

# FAST WIREFRAME-VISIBILITY ALGORITHM

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**Abstract:** In this paper, a fast wireframe-visibility algorithm is introduced. The algorithm's inputs are 3D wireframe model of an object, internal and external camera calibration parameters. Afterwards, the algorithm outputs the 2D image of the object with only visible lines and surfaces. 2D image of an object is constructed by using a camera model with the given camera calibration parameters and 3D wireframe object model. The idea behind the algorithm is finding the intersection points of all lines in 2D image of the object. These intersection points are called as critical points and the lines having them are critical lines. Lines without any critical points are regarded as normal lines. Critical and normal lines are processed separately. Critical lines are separated into smaller lines by its critical points and depth calculation is performed for the middle points of these smaller lines. For the normal lines, depth of the middle point of the normal line is calculated to determine if it is visible or not. As a result, the algorithm provides the minimum amount of point's depth calculation. Moreover, this idea provides much faster process for the reason that there aren't any resolution and memory problems like well-known image-space scan-line and z-buffering algorithms.

## 1 INTRODUCTION

In order to produce a display of a three-dimensional object, transformation of the modelling and world-coordinate descriptions to viewing coordinates, then to device coordinates; identification of visible lines; and the application of surface-rendering procedures should be processed. This study deals with determining visibility of object edges which is referred as wire-frame-visibility algorithms. They are also called visible-line detection or hidden-line detection algorithms (Hearn and Baker, 1997).

Visible line detection is one of the most difficult in problem computer graphics. Visible line detection algorithms attempt to determine the lines or edges that are visible or invisible to an observer located at a specific point in space. Algorithms are grouped in two parts; object-space and image-space algorithms. In the object-space algorithms, invisible lines are designated in 3D object model and then the visible parts are transformed into 2D image. On the other hand, in the image space algorithms, the invisible lines are removed directly in 2D image. In general, image-space algorithms are preferred since they are faster than the object-space algorithms. (Hearn and Baker, 1997)

Well-known image-space visible line detection algorithms are z-buffering, scan line methods, etc. Z-buffering works by testing pixel depth and comparing the current position with stored data in a buffer: z-buffer. On the other hand the scan line algorithms process the scene in scan line order (Dong, X., 1999).

In this study, a new and fast visible line detection algorithm is introduced for wireframe model. The algorithm is a type of image-space algorithms. It should be remembered that in wireframe model; objects are drawn as though made of wires with only their boundaries showing. In this study, object models in 3D are assumed to be represented by lines. This is reasonable since any curved shape can be approximated by defining some lines on the curve.

The algorithm is prepared as if it is much faster than the scan-line method for the reason that it does not have any resolution problem. Besides, memory requirement of the algorithm is much less than z-buffering method since it does not require pixel by pixel process.

This paper presents the implementation and the performance of the algorithm in its sections. In the second section of the paper, mathematical relation between 3D and 2D is explained in the aspects of 3D

object representation, 2D image construction and the depth calculation. In the third section, the algorithm is described step by step. The simulation of the algorithm is implemented in the Matlab programming and results are presented in the forth section. In the conclusion section, results of the study are evaluated.

## 2 2D TO 3D CONCEPT

In this section, 3D object representation, 2D image construction, perspective camera model and depth calculation method are explained.

### 2.1 3D Object Representation

In this study, 3D objects are represented in wireframe model. The model is represented by points, lines and surfaces correspondingly. As an example, let's consider a cube as shown in Figure 1. 3D coordinates of the points  $a, b, c, d, e, f, g$  and  $h$  are defined for a wireframe model representation. Next, the lines are expressed by its beginning and the ending points as line  $ab, ef, bd$ , etc. Finally, the surfaces are defined as  $abcd, acge$ , etc.

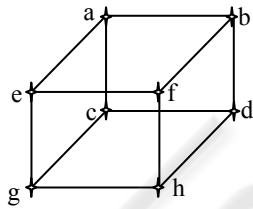


Figure 1: 3D model of a cube.

