# High Performance Computing Advanced MPI: Topologies

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# High Performance Computing: MPI topics and algorithms

- "Advanced" features of MPI (Message-Passing Interface): Collective operations, non-blocking collectives, sparse collectives, datatypes, one-sided communication, process topologies, MPI I/O, ...
- Efficient implementation of MPI collectives: Algorithms under (simplified) network assumptions
- · Useful for implementation of own algorithms, libraries, etc.



# MPI: the "Message-Passing Interface"

- Library with C and Fortran bindings that implements a message-passing model
- Current de facto standard in HPC and distributed memory parallel computing

PGAS competitors: UPC, CaF, OpenSHMEM, ...









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- Library with C and Fortran bindings that implements a message-passing model
- Current de facto standard in HPC and distributed memory parallel computing

## Open source implementations:

- mpich from Argonne National Laboratory (ANL)
- mvapich from Ohio State
- OpenMPI, community effort







from which many special purpose/vendor implementations are derived





## mpich(2) canonical references

William Gropp, Ewing L. Lusk, Nathan E. Doss, Anthony Skjellum: A High-Performance, Portable Implementation of the MPI Message Passing Interface Standard. Parallel Computing 22(6): 789-828 (1996)

William Gropp, Ewing L. Lusk: Sowing mpich: a Case Study in the Dissemination of a Portable Environment for Parallel Scientific Computing. IJHPCA 11(2): 103-114 (1997)

William Gropp, Ewing L. Lusk: A High-Performance MPI Implementation on a Shared-Memory Vector Supercomputer. Parallel Computing 22(11): 1513-1526 (1997)

Rajeev Thakur, Rolf Rabenseifner, William Gropp: Optimization of Collective Communication Operations in MPICH. IJHPCA 19(1): 49-66 (2005)





# Open MPI canonical references

Richard L. Graham, Brian Barrett, Galen M. Shipman, Timothy S. Woodall, George Bosilca: Open MPI: a High Performance, Flexible Implementation of MPI Point-to-Point Communications. Parallel Processing Letters 17(1): 79-88 (2007)

Edgar Gabriel, Graham E. Fagg, George Bosilca, Thara Angskun, Jack J. Dongarra, Jeffrey M. Squyres, Vishal Sahay, Prabhanjan Kambadur, Brian Barrett, Andrew Lumsdaine, Ralph H. Castain, David J. Daniel, Richard L. Graham, Timothy S. Woodall: Open MPI: Goals, Concept, and Design of a Next Generation MPI Implementation. PVM/MPI 2004: 97-104

Richard L. Graham, Timothy S. Woodall, Jeffrey M. Squyres: Open MPI: A Flexible High Performance MPI. PPAM 2005: 228-239





# Message-passing model

- Processors (in MPI: processes something executed by a physical processor/core) execute program on local data
- Processors exchange data and synchronize by explicit communication; only way to exchange information ("shared nothing")

Program: MIMD, SPMD

#### Communication:

- Asynchronous (non-blocking) or synchronous (blocking)
- In-order, out of order
- One-to-one, one-to-many, many-to-one, many-to-many





Strict message-passing (theoretical model):

Synchronous, in-order, one-to-one communication: Enforces event order, easier to reason about correctness, mostly deterministic execution, no race-conditions

Hoare: CSP

## Practical advantages:

- Enforces locality (no cache sharing, less memory per process)
- Synchronous communication can be implemented space efficiently (no intermediate buffering)

Recent: 60

May: OCCAM

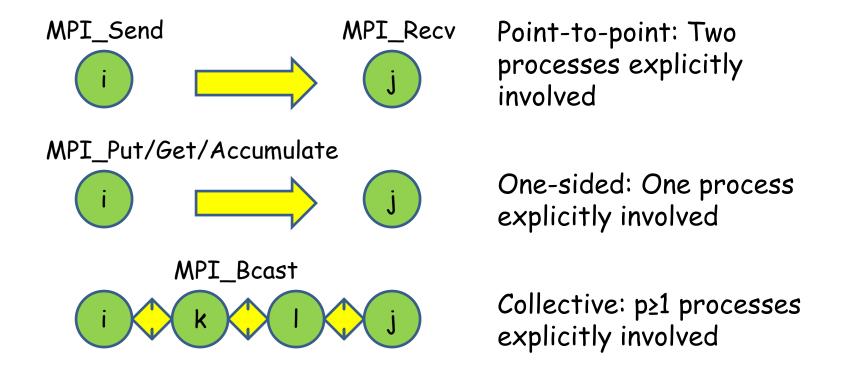
C. A. R. Hoare: Communicating Sequential Processes. Comm. ACM 21(8): 666-677 (1978)





# MPI message-passing model

#### Three main communication models:







# MPI message-passing model

All communication between named processes in same communication domain:

Communication domain ("communicator"): Ordered set of processes that can communicate (e.g., MPI\_COMM\_WORLD)

Name: For domain of p processes, rank between 0 and p-1







#### Note:

MPI is a specification; not an implementation (term "MPI library" refers to specific implementation of the standard)

MPI specification/standard prescribes (almost) nothing about the implementation, in particular:

- No performance model (thus no assumptions on communication network)
- No performance guarantees
- No prescribed algorithms (for collective communication, synchronization, datatypes etc.)
- But: Some assumptions about "High Quality implementation"





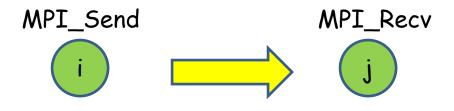


- Both sender and receiver explicitly involved in communication
- Communication is reliable (implementation and underlying communication infrastructure must guarantee this)
- Communication is ordered: Messages with same destination and same tag arrive in order sent (implementation must guarantee this)





(\*) Whether MPI\_Send can complete may depend on MPI implementation



i, j: name ("rank") of process

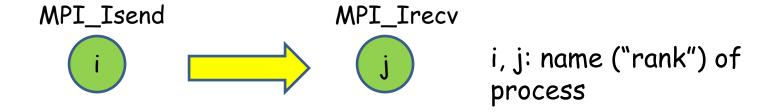
- Largely asynchronous model: MPI\_Send has semi-local completion semantics, when call completes buffer can be reused (\*)
- Blocking and non-blocking ("immediate") semantics

# Blocking MPI call:

Returns when operation locally complete, buffer can be reused, may or may not depend on actions of other processes, no guarantee regarding state of other processes







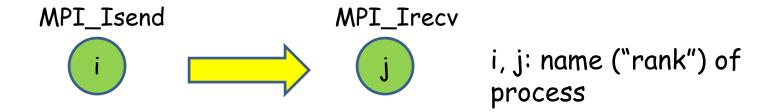
- Largely asynchronous model: MPI\_Send has semi-local completion semantics, when call completes buffer can be reused
- Blocking and non-blocking ("immediate") semantics

# Non-blocking MPI call:

Returns immediately, independent of actions of other processes, buffer may be in use. Completion as in blocking call must be enforced (MPI\_Wait, MPI\_Test, ...)







- Largely asynchronous model: MPI\_Send has semi-local completion semantics, when call completes buffer can be reused
- Blocking and non-blocking ("immediate") semantics

## Terminology note:

Non-blocking in MPI is not a non-blocking progress condition, but means that the calling process can continue; progress of communication eventually depends on progress of other process





MPI\_Ssend MPI\_Recv

i, j: name ("rank") of process

- Largely asynchronous model: MPI\_Send has semi-local completion semantics, when call completes buffer can be reused
- Blocking and non-blocking ("immediate") semantics
- Many send "modes": MPI\_Ssend (MPI\_Bsend, MPI\_Rsend)

```
int x[count];
MPI_Ssend(x,count,MPI_INT,dest,tag,comm);
```





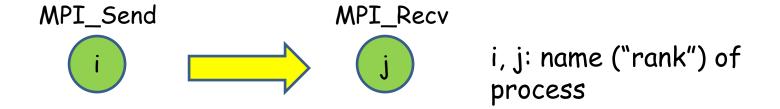
MPI\_Issend MPI\_Irecv

i, j: name ("rank") of process

- Largely asynchronous model: MPI\_Send has semi-local completion semantics, when call completes buffer can be reused
- Blocking and non-blocking ("immediate") semantics
- Many send "modes": MPI\_Ssend (MPI\_Bsend, MPI\_Rsend)







- Largely asynchronous model: MPI\_Send has semi-local completion semantics, when call completes buffer can be reused
- Blocking and non-blocking ("immediate") semantics
- Non-determinism only through "wildcards" in MPI\_Recv





```
MPI_Send_init

MPI_Recv_init

i, j: name ("rank") of process
```

- Persistent versions of all blocking point-to-point operations
- Optimization potential: All parameters bound at initialization call (in MPI\_Request object), can be reused, algorithm can be adapted to use case (parameters, process mapping, etc.)





