002826414	-0.0041398690	0.0044551684
pilot	6.612933e-05	0.0281784736	-0.0383119118	-0.0311400514	-0.0038757813
architect	4.170209e-02	-0.0286279377	-0.0341354438	0.0157678821	0.0326894586
author	2.197543e-02	-0.0552400023	0.0364413281	-0.0040227081	0.0228139451
chemist	-2.767360e-02	0.0265741751	0.0123821619	0.0168462198	-0.0227471508
minister	4.506174e-01	0.5717133326	-1.5631368858	0.5344009789	0.2306228944
professor	-6.480716e-02	0.0851320168	-0.0020435183	-0.0240350601	-0.0564568943
dentist	2.002024e-01	-0.1680138239	-0.1224283788	0.1307820534	0.1611898874
reporter	1.024701e-01	-0.1114732718	-0.0281267433	0.1144600529	-0.0305533856
engineer	9.252980e-03	-0.0029630024	-0.0121679614	-0.0016972918	0.0067816128
undertaker	-1.246416e-01	0.0428826160	0.1598822220	-0.2009754879	-0.0917645066

lawyer	1.290343e-01	-0.1158368049	-0.0686752642	0.0783167261	0.1049404981
physician	-2.943468e-02	0.0254687837	0.0169608283	-0.0170078556	-0.0238055629
welfare.worker	-5.217496e-02	-0.1013579248	0.2286508212	-0.1101793329	-0.0218096868
teacher	1.204974e-02	-0.0575660000	0.0569552369	-0.0048688903	0.0163052822
conductor	-7.934558e-02	0.2680518880	-0.2245644238	-0.1397258237	-0.2757971028
contractor	4.638609e-01	-0.6683541646	0.0946248208	0.6919297623	0.4123061476
factory.owner	2.018532e-01	-0.3302683244	0.0946219725	0.3431930395	0.1849054979
store.manager	-5.134601e-01	0.6154798268	0.0638008294	-0.7124709947	-0.4390898166
banker	-4.130514e-03	-0.0026367364	0.0107987901	0.0025411286	-0.0024763100
bookkeeper	-8.658934e-03	0.2841270583	-0.3699921949	-0.0869080868	0.3749391930
mail.carrier	7.852925e-03	-0.0098558051	-0.0003758578	0.0091409222	0.0448944429
insurance.agent	1.543265e-02	-0.0188192686	-0.0014834499	0.0178093539	-0.0371347710
store.clerk	-7.269452e-02	0.0369785066	0.0770248695	-0.0693628533	-0.1831197847
carpenter	2.080036e-01	-0.0275388241	-0.0241931885	-0.0555106290	-0.0770260428
electrician	-6.214296e-02	0.1307454171	0.2231658648	-0.3110105778	-0.2771543823
RR.engineer	-1.053853e-01	-0.1645379677	0.7067141803	-0.2689384305	-0.2715007418
machinist	1.120353e-01	0.1324492044	0.2620900655	-0.4356321537	-0.4138405878
auto.repairman	8.984081e-02	-0.0199195648	-0.0028886382	-0.0191743410	-0.0296939031
plumber	-3.990263e-02	0.0365308288	-0.1189267476	0.0641829535	0.0706150615
gas.stn.attendant	-1.212830e-01	-0.0630042739	0.0885955746	0.0796738851	0.0801344153
coal.miner	3.012351e-01	-0.1652375665	-0.0898295167	0.1272927869	0.0724329684
streetcar.motorman	-1.043407e-01	0.0680911623	-0.2780522059	0.1788112838	0.1916233016
taxi.driver	-4.132987e-02	0.0066309817	0.0217130168	-0.0034749067	0.0014499212
truck.driver	-1.019320e-01	0.0556547715	-0.0046127287	-0.0158666625	0.0017316476
machine.operator	8.702268e-02	-0.0278151698	-0.0037569693	-0.0065785444	-0.0183064649
barber	2.555052e-02	0.0049827799	-0.0129366367	-0.0102257842	-0.0116446792
bartender	-1.724084e-01	-0.0680241134	0.1056488630	0.1003716513	0.1043185648
shoe.shiner	-2.047237e-01	0.0514555005	0.0949963779	-0.0321662961	-0.0047964496
cook	4.288972e-03	-0.0003118063	-0.0018677372	-0.0004324751	-0.0008665912
soda.clerk	-1.620793e-01	-0.1071333797	0.1548225122	0.1088283319	0.1063269882
watchman	-1.219807e-01	-0.0114483703	0.0498727897	0.0416361507	0.0502893942
janitor	-6.854836e-02	0.0056691683	0.0430048618	-0.0040880156	0.0033174078
policeman	-3.268975e-02	0.1190515958	0.0111418063	-0.1484979903	-0.1245500038
waiter	-7.997160e-02	-0.0746512229	0.1043973168	0.0609984219	0.0566298249

Upon evaluating the dfbetas for the regression model, we identify the cases that exert the greatest influence on the coefficients of each predictor. For each predictor, the case with the largest absolute dfbeta value indicates the observation that, if omitted, would lead to the most significant change in the estimated coefficient for that predictor.

For the `education` predictor, the occupation with the largest influence appears to be 'contractor,' as indicated by the dfbeta matrix. This could suggest that contractors, perhaps due to their unique combination of education and occupational prestige, either increase or decrease the slope of the `education` predictor more than any other occupation.

In the case of `income`, the 'minister' role shows a substantial effect. Given that ministers might have a different relationship between their income and occupational prestige compared to other occupations, their inclusion or exclusion in the model could significantly affect the income coefficient.

Finally, for the categorical predictor `type`, we combine the effects for the levels 'prof' and 'wc' to understand the influence on the overall variable. The occupation 'contractor' again stands out, indicating that the way the type of occupation is related to prestige for contractors is significantly different from the general trend.

These observations suggest that the contractor and minister roles have unique characteristics that influence the model's understanding of the relationship between predictors like education, income, and occupational type with prestige. These unique characteristics could be due to outlier values, leverage points, or a non-representative distribution of these roles in comparison to the general trends in the dataset.