STP 530: Applied Regression Analysis

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Homework 7

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Due Date : 24th Oct 2023, 10:30AM

Question 8.21 a. Develop the response function for each type of protection category.
b. For each of the following questions, specify the alternatives Ho and Ha for the appropriate test: (I) With X1 fixed, does wearing a bump cap reduce the expected severity of injury as compared with wearing no protection? (2) With X1 fixed, is the expected severity of injury the same when wearing a hard hat as when wearing a bump cap?

Answer:
Part a: Develop the response function for each type of protection category.

The response functions indicate the expected severity of injury under different conditions, given by the type of protection worn. Each equation is a linear combination of the predictors and their respective coefficients, representing different scenarios in the population. The beta coefficients are unknown parameters that we aim to estimate from the data.

1. Hard Hat:
   - When a hard hat is worn, X2 = 1, and X3 = 0. The response function (i.e., the expected value of Y) is:
     - E(Y) = beta_0 + beta_1*X1 + beta_2*1 + beta_3*0
     - E(Y) = beta_0 + beta_1*X1 + beta_2

2. Bump Cap:
   - When a bump cap is worn, X2 = 0, and X3 = 1. The response function is:
     - E(Y) = beta_0 + beta_1*X1 + beta_2*0 + beta_3*1
     - E(Y) = beta_0 + beta_1*X1 + beta_3

3. None (No protection):
   - When no protection is worn, both X2 and X3 are 0. The response function is:
     - E(Y) = beta_0 + beta_1*X1 + beta_2*0 + beta_3*0
     - E(Y) = beta_0 + beta_1*X1

Part b: Specify the null and alternative hypotheses.

Question 1:
"With X1 fixed, does wearing a bump cap reduce the expected severity of injury as compared with wearing no protection?"

For this question, we are comparing the "Bump cap" category (E(Y) = beta_0 + beta_1*X1 + beta_3) to the "None" category (E(Y) = beta_0 + beta_1*X1). Specifically, we are interested in whether beta_3 is negative, indicating a reduction in the expected severity.

- Ho (Null Hypothesis): beta_3 = 0 (Wearing a bump cap does not change the expected severity of injury compared with no protection.)
- Ha (Alternative Hypothesis): beta_3 < 0 (Wearing a bump cap reduces the expected severity of injury compared with no protection.)

Question 2:
"With X1 fixed, is the expected severity of injury the same when wearing a hard hat as when wearing a bump cap?"

Here, we are comparing the "Hard hat" category ($E(Y) = \beta_0 + \beta_1 X1 + \beta_2$) to the "Bump cap" category ($E(Y) = \beta_0 + \beta_1 X1 + \beta_3$). We want to know if the difference, represented by ($\beta_2 - \beta_3$), is zero.

- Ho (Null Hypothesis): $\beta_2 = \beta_3$ (The expected severity of injury is the same for both a hard hat and a bump cap.)
- Ha (Alternative Hypothesis): $\beta_2 \neq \beta_3$ (The expected severity of injury is not the same for a hard hat and a bump cap.)