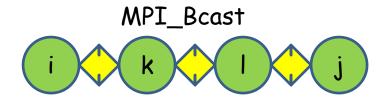
#### One-sided communication



- Only one process explicitly involved in communication; origin supplies all information
- Communication is reliable, but not ordered
- Non-blocking communication, completion at synchronization point
- Different types of synchronization (active/passive; collective/localized)
- Target memory in exposed/accessible window







News in MPI 3.0

- All processes explicitly involved in data exchange or compute operation (one-to-many, many-to-one, many-to-many)
- All collective operations blocking in the MPI sense: Return when operation is locally complete, buffers can be reused
- Operations are reliable
- Collectives (except for MPI\_Barrier) are not synchronizing

```
float *buffer = malloc(count*sizeof(float));
MPI_Bcast(buffer,count,MPI_FLOAT,root,comm);
```







Synchronizing:

MPI\_Barrier

Exchange, regular, rooted

MPI\_Bcast

MPI\_Gather/MPI\_Scatter

Non-rooted (symmetric)

MPI\_Allgather

MPI\_Alltoall

Reduction, regular, rooted

MPI\_Reduce

Non-rooted (symmetric)

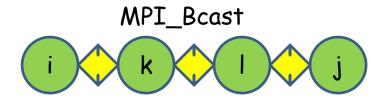
MPI\_Allreduce

MPI\_Reduce\_scatter\_block

MPI\_Scan/MPI\_Exscan







Exchange, irregular (vector)

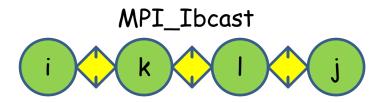
MPI\_Gatherv/MPI\_Scatterv
MPI\_Allgatherv
MPI\_Alltoallv/MPI\_Alltoallw

Reduction, irregular (vector)

MPI\_Reduce\_scatter

Irregular collective: Size and basetype (technically: type signature) of buffers must match pairwise between processes, but can differ between different pairs

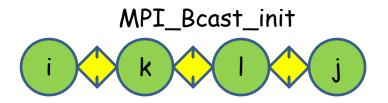




With MPI 3.0

- All 17 collectives in non-blocking versions
- Call returns immediately, independent of actions of other processes
- Completion with same semantics as blocking counterpart must be explicitly enforced (MPI\_Wait, MPI\_Waitall, ...)





With MPI 4.0

- All 17 collectives in persistent versions
- Optimization potential: All parameters bound at initialization call (in MPI\_Request object), can be reused, algorithm can be adapted to use case (parameters, process mapping, etc.)

```
MPI_Barrier_init(comm,MPI_INFO_NULL,&request);
while (1) {
   MPI_Start(&request); // start the barrier
   MPI_Wait(&request); // now enforce the barrier
}
MPI_Request_free(&request);
```



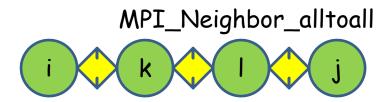


- Although semantically equivalent, blocking, non-blocking, and persistent collective cannot be used together
- Different from persistent point-to-point: modes can be mixed

```
Process i: Process j: MP. Bcast_init(...,&req)
MPI_Bcas (); MPI_Ibca (); MPI_Start(&req);
```







With MPI 3.0 and MPI 4.0

 5 sparse (neighbor) collective operations, in blocking, nonblocking and persistent versions

Non-rooted (symmetric)

MPI\_Neighbor\_allgather
MPI\_Neighbor\_alltoall

Exchange, irregular (vector)

MPI\_Neighbor\_allgatherv

MPI\_Neighbor\_alltoallv

MPI\_Neighbor\_alltoallw





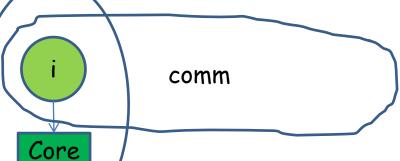
#### All models

Communication is wrt. communicator: Ordered set of processes, mapped to physical nodes/processors/cores

In one-sided model, communicator is part of memory-window

Process

MPI processes (mostly) statically bound to processors/cores



```
MPI_Comm_rank(comm,&rank);
MPI_Comm_size(comm,&size);
```

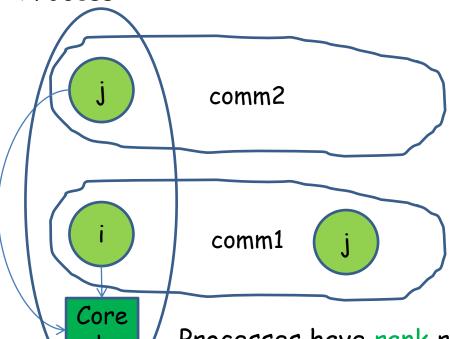
Processes have rank relative to communicator; the same process can have different ranks in different communicators





MPI has multi-purpose (collective) operations for creating new communicators out of old ones. If a different mapping is needed, new communicator must be created (MPI objects are static)

#### Process



Process with rank j in comm1 may have to send state/data to the process with rank j in comm2

Processes have rank relative to communicator; the same process can have different ranks in different communicators





For discussion on collective interfaces, sparse (neighbor) and global communication, and the relation to communicators, see

Jesper Larsson Träff, Sascha Hunold, Guillaume Mercier, Daniel J. Holmes: MPI collective communication through a single set of interfaces: A case for orthogonality. Parallel Comput. 107: 102826 (2021)

Many interesting things to do for (paid) Master's thesis...





#### All models

- Data in buffers can be arbitrarily structured, not necessarily only consecutive elements
- Data structure/layout communicated to MPI implementation by datatype handle
- Communication buffers: Buffer start (address), element count, element datatype







e.g. indexed datatype

MPI terms: Derived datatypes, user-defined datatypes, ... (MPI 3.1 standard, Chapter 4)





# Beyond message-passing MPI features

- MPI-IO: Serial and parallel IO, heavy use of derived datatypes in specification
- MPI process management (important concept: intercommunicators): Coarse grained management (spawn, connect) of additional processes
- <u>Topologies</u>: Mapping application communication patterns to communication system
- Tools support: Profiling interface and library information





## Concrete example (MPI code recap)

- C code (could also be "mild" C++)
- What to include, what to define
- Using assertions
- Good SPMD style: program works for all numbers of processes
- Compiling and running (environment dependent)





```
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <mpi.h>
//#define NDEBUG
                         Good practice: Make assertions
#include <assert.h>
#define SRTAG 777
int main(int argc, char *argv[])
 MPI Comm comm;
  int rank, size;
  int root;
 MPI_Init(&argc, &argv); MPI library must be initialized
  // [...] Here comes the code
 MPI_Finalize();
  return 0;
```





# The code, part 1: Two processes communicate (first and last)

```
New comm for this
MPI Comm dup (MPI COMM WORLD, &comm);
MPI Comm rank(comm, &rank);
MPI Comm size (comm, &size)
                              Assert simple assumptions
assert(rank<size);
           Buffers always have this structure: Address, count, type
int r:
if (rank==0) {
  MPI Sendrecv (&rank, 1, MPI INT, size-1, SRTAG,
                &r, 1,MPI INT, size-1,SRTAG,
                comm, MPI STATUS IGNORE);
  assert(r==size-1);
} else if (rank==size-1) { Good SPMD style: All sizes
  MPI Sendrecv (&rank, 1, MPI INT, 0, SRTAG,
                &r, 1, MPI INT, 0, SRTAG,
                comm, MPI STATUS IGNORE);
  assert (r==0);
```



