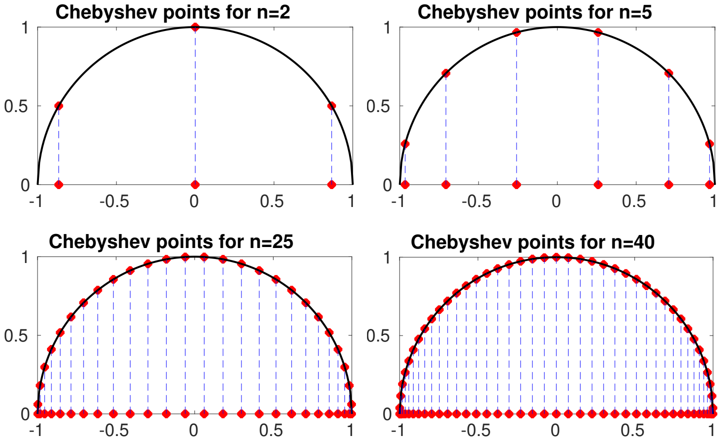
1000 ways to spell Chebyshev

Pafnuty Lvovitch Chebyshev (1821–1894)
Pafnutii L'vovitsch Tschebysheff
Paphnutij L'vovič Tchebicheff
Pafnutii L'vovitsch Čebysev
Pafnuty L'vovič Tschebysheff
Pafnutii Lvovitch Chebyshev
Pafnuty L'vovič Chebishev
Pafnutii Lvovitch Tchebicheff
Pafnutii L'vovič Chebyshev
Pafnuty L'vovitsch Čebysev
Pafnuty L'vovitsch Tschebyshev
Paphnutij L'vovitsch Chebishev
Pafnuty L'vovitsch Chebishev
Pafnutii Lvovitch Tschebyshev
Paphnutij L'vovič Tschebysheff
Pafnuty L'vovič Tchebicheff
Pafnutii L'vovitsch Tschebyshev
Pafnuty L'vovitsch Chebyshev
Paphnutij L'vovitsch Tschebyshev
Paphnutij Lvovitch Chebishev
Pafnuty L'vovitsch Tchebicheff

Chebyshev points

$$x_i = \cos\left(\frac{2i+1}{2n+2}\pi\right), \qquad i = 0, \ldots, n.$$



Error estimates for Chebyshev interpolation

$$\boxed{\|u - I_n^{Cheb} u\|_{\infty,-1,1]} \leq (1+\Lambda_n) \inf_{v \in \mathbb{P}_n} \|u - v\|_{\infty,[-1,1]}}$$

The Lebesgue constant Λ_n satisfies

$$\frac{2}{\pi}\ln(n+1) + 0.53\ldots \leq \Lambda_n \leq \frac{2}{\pi}\ln(n+1) + 1.$$

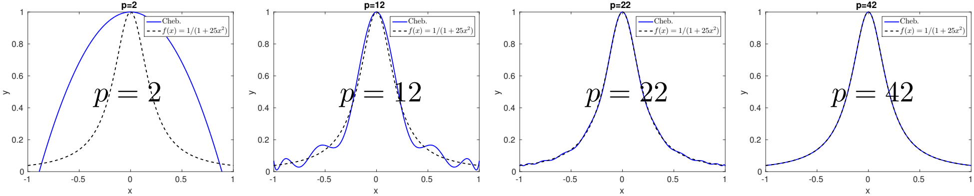
n	Λ_n
5	2.104
10	2.489
15	2.728
20	2.901

explicit formula: $\Lambda_n = \frac{1}{n+1}\sum_{i=0}^{n} \cot\frac{(2i+1)\pi}{4(n+1)} = \frac{2}{\pi}\ln(n+1) + \frac{2}{\pi}\left(\gamma + \ln\frac{8}{\pi}\right) + O\left(\frac{1}{n^2}\right),$ $\gamma = 0.57\ldots$

Choice of interpolation points matters

$$\text{interpolation of} \quad f(x) = \frac{1}{1+25x^2} \quad \text{on } [-1,1]$$

interpolation in Chebyshev points



equidistributed interpolation points