Understanding of the 3D geometry of the object is necessary since the 2D view will be considered. Any plane surface in 3D can be expressed as following (Hearn and Baker, 1997):

$$Ax + By + Cz + D = 0 \quad (1)$$

where  $(x, y, z)$  is any point on the plane and the coefficients  $A, B, C$  and  $D$  are constants describing the spatial properties of the plane. If three successive polygon vertices  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  are selected  $A, B, C$  and  $D$  values can be obtained as following (Hearn and Baker, 1997):

$$A = y_1(z_2 - z_3) + y_2(z_3 - z_1) + y_3(z_1 - z_2) \quad (2)$$

$$B = z_1(x_2 - x_3) + z_2(x_3 - x_1) + z_3(x_1 - x_2) \quad (3)$$

$$C = x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \quad (4)$$

$$D = -x_1(y_2z_3 - y_3z_2) - x_2(y_3z_1 - y_1z_3) - x_3(y_1z_2 - y_2z_1) \quad (5)$$

### 2.2 2D Image Construction

Camera model is the main issue to discuss during the 2D image construction process. Because 3D world coordinates are transformed into the image coordinates according to the camera model. In this paper, the perspective camera model is employed which is applicable to CCD cameras, IIR systems, X-ray images and etc. A point in the world is transformed into pixel coordinates after three steps using perspective camera model which is described in Zisserman and Hartley, 2003.

First of all, a point  $N = [N_x \ N_y \ N_z]^T$  in the world coordinate frame (WCF) is related to the camera coordinate frame (CCF) as:

$$p_c = [X_c \ Y_c \ Z_c]^T = R_w^c (N - T) \quad (6)$$

where  $T = [T_x \ T_y \ T_z]^T$  is the camera position expressed in the WCF and  $R_w^c$  is the rotation matrix relating the two coordinate frames. Recall that point  $p_c$  satisfies the plane equation as following:

$$AX_c + BY_c + CZ_c + D = 0 \quad (7)$$

For the second step,  $p_c$  is expressed in the image coordinates by using "similar triangles", a point in the CCF is transformed to the image coordinate frame (ICF) as:

$$p_i = [u \ v]^T = \left[ f \frac{X_c}{Z_c} \ f \frac{Y_c}{Z_c} \right]^T \quad (8)$$

where  $f$  is the effective focal length.  $p_i$  is also affected by the distortion coefficient of the lens and becomes:

$$p_d = [u_d \ v_d]^T = \left[ \frac{2u}{1 + \sqrt{1 - 4\kappa(u^2 + v^2)}} \ \frac{2v}{1 + \sqrt{1 - 4\kappa(u^2 + v^2)}} \right]^T \quad (9)$$

where  $\kappa$  is the distortion coefficient of the total lens system. Finally,  $p_d$  is expressed in terms of pixel coordinates by taking pixel size and image centre into account as:

$$p_{pix} = [x \ y]^T = \left[ \frac{u_d}{S_x} + C_x \ \frac{v_d}{S_y} + C_y \right]^T \quad (10)$$

Therefore, camera coordinates of a point is projected into the image coordinates by using the camera calibration parameters. Method for finding the camera calibration parameters is out of scope of this study. The parameters are assumed to be known precisely. Camera model block diagram is shown in Fig. 2.

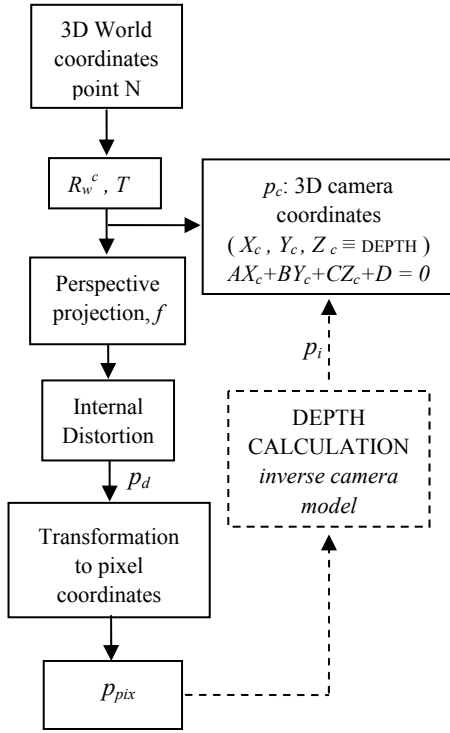


Figure 2: Perspective Camera Model.