# The code, part 2: All processes communicate collectively

```
int *allrank;
                                                No stack allocation
allrank = (int*)malloc(size*sizeof(int));
for (root=0; root<size; root++) {</pre>
  allrank[root] = rank;
  MPI Bcast(&allrank[root],1,MPI INT,root,comm);
  assert(allrank[root] == root);
                Buffer structure, but beware: one block
int *ar;
ar = (int*)malloc(size*sizeof(int));
MPI Allgather (&rank, 1, MPI INT,
                ar, 1, MPI INT, comm);
for (root==0; root<size; root++)
                                          Allgather is Bcast from all
  assert(ar[root] == allrank[root]);
                      Free after use, don't forget
free (allrank);
free (ar);
```





# Running the code (on hydra):

```
> scp mpiintro.c <account>@hydra:~/
> ssh hydra
> module avail
> module load mpi/openmpiS
> mpicc -o intro -03 mpiintro.c
> srun -N 10 --tasks-per-node=32 ./intro
>
```

See <u>doku.par.tuwein.ac.at</u> for more





# Example: Three algorithms for matrix-vector multiplication

mxn matrix A and n-element vector x distributed evenly across p MPI processes: compute

$$y = Ax$$

with y the m-element result vector

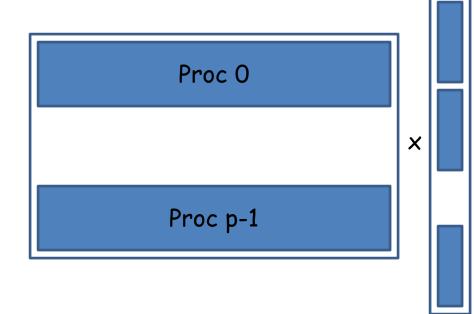
### Even distribution:

- Each of p processes has an mn/p element submatrix, an n/p element subvector, and computes an m/p element result vector.
- Algorithms should respect/preserve distribution





 $Ai: (m/p) \times n \text{ matrix}$ 



# Algorithm 1:

- Row-wise matrix distribution
- Each process needs full vector:

MPI\_Allgather(v)

 Compute blocks of result vector locally

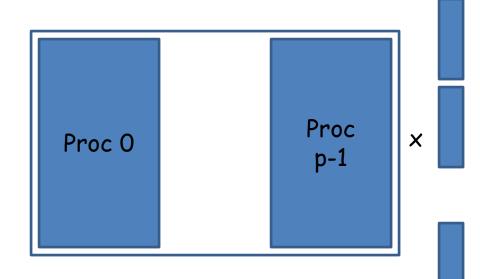
Conjecture:  $T(m,n,p) = O((m/p)n + n + log p), p \le m$ 

Why?





$$A = (A0 | A1 | A2), Ai m \times (n/p)$$
 matrix



# Algorithm 2:

- Column-wise matrix distribution
- Compute local partial result vector
- MPI\_Reduce\_scatter
   to sum and distribute
   partial results

Conjecture:  $T(m,n,p) = O(m(n/p) + m + \log p), p \le n$ 

Why?





### Conjecture:

- MPI\_Allgatherv(n) can be done in O(n+log p) communication steps
- MPI\_Reduce\_scatter(n) can be done in O(n+log p) communication steps

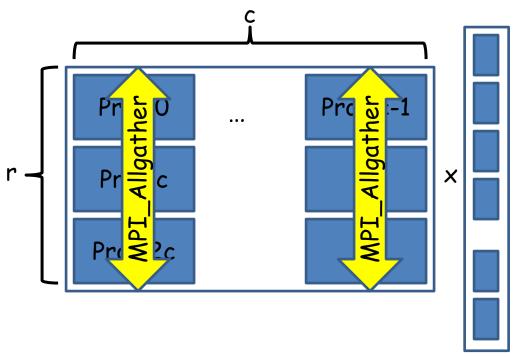
for certain types of networks, and not better (regardless of network)

### This lecture:

Will substantiate the conjectures (constructions and proofs)







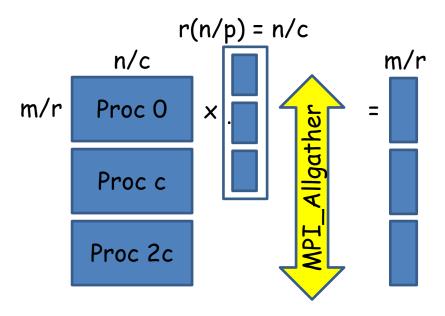
# Algorithm 3:

- Matrix distribution into blocks of m/r x n/c elements
- Algorithm 1 on columns
- Algorithm 2 on rows

Step 1: Simultaneous MPI\_Allgather(v) on all c process columns



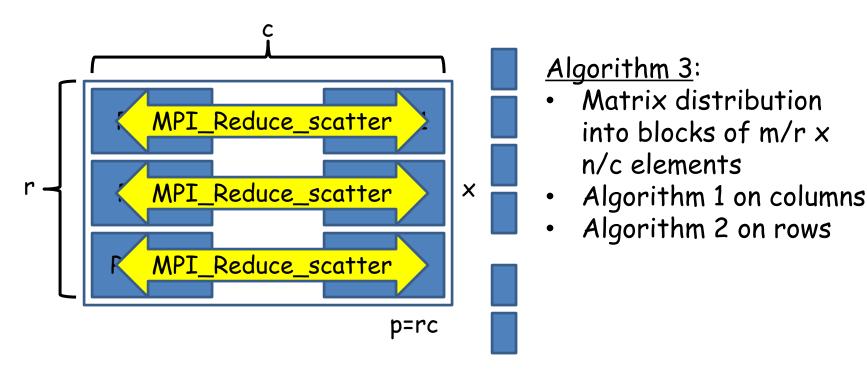




Step 1: Simultaneous MPI\_Allgather(v) on all c process columns



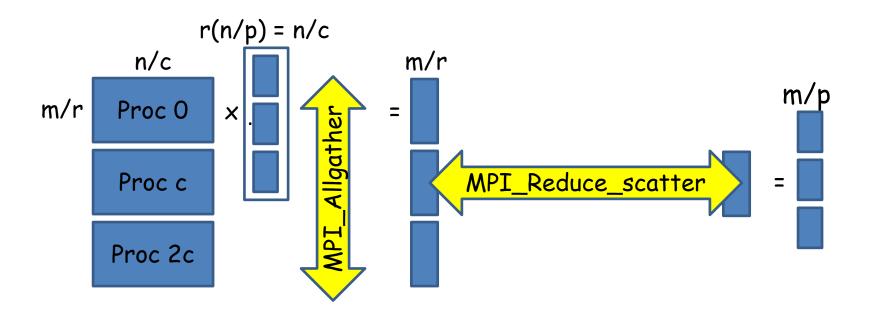




Step 4: Simultaneous MPI\_Reduce\_Scatter(\_block) on process row communicators



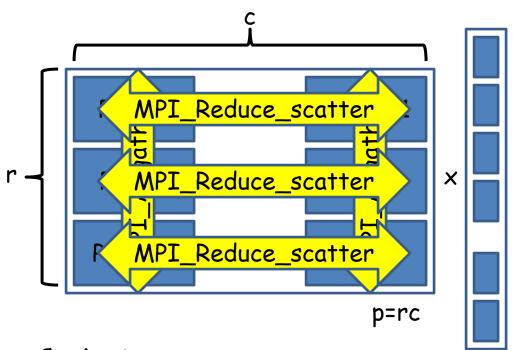




Step 4: Simultaneous MPI\_Reduce\_Scatter(\_block) on process row communicators







# Algorithm 3:

- Matrix distribution into blocks of m/r x n/c elements
- Algorithm 1 on columns
- Algorithm 2 on rows

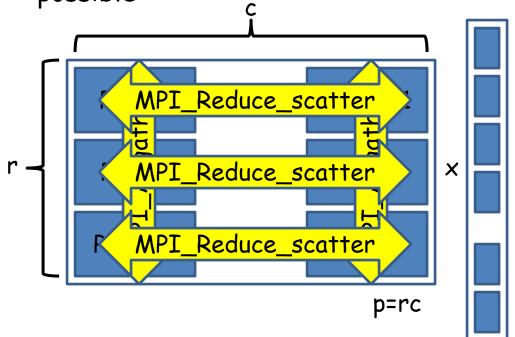
Conjecture:

 $T(m,n,p) = O(mn/p + n/c + m/r + log p), p \leq min(mc,nr)$ 





Algorithm 3 is more scalable. To implement the algorithm, essential that independent, simultaneous, collective communication on (column and row) subsets of processes is possible



### Algorithm 3:

- Matrix distribution into blocks of m/r x n/c elements
- Algorithm 1 on columns
- Algorithm 2 on rows

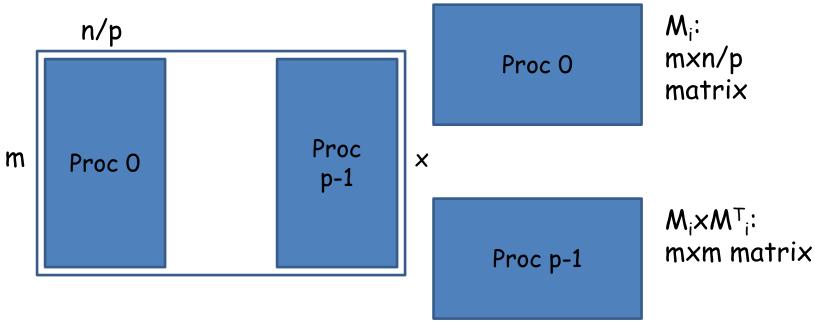
Note:

Interfaces that do not support collectives on subsets of processes cannot express Algorithm 3 (e.g., UPC, CaF)





<u>Algorithm 2</u> can also be used for computing  $M \times M^T$ , the product of mxn matrix M with its own transpose (also called SYRK: Symmetric Rank K update); result is a symmetric mxm matrix



Compute for processor i:

 $(M \times M^T)_i = \sum_{0 \le i < p} M_i \times M^T_i$  with local SYRK and MPI\_Reduce\_scatter, result distributed column- or row-wise (use-case for datatypes)



This algorithm is good (optimal) for  $m \le n$  (short, wide matrices) and smaller  $p, p \le n/\sqrt{m(m-1)}$ 

Cost analysis (conjecture):  $T(m,n,p) = O(m^2n/p + m^2 + \log p)$ 



Real communication volume is m(m+1)/2 (p-1)/p





# MPI Process topologies (MPI 3.1, Chapter 7)

### Observation:

For Algorithm 3, row and column subcommunicators are needed; must support concurrent collective operations

Convenient to organize processors into a Cartesian rxc mesh with MPI processes in row-major.

Use this Cartesian (x,y) naming to create subcommunicators for the matrix-vector example

Cartesian naming useful for torus/mesh algorithms, e.g., for d-dimensional stencil computations (Jacobi, Life, ...)





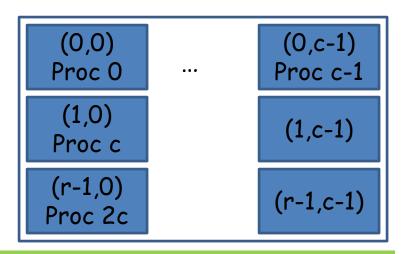
```
rcdim[0] = c; rcdim[1] = r;
period[0] = 0; period[1] = 0; // no wrap around
reorder = 0; // no attempt to reorder
MPI_Cart_create(comm, 2, rcdim, period, reorder, &rccomm);
```

Collective communicator creation call:
All processes belonging to old communicator (comm) must
perform call, each process becomes new communicator (recomm,
or MPI COMM NULL), possibly with new rank and size





```
rcdim[0] = c; rcdim[1] = r;
period[0] = 0; period[1] = 0; // no wrap around
reorder = 0; // no attempt to reorder
MPI_Cart_create(comm, 2, rcdim, period, reorder, &rccomm);
```



Row major process numbering

Same processes in rccomm communicator as in comm (rc must be at most size(comm)). But in rccomm, processes also have a d-dimensional coordinate as "name".

But: Ranks may not be bound to the same processors (if reorder=1)





```
rcdim[0] = c; rcdim[1] = r;
period[0] = 0; period[1] = 0; // no wrap around
reorder = 0; // no attempt to reorder
MPI_Cart_create(comm, 2, rcdim, period, reorder, &rccomm);
```

# (0,0) Proc 0 ... (0,c-1) Proc c-1 (1,0) Proc c (r-1,0) Proc 2c (r-1,c-1)

# $rank = \sum_{0 \le i < d} (coord[i] \prod_{0 \le j < i} d[j])$

### Translation functions

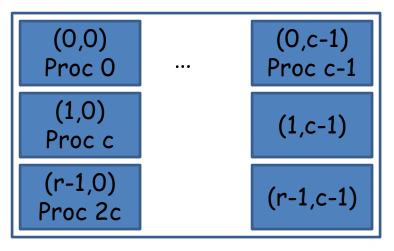
```
MPI_Cart_rank(rccomm, rccoord, &rcrank);
```

E.g., 
$$rank(2,c-2) = c-2+2c = 3c-2$$





```
rcdim[0] = c; rcdim[1] = r;
period[0] = 0; period[1] = 0; // no wrap around
reorder = 0; // no attempt to reorder
MPI_Cart_create(comm, 2, rcdim, period, reorder, &rccomm);
```



Communication in recomm uses ranks (not coordinates).

Communication between any ranks possible/allowed.

Processes are neighbors if they are adjancent in Cartesian mesh/torus

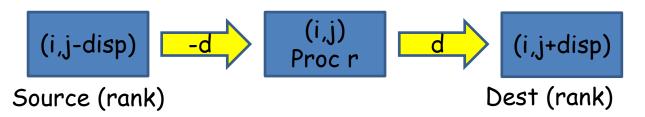




Informatics

```
rcdim[0] = c; rcdim[1] = r;
period[0] = 0; period[1] = 0; // no wrap around
reorder = 0; // no attempt to reorder
MPI_Cart_create(comm, 2, rcdim, period, reorder, &rccomm);
```

# Rank "shifting" function (no communication)



Shift along dimension 0, displacement 2 (e.g.). What if j-disp<0 or j+disp>=size? Answer: MPI\_PROC\_NULL if (!period[0]),

otherwise cyclic shift

Paralel W523

©Jesper Larsson Träff

MPI convenience functionality for factoring p into "best" (closest??) r and c... (in d dimensions)

```
int dimensions[d] = {0, ..., 0}; // all dimensions free
MPI_Dims_create(p,d,dimensions);
```

Factor p into d dimensions with order (size) as close to each other as possible... (MPI 3.1, p.293)

Much natural functionality is not in MPI (what can you think of?)

Jesper Larsson Träff, Sascha Hunold, Guillaume Mercier, Daniel J. Holmes: MPI collective communication through a single set of interfaces: A case for orthogonality. Parallel Comput. 107: 102826 (2021)





```
int dimensions[d] = {0, ..., 0}; // all dimensions free
MPI_Dims_create(p,d,dimensions);
```

Factor p into d dimensions with order (size) as close to each other as possible... (MPI 3.1, p.293). Why should this be best?

Note: Seemingly innocent interface entails integer factorization. Complexity of factorization is unknown (probably high)

Note: This algorithm for 2-dimensional balancing is exponential:

```
f = (int)sqrt((double)p);
while (p%f!=0) f--;
```

Pseudo-polynomial: Polynomial in the magnitude, but not the size of p. It takes only log p bits to represent p.





But... factorization of small numbers (p in the order of millions) that fulfill the MPI specification (dimension sizes in increasing order) is not expensive (for p up to millions):

Jesper Larsson Träff, Felix Donatus Lübbe: Specification Guideline Violations by MPI\_Dims\_create. EuroMPI 2015: 19:1-19:2

### Ca. 2015:

Many (all?) MPI libraries had (severely) broken implementations of MPI\_Dims\_create

(SE) Problem: How can quality of MPI library implementation be assessed? Ensured?





