Composite Newton-Cotes formulas (smooth integrands)

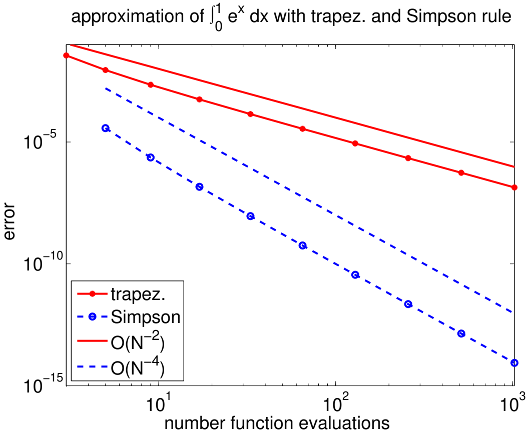
goal: compute $\int_0^1 e^x\,dx$ with composite trapezoidal and Simpson rule.

The errors E_{trap}, $E_{Simpson}$ satisfy (F denotes the number of function evalulations):

$$E_{trap}(h) \le Ch^2 \sim CF^{-2}, \qquad E_{Simpson} \le Ch^4 \sim CF^{-4},$$

h	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}
$F_{trap} \sim 1/h$	2	3	5	9	17	33	65	129	257
E_{trap}	1.4_{-1}	3.6_{-2}	8.9_{-3}	2.2_{-3}	5.6_{-4}	1.4_{-4}	3.5_{-5}	8.7_{-6}	2.2_{-6}
$F_{Simpson} \sim 1/h$	3	5	9	17	33	65	129	257	513
$E_{Simpson}$	5.8_{-4}	3.7_{-5}	2.3_{-6}	1.5_{-7}	9.1_{-9}	5.7_{-10}	3.6_{-11}	2.2_{-12}	1.4_{-13}

Composite Newton-Cotes formulas (smooth integrands)



conclusion: for smooth integrands more efficient to employ higher order quadrature formulas

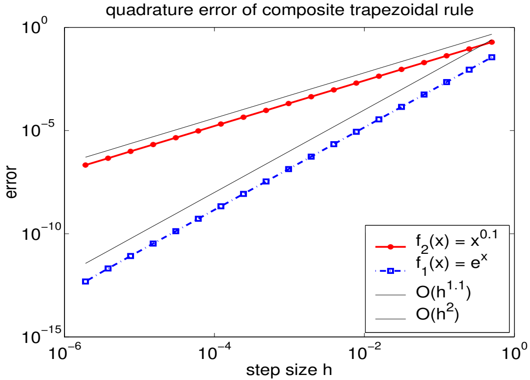
Composite Newton-Cotes formulas (non-smooth integrands)

$$f_1(x) := e^x, \qquad f_2(x) := x^{0.1}$$

$E_i(h) :=$ quadrature error of composite trapezoidal rule for $\int_0^1 f_i(x)\,dx$.

$$f_1(x) = e^x \Longrightarrow f_1 \in C^2([0,1]) \Longrightarrow E_1(h) \leq Ch^2,$$

$$f_2(x) = x^{0.1} \Longrightarrow f_2 \notin C^2([0,1]) \Longrightarrow \text{ cannot expect } E_2(h) \leq Ch^2$$



Remark: one can show that

$$E_2(h) \leq Ch^{1.1}$$