### 2.3 Depth Calculation

Transformation from world frame to the camera frame results in alignment of the location of the object with the line of sight direction of the camera. Thus, visible or invisible parts should be decided in camera coordinates. At the same time, surface coefficients  $A$ ,  $B$ ,  $C$  and  $D$  in equation (1) should be calculated in camera coordinate frame by using the equations (2), (3), (4) and (5). Since the camera is located at the origin of the camera coordinate frame,  $z$  coordinate of any point is nothing but its depth according to the camera. If any two points having same  $x$  and  $y$  coordinates are investigated, visible point has smaller  $z$  value than the invisible point since it is closer to the camera.

In Figure 2, inverse camera model process is also shown in dashed lines, which converts the image pixel coordinates to the camera coordinates. Consequently, depth is calculated by this inverse process.

Let's explain the depth calculation step by step. First,  $p_{pix}$  in equation (10) is known and  $p_d$  can be obtained directly from  $p_{pix}$  as below:

$$p_d = [u_d \ v_d]^T = [S_x(x - C_x) \ S_y(y - C_y)]^T \quad (11)$$

From equation (9), distorted image coordinates are known as:

$$\begin{aligned} u_d &= \frac{2u}{1 + \sqrt{1 - 4\kappa(u^2 + v^2)}} \\ v_d &= \frac{2v}{1 + \sqrt{1 - 4\kappa(u^2 + v^2)}} \end{aligned} \quad (12)$$

From the relations in equation (12),  $u$  and  $v$  can be obtained simply by iteration in which known  $u_d$  and  $v_d$  values are used. After that, by considering the equation (8),  $X_c$  and  $Y_c$  are calculated as below:

$$X_c = u \frac{Z_c}{f} \quad Y_c = v \frac{Z_c}{f} \quad (13)$$

Thus, we obtain the  $x$  and  $y$  coordinates of the camera coordinates. By substituting them into equation (7), the value of  $Z_c$ , which is exactly equal to the depth of the point, is found by using the equation below:

$$A(u \frac{Z_c}{f}) + B(v \frac{Z_c}{f}) + C(Z_c) + D = 0 \quad (14)$$

$$Depth = Z_c = \frac{-D}{A \frac{u}{f} + B \frac{v}{f} + C} \quad (15)$$

Therefore, depth calculation is succeeded as in equation (15) for given  $x$  and  $y$  pixel coordinates of the point. Coefficients of the plane to which the point corresponds in 3D should be known, too.

### 3 THE ALGORITHM

The visible line detection algorithm introduced in this paper is based on finding the critical points in the image. Critical points are the intersection points of the lines in the image. Critical points are simply found as calculation of intersection of the lines with known beginning and ending points, slopes and constant terms. Remember that any line can be expressed as:

$$y = m_i x + c_i \quad (16)$$

where  $m_i$  is the slope and  $c_i$  is the constant term of the line. Parameters  $m_i$  and  $c_i$  are calculated by using the beginning  $(x_b, y_b)$  and the ending  $(x_e, y_e)$  points of the line as following:

$$m = \frac{y_b - y_e}{x_b - x_e} \quad (17)$$

$$c = y_b - mx_b \text{ or } c = y_e - mx_e \quad (18)$$

Critical point calculation starts with finding the intersection point  $(x_{intersection}, y_{intersection})$  of any two

lines having line parameters  $m_1, c_1, m_2$  and  $c_2$ .

$$x_{intersection} = \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} \quad (19)$$

$$y_{intersection} = \frac{c_2 - c_1}{m_1 - m_2} \quad (20)$$

If the intersection points coordinates  $x_{intersection}$  and  $y_{intersection}$  are in the range of the lines beginning and ending points, it is regarded as critical points. Otherwise, it is not a critical point.