### MPI\_Cart\_create operation

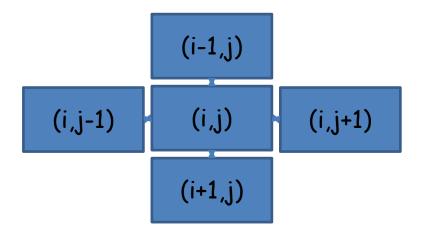
- Defines Cartesian naming, creates new communicator, and caches information with this (number and sizes of dimensions ...)
- Row-major ordering of processes (dimension order 0, 1, 2, ...)
- Neighbors: The 2d adjacent processes in each dimension
- Defines order of neighbors (dimension-wise), important for MPI 3.0 neighborhood collectives

  See later
- May (reorder=1) attempt to remap processes, such that processes that are neighbors in virtual application topology are neighbors in physical system





### Cartesian neighborhoods in two dimensions



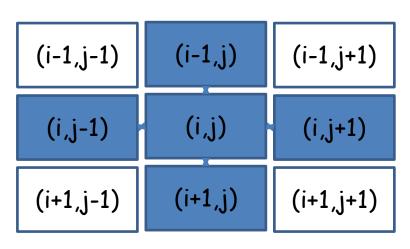
Pattern aka 5-point stencil

Process with coordinates (i,j) has 2d direct neighbors (for Cartesian communicator in d dimensions) with which it is implicitly supposed to communicate. This is the neighborhood (graph) used with MPI neighborhood collectives Process (0,j) neighbor of process (r-1,j) if periodic in dimension





### Cartesian neighborhoods in two dimensions



### MPI restrictions:

- Not possible to specify other patterns, e.g., 9point stencil
- Neighborhood is unweighted

Process with coordinates (i,j) has 2d direct neighbors (for Cartesian communicator in d dimensions) with which it is implicitly supposed to communicate. This is the neighborhood (graph) used with MPI neighborhood collectives Process (0,j) neighbor of process (r-1,j) if periodic in dimension



For discussion on communicators, naming schemes and topological information associated with communicators, see again

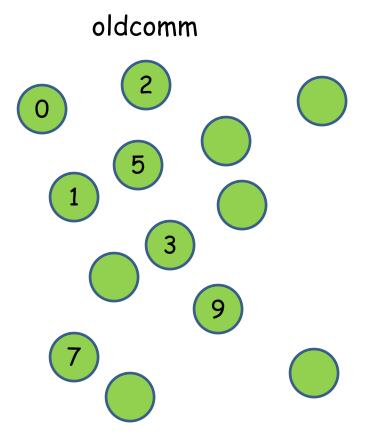
Jesper Larsson Träff, Sascha Hunold, Guillaume Mercier, Daniel J. Holmes: MPI collective communication through a single set of interfaces: A case for orthogonality. Parallel Comput. 107: 102826 (2021)

Many interesting things to do for (paid) Master's thesis...





# Reordering processes via new communicators



MPI assumption:

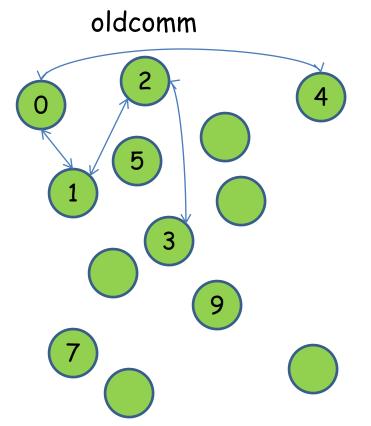
Application programmer knows which processes will communicate (heaviest communication, most frequent communication): Virtual topology

### Idea:

Convey this information to MPI library; library can suggest a good mapping of MPI processes to processors that fits communication system





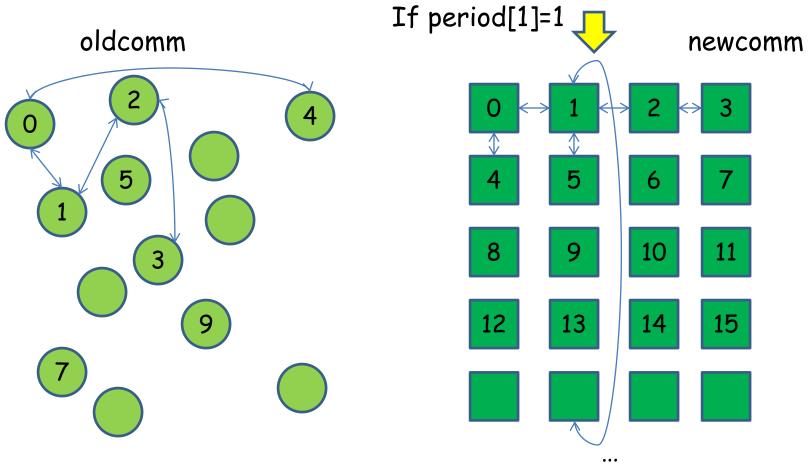


### MPI\_Cart\_create:

Implicit assumption: Process with rank i in old communicator will likely communicate with neighboring processes in implicit Cartesian neighborhood







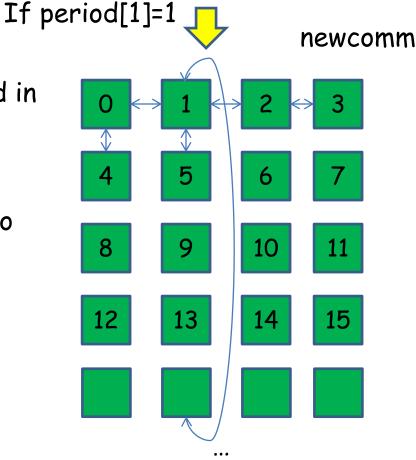




Ranks in newcomm are organized in a (virtual) d-dimensional mesh/torus

Good order: Ranks correspond to processor id's in physical torus system

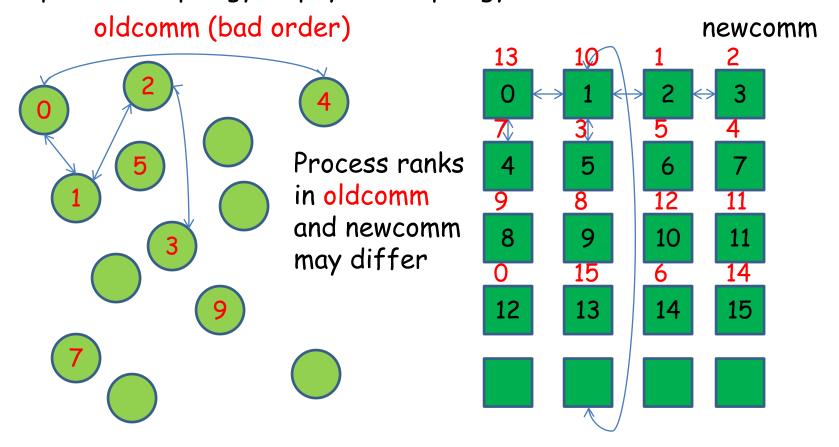
Physical torus systems: 3-dim, 5-dim, 6-dim; BlueGene, Cray, Fujitsu K, Fugaku







Topology creation with reorder=1: MPI library may attempt to map virtual topology to physical topology: newcomm



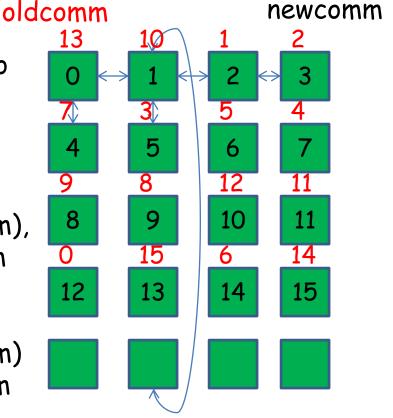




Processes in oldcomm remapped to other processes in newcomm

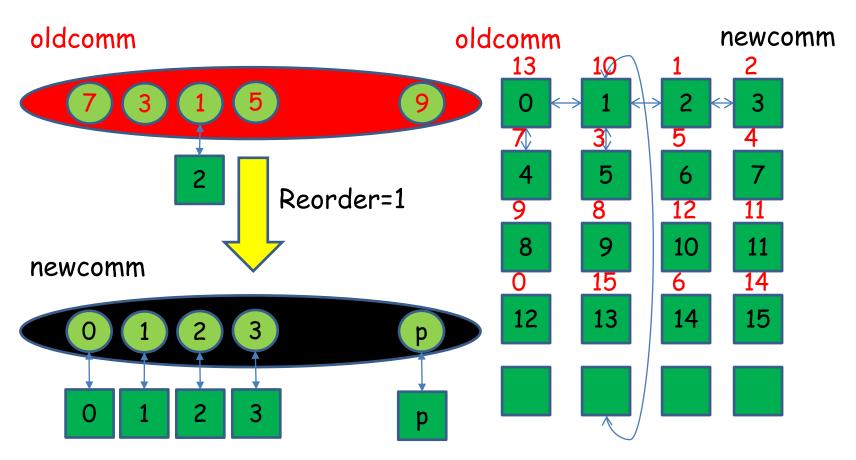
### Example:

- Rank 1 in oldcomm mapped to processor 2 (rank 2 in newcomm), remapped to rank 1 in newcomm (which was rank 10 in oldcomm)
- Rank 4 in oldcomm mapped to processor 7 (rank 7 in newcomm) remapped to rank 4 in newcomm













If reordering has been done (ranks in newcomm # ranks in oldcomm)

- Has reordering been done? All processes: Check whether rank in the two communicators differ, allreduce to inform all (or use MPI\_Comm\_compare)
- Application may have to transfer data to same rank in new communicator; no MPI support for this
- Need to be able to translate between ranks in different communicators

Cartesian communicators can capture only limited Cartesian communication patterns: 2d neighborhoods





MPI Comm compare(comm1,comm2,&result)

### Result either of

- MPI\_IDENT (communicators identical, same)
- MPI\_CONGRUENT (same processes in same order, nevertheless different communicator)
- MPI\_SIMILAR (same processes, different order)
- MPI\_UNEQUAL (otherwise)





### Interface ideas for general stencil patterns

Jesper Larsson Träff, Felix Donatus Lübbe, Antoine Rougier,

Sascha Hunold: Isomorphic, Sparse MPI-like Collective

Communication Operations for Parallel Stencil Computations.

EuroMPI 2015: 10:1-10:10

Jesper Larsson Träff, Sascha Hunold: Cartesian Collective

Communication. ICPP 2019: 48:1-48:11

Jesper Larsson Träff, Sascha Hunold, Guillaume Mercier, Daniel J. Holmes: MPI collective communication through a single set of interfaces: A case for orthogonality. Parallel Comput. 107: 102826 (2021)

# Interesting Master thesis projects here





#### Recent improvements to MPI implementations

- Remapping for network topology: Place links in communication graph (e..g. Cartesian neighborhoods) on links in hardware network
- Remapping for hierarchical systems: Map as many neighbors as possible inside compute nodes

William D. Gropp: Using node and socket information to implement MPI Cartesian topologies. Parallel Computing 85: 98-108 (2019)



- Remapping for hierarchical systems: Heuristically map as many neighbors as possible inside compute nodes
- Challenge: balance bandwidth inside nodes with (multi-lane) bandwidth across nodes

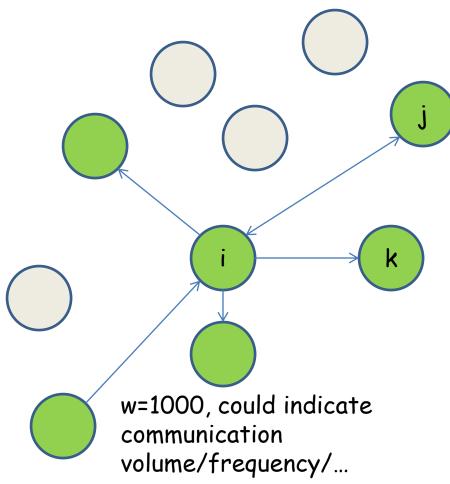
Konrad von Kirchbach, Markus Lehr, Sascha Hunold, Christian Schulz, Jesper Larsson Träff: Efficient Process-to-Node Mapping Algorithms for Stencil Computations. CLUSTER 2020: 1-11

TU Wien: New FWF project on process mapping





# Fully general application communication patterns



- Weighted, directed (multi)graph to specify application communication pattern (whom with whom, how much)
- Hints can be provided (info)
- Distributed description, any MPI process can specify any edges





Example: Process i may specify (what it knows):

sources: [0,5,7,2,...]

(out)degrees: [2,2,2,2,...]

destinations: [1,2,18,36,4,5,8,117,...]

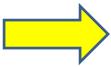
weights: [100,1000,10,10,1000,1000,100,100,...]

MPI (3.0) uses graph structure for descriptive purposes (neighborhood collectives). MPI library can attempt to remap communication graph to fit target communication system. Other optimizations are possible.





If reordering is requested, rank i in new graphcomm may be bound to different processor than rank i in comm



Data redistribution (from rank i in comm to rank i in graphcomm) can be necessary; must be done explicitly by application (first: Check if reordering has taken place)

#### Note

It is legal for an MPI library to return a graphcomm with the same mapping as comm





# If each process knows its incoming and outgoing communication edges

may be more convenient and efficient

Furthermore, the operation does not require communication in order to support the neighborhood query functions:

```
MPI_Dist_graph_neighbors_count(graphcomm, ...)
MPI_Dist_graph_neighbors(graphcomm, ...)
```





#### MPI 3.1 Virtual topologies

- "info" can be provided; not (yet) specified in MPI standard, implementation dependent
- Interpretation of weights unspecified (when? How much? How often? ...)
- Differences between Cartesian and distributed graph topologies: Cartesian creation function (MPI\_Cart\_create) takes no weights, no info

Do MPI libraries perform non-trivial mapping?

Try it





Distributed graph interface since MPI 2.2.

Old MPI 1.0 interface is badly non-scalable: Full graph required at each process

Don't use MPI\_Graph\_create!

#### <u>Definition</u>:

An MPI construct is non-scalable if it entails  $\Omega(p)$  memory or time overhead(\*)

(\*): that cannot be accounted for/amortized in the application

T. Hoefler, R. Rabenseifner, H. Ritzdorf, B. R. de Supinski, R. Thakur, Jesper Larsson Träff: The scalable process topology interface of MPI 2.2. Concurrency and Computation: Practice and Experience 23(4): 293-310 (2011)





# Implementing topology mapping

#### Step 1:

Application specifies communication patterns as weighted directed (guest) graph (implicitly: Unweighted, d-dimensional Cartesian pattern)

#### Step 2:

MPI library knows physical communication structure; model subgraph of system spanned by communicator as weighted, directed (host) graph

#### Step 3:

Map/embed communication (guest) graph onto system (host) graph, subject to... The map (embedding) is an injective function  $\Gamma$  from guest graph vertices to host graph vertices





Mapping function can be injective, surjective, bijective

Some optimization criteria:

- As many heavy communication edges on physical edges as possible
- Minimize communication costs (volume)
- Small congestion (bandwidth)
- Small dilation (latency)
- ...

Meta-Theorem: Most graph embedding problems are NP-hard

Depending on physical system (e.g., clustered, hierarchical), problem can sometimes be formulated as a graph partitioning problem (e.g., clusters, partition into nodes, minimize weight of inter-node edges), ...





Guest graph G=(V,E), let w(u,v) denote the volume of data from process (vertex) u to process (vertex) v

Host graph H=(V',E'), let c(u',v') denote the network capacity between processors u' and v'. Let R(u',v') be a function determining the path from processor u' to v' in the system (Routing algorithm)

Let  $\Gamma$ :  $V \rightarrow V'$  be an injective mapping (embedding). The congestion of link e in E' wrt.  $\Gamma$  is defined as

Cong(e) =  $[\sum w(u,v) \text{ over } (u,v) \text{ in E where e in R}(\Gamma(u),\Gamma(v))]/c(e)$ 

The congestion of  $\Gamma$  is defined as the maximum congestion of any link e

Call the problem of determining whether an embedding exists that has congestion less than C for CONG





<u>Concrete Theorem</u>: For given guest and host graphs G and H, determining whether an embedding exists that has congestion at most C is NP-complete.

