# Sheet 6

Discussion of the sheet: Tue./Wed., December 6/7

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This exercise sheet is concerned with the topics

- Stokes' theorem
- Fourier series

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**1.** Use Stokes' theorem to compute the line integral $\int_{\partial S} \psi \cdot ds$, where $S$ is the rectangle with the vertices $(0,1,0)$, $(1,1,0)$, $(1,0,0)$, $(0,0,0)$ and $\psi = (3x^2y, z+x, xyz)^T$.

**2.** Consider the surface given by the parametrisation $r(u,v) = (u, v, u^3 - 3uv^3)^T$ on the domain $G = \{(u,v) : u^2 + v^2 \leq 1\}$.

**a)** Sketch the surface $S$;

**b)** Give a parametrisation of its boundary $\partial S$;

**c)** Use Stokes' theorem to the function $\psi = (0,0,x)^T$, computing first a line integral, and conclude something about a surface integral.

**3.** Let $f(x) = x^2$ on $I = [-1/2, 1/2]$. Sketch the periodic extension of $f$ and compute its complex Fourier series. Evaluate the series at $x = 0$ and derive an identity for $\pi^2$ as an infinite sum.

**4.** Use the same $f$ from the previous exercise. Write Parseval's identity for the complex Fourier series on $I$

$$\|f\|^2_{L^2(I)} = \sum_{k \in \mathbb{Z}} |c_k|^2$$

and derive an identity for $\pi^4$ as an infinite sum.

**5.** Let $f(t) = \exp(-t)$ on $I = [0,1]$. Provide different periodic continuations of $f$ such that its (real) Fourier series

**a)** has only cosine terms;

**b)** has only sine terms;

**c)** has a period of 1;

**d)** has a period of 2.

Provide the expansions of a) and b). Which values do the Fourier series give at $t = 0$?