To make the importance and the necessity of the critical points more clear, let's work on an example. In Figure 3, 2D image of a cube is shown without visible line detection. Camera view side is also illustrated in the figure. Critical points are marked as well. It should be noted that the beginning and the ending points of the lines are not marked even they are also intersection points. The lines without any critical points will be investigated after the investigation of the critical lines.

There is one more important subject one should notice that any line of a cube is a member of two surfaces of the cube. This can be shown in Figure 3 since the line  $fg$  is a member of the surface  $afgk$  and  $fgmh$ . Thus, if we name the surfaces  $afgk$  and  $fgmh$  as native surfaces, visibility of the line  $fg$  should be decided according to its native surfaces visibilities.

As observed in Figure 3, point  $b$  and  $d$  are the critical points. They are included in the lines  $ac, fg, ce, gh$  which are also called as critical lines. Therefore, visibility check should be applied to lines  $ab, bc, fb, bg, gd, dh, cd, de$ . It is achieved by taking the middle points of these lines for two times since the lines have two native surfaces. Middle points are also marked in Figure 3. Let's call these middle points as  $m_i$  where  $i$  is from 1 to 8 for this case. First of all, the surfaces which include the point  $m_i$ 's should be listed. This is simply achieved by checking the coordinates of the point according to the surface borders coordinates. Note that the native surfaces will be included in this surface list as well. Afterwards, at each  $m_i$  the depths of the all surfaces in the list are calculated as explained in section 2 and are written in the depth matrix. If the minimum member of the depth matrix is equal to the depth of the currently selected native surface, the line is visible and its state should be set to 1. Otherwise it is hidden and its state should be set to 0. Thus, the

visibilities of the critical lines are decided. One should recall that this process is done for two times since a line has two native surfaces for this example.

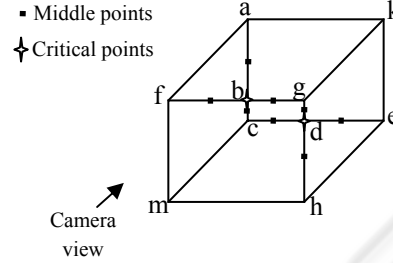


Figure 3: Critical points of a cube.

When it comes to the visibilities of the normal lines which do not have any critical points on it, it is decided in a similar way of the critical lines. In this case the middle points of the normal lines are taken and the visibility of that point is investigated just like critical lines.

To sum up, algorithms steps are presented as following:

- Step 1* Convert the 3D wire-frame from World to 3D camera frame
- Step 2* Calculate the coefficients ( $A, B, C$  and  $D$ ) of all the surfaces on the object
- Step 3* Convert the coordinates from camera to image frame.
- Step 4* Calculate the parameters ( $m$  and  $c$ ) of all lines.
- Step 5* Calculate the critical points in the image
- Step 6* Investigate the visibility of the critical lines one by one by separating the line according to its critical points.
- Step 7* Investigate the visibility of the normal lines.
- Step 8* Plot the visible lines

## 4 SIMULATION RESULTS

The algorithm is implemented in Matlab. The results are presented in this section for two experiments. First, two surfaces with different depths is presented. In the second experiment, 3D wireframe model is defined for five prisms. Camera parameters, rotations and translations are kept same for both of the experiments.

Figure 4, 5 and 6 are the simulation results for the first experiment. One triangle and one parallelogram are defined in 3D wireframe model. Thus, corner point's coordinates and the points that construct the lines and the surfaces are defined. In



Figure 4, two surfaces are illustrated in the World coordinate system. Star in the figure shows the camera location. It is obviously seen in Figure 4 that the triangle is in front of the parallelogram. After the transformation from 3D to 2D is applied by using the camera calibration parameters, Figure 5 is obtained. Critical points are also marked by stars in Figure 5. The algorithm runs for 4 critical and 3 normal lines. Figure 6 is the result as expected. 11 pieces of lines are investigated during the algorithm. Therefore, depth calculation is performed for only 11 times in this experiment.