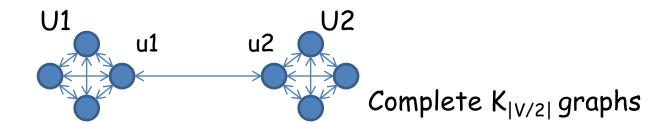
Proof: CONG is in NP. Non-deterministically choose an embedding, and check whether its congestion is at most C; this can be done in polynomial time

Completeness is by reduction from MINIMUM CUT INTO BOUNDED SETS (Garey&Johnson ND17): Given a graph G=(V,E), two vertices s,t in V, integer L, determine whether there is a partition of V into subsets V1 and V2 with s in V1, t in V2, |V1|=|V2|, such that the number of edges between V1 and V2 is at most L





Let (G,s,t,L) be an instance of MINIMUM CUT INTO BOUNDED SETS. Let the host graph H be constructed as follows



c(u1,u2) = |V|, c(e) = 1 for all other edges of H. The routing function R selects for u,v in V' a shortest path from u to v

Let the guest graph be G with w(e)=1 for e in E, except for the edge (s,t) for which  $w(s,t) = |V|^4$  if (s,t) is not in E, and  $w(s,t)=|V|^4+1$  if (s,t) in E. That is, the guest graph may have one edge not in G, namely (s,t), and each edge in G contributes a volume of at least 1





Any injective mapping  $\Gamma$  from G to H determines a partition of V into V1 and V2 with |V1|=|V2|, namely  $Vi=\{v\mid \Gamma(v) \text{ in } Ui\}$ 

Any mapping that minimizes the congestion must map (s,t) to (u1,u2). This gives a congestion of at most  $|V|^3+|V|$  (the volume on (u1,u2) is at most  $|V|^4+|V|^2/4$ ); any other mapping has congestion at least  $|V|^4$ .

In such a mapping, the most congested edge is (u1,u2), with congestion

 $C = |V|^3 + |\{(v1,v2) \text{ in } E \mid v1 \text{ in } V1 \text{ and } v2 \text{ in } V2 \}|/|V|$ 

fractional

Thus, a solution to CONG with congestion at most |V|3+L/|V| gives a solution to the MINIMUM CUT INTO BOUNDED SETS instance





A nicer construction (15.11.2021)

Host graph H as before (dumbbell)

Let the guest graph be G with w(e)=|V| for e in E, except for the edge (s,t) for which  $w(s,t)=|V|^4$  if (s,t) is not in E, and  $w(s,t)=|V|^4+|V|$  if (s,t) in E. That is, the guest graph may have one edge not in G, namely (s,t), and each edge in G contributes a volume of at least |V|





Any injective mapping  $\Gamma$  from G to H determines a partition of V into V1 and V2 with |V1|=|V2|, namely  $Vi=\{v\mid \Gamma(v) \text{ in } Ui\}$ 

Any mapping that minimizes the congestion must map (s,t) to (u1,u2). This gives a congestion of at most  $|V|^3+|V|^2$  (the volume on (u1,u2) is at most  $|V|^4+|V||V|^2/4$ ); any other mapping has congestion at least  $|V|^4$ .

In such a mapping, the most congested edge is (u1,u2), with congestion

 $C = |V|^3 + |\{(v1,v2) \text{ in } E \mid v1 \text{ in } V1 \text{ and } v2 \text{ in } V2\}|$ 

Thus, a solution to CONG with congestion at most  $|V|^3+L$  gives a solution to the MINIMUM CUT INTO BOUNDED SETS instance





Conversely, from a solution to the MINIMUM CUT INTO BOUNDED SETS an embedding with this congestion can easily be constructed (check!)

#### Proof (simplified) from

T. Hoefler, M. Snir: Generic topology mapping strategies for large-scale parallel architectures. ICS 2011: 75-84

#### ND17, see

M. R. Garey, D. S. Johnson: Computer and Intractability. A guide to the theory of NP-Completeness. W. H. Freeman, 1979 (update 1991)





CONG is a special case of the simple graph embedding problem

For given (complete) guest graph G=(V,E) with edge weights w(u,v), and host graph H=(V',E') with edge costs c(u',v'), find an injective mapping (embedding)  $\Gamma: V \rightarrow V'$  that minimizes

 $\sum (u,v)$  in E:  $w(u,v)c(\Gamma(u) \Gamma(v))$ 

This is the quadratic assignment problem (QAP), which is NP-complete (Garey&Johnson ND43)

w(u,v): volume of data to be transferred between processes u and v

c(u,v): cost per unit; could model pairwise communication costs (without congestion) in network





#### Some MPI relevant solutions to the topology mapping problem

- S. H. Bokhari: On the Mapping Problem. IEEE Trans. Computers (TC) 30(3):207-214 (1981)
- T. Hatazaki: Rank Reordering Strategy for MPI Topology Creation Functions. PVM/MPI 1998: 188-195
- J. L. Träff: Implementing the MPI process topology mechanism. SC 2002:1-14
- H. Yu, I-Hsin Chung, J. E. Moreira: Blue Gene system software Topology mapping for Blue Gene/L supercomputer. SC 2006: 116
- A. Bhatele: Topology Aware Task Mapping. Encyclopedia of Parallel Computing 2011: 2057-2062
- A. Bhatele, L. V. Kalé: Benefits of Topology Aware Mapping for Mesh Interconnects. Parallel Processing Letters 18(4): 549-566 (2008)

Emmanuel Jeannot, Guillaume Mercier, Francois Tessier: Process Placement in Multicore Clusters: Algorithmic Issues and Practical Techniques. IEEE Trans. Parallel Distrib. Syst. 25(4): 993-1002 (2014)





Heuristics for Cartesian topology mapping (with arbitrary neighborhoods)

Sascha Hunold, Konrad von Kirchbach, Markus Lehr, Christian Schulz, Jesper Larsson Träff: Efficient Process-to-Node Mapping Algorithms for Stencil Computations. IEEE Cluster (2020)

William D. Gropp: Using node and socket information to implement MPI Cartesian topologies. Parallel Comput. 85: 98-108 (2019)





# Graph partitioning software for process topology embedding

KaHIP, Karlsruhe High Quality Parititoning, recent, state-of-the art graph partitioning (heuristic, multi-level) for large graphs; see <a href="http://algo2.iti.kit.edu/documents/kahip/">http://algo2.iti.kit.edu/documents/kahip/</a>

Peter Sanders, Christian Schulz:

Think Locally, Act Globally: Highly Balanced Graph Partitioning.

SEA 2013: 164-175

VieM, Vienna Mapping and Quadratic Assignment, solves the embedding problem for hierarical systems as a special quadratic assignment problem; see <a href="http://viem.taa.univie.ac.at/">http://viem.taa.univie.ac.at/</a>

Christian Schulz, Jesper Larsson Träff: Better Process Mapping and Sparse Quadratic Assignment. SEA 2017: 4:1-4:15





# Graph partitioning software for process topology embedding

METIS, ParMETIS, ...: Well-known (early) software packages for (hyper)graph and mesh partitioning; see <a href="http://glaros.dtc.umn.edu/gkhome/projects/gp/products?q=views/metis">http://glaros.dtc.umn.edu/gkhome/projects/gp/products?q=views/metis</a>

George Karypis: METIS and ParMETIS. Encyclopedia of Parallel Computing 2011: 1117-1124

Scotch, a "software package and libraries for sequential and parallel graph partitioning, static mapping and clustering, sequential mesh and hypergraph partitioning, and sequential and parallel sparse matrix block ordering"; see <a href="http://www.labri.fr/perso/pelegrin/scotch/">http://www.labri.fr/perso/pelegrin/scotch/</a>

Cédric Chevalier, François Pellegrini: PT-Scotch. Parallel Computing 34(6-8): 318-331 (2008)





# Graph partitioning software for process topology embedding

Jostle, originally for partitioning and load balancing for unstructured meshes; no longer open source (2017); see <a href="http://staffweb.cms.gre.ac.uk/~wc06/jostle/">http://staffweb.cms.gre.ac.uk/~wc06/jostle/</a>

C. Walshaw and M. Cross. JOSTLE: Parallel Multilevel Graph-Partitioning Software - An Overview. In F. Magoules, editor, Mesh Partitioning Techniques and Domain Decomposition Techniques, pages 27-58. Civil-Comp Ltd., 2007. (Invited chapter)



# Lots of possibilities for projects/Master's Thesis

#### Topology mapping: Discussion

- For scalability, distributed specification of communication graph
- Requires distributed algorithm (heuristic) to solve/approximate hard optimization problem
- Can the mapping overhead be amortized?
- Does it make sense to look for an optimum? Is the MPI interface sufficiently rich to provide all relevant information? Does the application have this information?
- Is the application programmer prepared to use the complex interface?