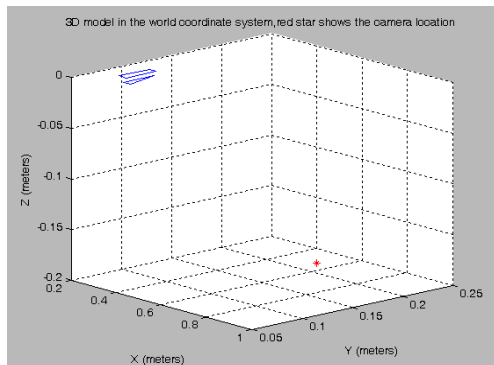


Figure 4: Two surfaces in the world coordinate frame.

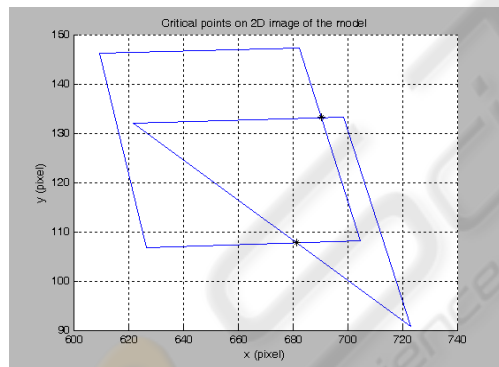


Figure 5: 2D image and the critical points.

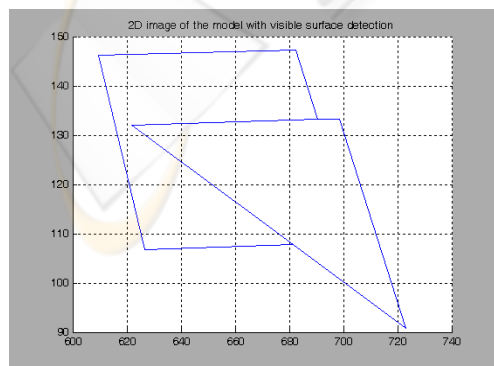


Figure 6: 2D image after visible line detection.

For the second experiment, the results are presented in Figure 7, 8, 9 and 10.

Five prisms are defined having different heights, widths and lengths. These prisms are shown in Figure 7 in World coordinate system. Figure 8 is also put to present the camera coordinate system. Stars in the figures stand for the camera location. As shown in Figure 8, camera is at the origin of the camera coordinate system.

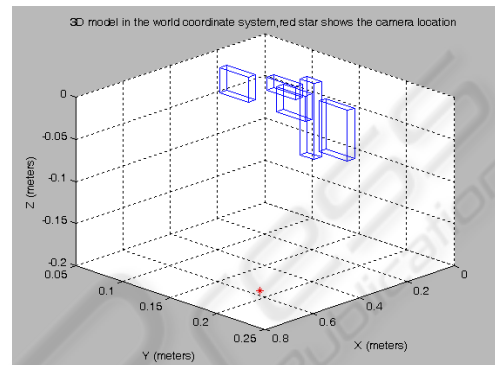


Figure 7: Five prisms in the world coordinate frame.

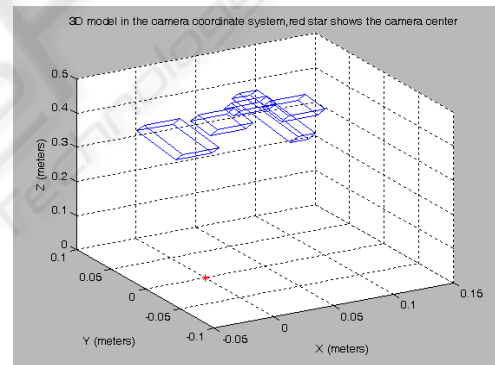


Figure 8: Five prisms in the camera coordinate frame.

Figure 9 is the 2D image of the prisms without considering the invisible lines under the camera parameters. Critical points are also marked in the figure. Figure 10 is the result of the visible line detection algorithm. In this experiment, there exist 200 critical points and 60 normal lines. Depth calculation is done for 321 times. Even though 321 depth calculations seem to be a large number, duration for the program is shorter than z-buffer or scan-line method in Matlab. Furthermore, there isn't any resolution or memory troubles during the algorithm.

If the prisms numbers are to be increased, time for visible line detection will be increased obviously. However, it would be still faster than the scan-line or z-buffering methods.

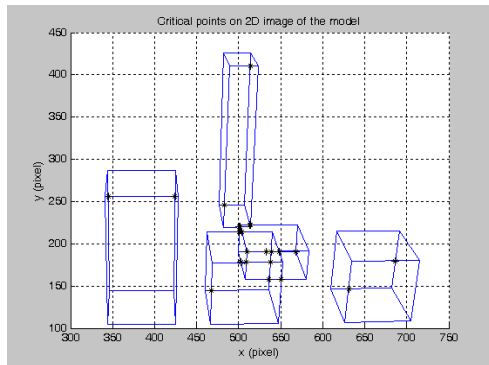


Figure 9: 2D image of prisms and the critical points.

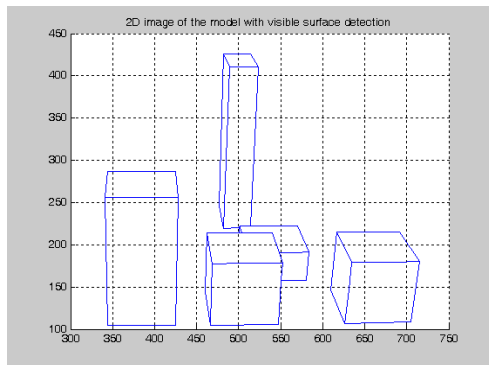


Figure 10: 2D image after visible line detection.

## 5 CONCLUSIONS

In this paper, a visible line detection algorithm is introduced for wireframe model. The algorithm is designed as fast as possible. Such algorithm would be applicable for air or land vehicles since the rate of the performance is the most critical term in those applications.

The algorithm performance should be compared with the well-known image-space algorithms like z-buffering and scan-line method in order to conclude the subject.

The z-Buffer algorithm is one of the most common hidden-line algorithms to implement in either software or hardware. The algorithm is an image-space algorithm. The z-buffer is extension of the frame buffer idea. Frame buffer is used to store the attributes (intensity) of each pixel in. The z buffer is a separate depth buffer, with the same number of entries as the frame buffer, used to store the z coordinate or depth of visible pixel in image-space. Visibility of a pixel is decided by comparing the z-values of the saved x, y values (Dong, X., 1999). Conceptually, the algorithm is a

search over x and y and visibility decision is done pixel by pixel.

When it comes to the scan line algorithms process the scene in scan line order. Scan line algorithms operate in image-space. They process the image one scan line at a time rather than one pixel at a time. By using area coherence of the polygon, the processing efficiency is improved over the pixel oriented method (Dong, X., 1999). However, resolution of the scan line makes troubles in time and precision.

The algorithm explained in this paper does not have any resolution and memory problems. This algorithm bases on finding the intersection points of the all lines of the object in 2D image coordinates. These intersection points are called as critical points and the lines having them are called as critical lines. Lines without any critical points are called as normal lines. Depth calculation is achieved for the separated critical lines by critical points and the normal lines. This approach provides much faster process especially when it is compared with scanning line or z-buffer methods. The only constraint for this algorithm is wireframe 3D model representation. This constraint will not cause any problem in real-life experiments since any curved shape can be approximated by defining some lines on that curve.

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