#### Creating the communicators for MV example (Method 1)

```
int rcrank;
int rccoord[2];

MPI_Comm_rank(rccomm, &rcrank);
MPI_Cart_coords(rccomm, rcrank, 2, rccoord);

MPI_Comm_split(rccomm, rccoord[0], rccoord[1], &ccomm);
MPI_Comm_split(rccomm, rccoord[1], rccoord[0], &rcomm);
```

color: all processes calling with same color, will belong to same communicator

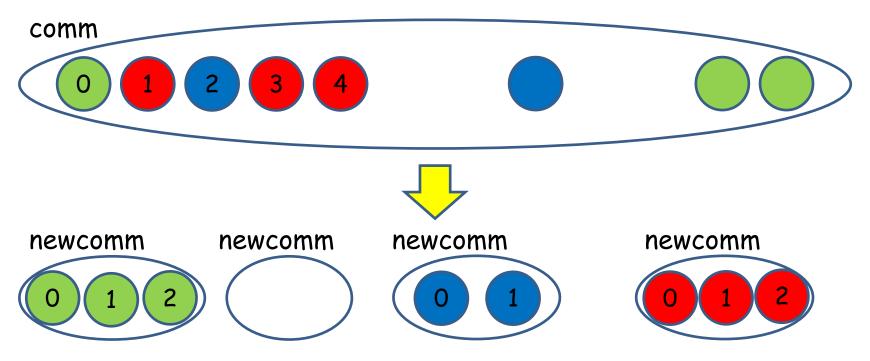


key: determines relative process ordering in new comminicator





MPI Comm split (comm, color, key, &newcomm);

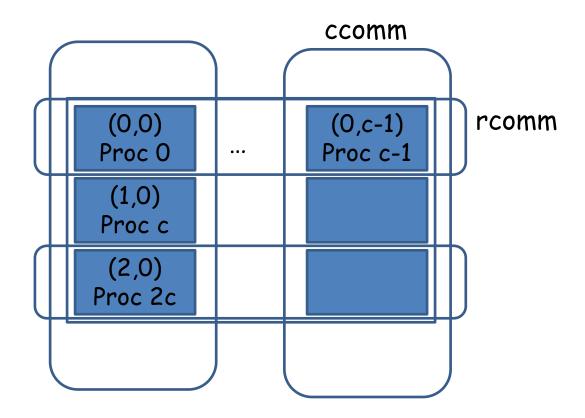


Processes in comm are sorted after key; determines relative order in newcomm





```
MPI_Comm_split (color=rccoord[0]);
MPI_Comm_split (color=rccoord[1]);
```







#### Creating the communicators for MV example (Method 2)

MPI\_Comm\_split() is an expensive collective operation. MPI process groups can be used for possibly more efficient communicator creation

```
MPI Comm group (rccomm, &rcgroup); // get group out
MPI Comm rank(rccomm, &rcrank);
MPI Cart coords (rccomm, rcrank, 2, rccoord);
for (i=0; i<rcsize; i++) {
  MPI Cart coords (rccomm, i, 2, coord);
  if (coord[0] == rccoord[0]) cranks[c++] = i;
                                                    Process
  if (coord[1] = rccoord[1]) rranks[r++] = i;
                                                    group
                                                    operations
MPI Group include (rcgroup, c, cranks, &cgroup);
MPI Group include (rcgroup, r, rranks, &rgroup);
MPI Comm create (rccomm, cgroup, &ccomm);
                                               Better?
MPI Comm create (rccomm, rgroup, &rcomm);
NS23
                     ©Jesper Larsson Träff
```



# MPI process groups (MPI 3.1, Chapter 6)

Process local MPI objects representing ordered sets of processes

MPI\_Comm\_group: Get group of processes associated with communicator

Processes in group ranked from 0 to size of group-1, a process can locally determine if it's a member of some group, and its rank in this group

#### Operations:

- Comparing groups (same processes, in same order)
- Group union, intersection, difference, ...
- Inclusion and exclusion of groups
- Range groups
- Translation of ranks between groups





#### Creating the communicators (Method 2')

#### MPI\_Comm\_split() is an expensive collective operation. MPI process groups can be used for possibly more efficient communicator creation

```
MPI Comm group (rccomm, &rcgroup); // get group out
     MPI Comm rank(rccomm, &rcrank);
     MPI Cart coords (rccomm, rcrank, 2, rccoord);
     for (i=0; i<rcsize; i++) {
       MPI Cart coords (rccomm, i, 2, coord);
       if (coord[0] == rccoord[0]) cranks[c++] = i;
       if (coord[1] = rccoord[1]) rranks[r++] = i;
Even better?
     MPI Group include (rcgroup, c, cranks, &cgroup);
     MPI Group include (rcgroup, r, rranks, &rgroup);
     MPI Comm create group (rccomm, cgroup, tag, &ccomm);
```



MPI Comm create group (rccomm, rgroup, tag, &rcomm);

# MPI 3.1 standard: Many variations, many subtleties: Read it

- MPI\_Comm\_create(comm, ...): Collective over all processes in comm
- MPI\_Comm\_create\_group(comm,group, ...): Collective over all processes in group, processes in group subset of processes in group of comm

MPI\_Comm\_create\_group: Smaller groups can work independently to create their communicator. Tag arguments for multi-threaded uses (many news in MPI 3.1)



Why should MPI\_Comm\_create be better than MPI\_Comm\_split?

Reasonable to expect

"not slower than"

MPI\_Comm\_create(...,group,...) ≤ MPI\_Comm\_split(...,color...)

all other things (communicator) being equal

MPI\_Comm\_create() straightforward by MPI\_Comm\_split():
 take color as first process in group, key as rank in group, for
 processes in group (MPI\_UNDEFINED otherwise). More
 specialized operation

Exercise: Implement

Reasonable to expect MPI\_Comm\_create() in O(log p)? Or O(log p+p)? Probably not! Recall: MPI has no performance model or guarantee oJesper Larsson Träff

MPI\_Comm\_create(...,group,...) ≤ MPI\_Comm\_split(...,color...)

Example of performance guideline that can be verified by benchmarking.

If not fulfilled, something wrong with MPI library

Jesper Larsson Träff, William D. Gropp, Rajeev Thakur: Self-Consistent MPI Performance Guidelines. IEEE Trans. Parallel Distrib. Syst. 21(5): 698-709 (2010)





All group operations can be implemented in O(s) operations, where s is the size of the largest group in the operation. Space consumption is O(U), where U is the maximum number of MPI processes in the system (size of MPI COMM WORLD)

#### Questions:

- Group (and communicator) operations permit construction of unrestricted rank->process mappings; this is costly in space.
   Possible to define a useful set of more restricted operations?
- Which group operations can be implemented in o(s) operations?

Jesper Larsson Träff: Compact and Efficient Implementation of the MPI Group Operations. EuroMPI 2010: 170-178





Problem still to be solved:

Finding out where data from some rank (in oldcomm) have to be sent to: Where is the rank in newcomm?

Problem: Find from rank, torank, such that process in (in oldcomm) can receive data from from rank, and send its own data to torank

Recall: Cannot send from rank in oldcomm to rank in newcomm; communication is always relative to one, same communicator





```
int torank, fromrank; // to be computed, in oldcomm
int oldrank, newrank;
MPI Group oldgroup, newgroup;
MPI Comm group (oldcomm, &oldgroup);
MPI Comm group (newcomm, &newgroup);
// where has rank been mapped to?
MPI Comm rank(oldcomm, &oldrank); // rank in old
MPI Group translate rank (newgroup, 1, &oldrank,
                          oldgroup, &torank);
// torank may be MPI UNDEFINED
// if newcomm smaller than oldcomm
MPI Comm rank(newcomm, &newrank);
MPI Group translate ranks (newgroup, 1, & newrank,
                           oldgroup, &fromrank);
```





#### Scalability of MPI communicator concept

- A communicator must support translation from MPI rank to physical core in system
- Storing translations explicitly as p-element lookup array (per core) is a non-scalable implementation. But still the most common

Example: 250.000 cores, 4-Byte integers  $\approx$  1MByte space per communicator (per core). Applications on systems with limited memory, and many communicators can be trouble

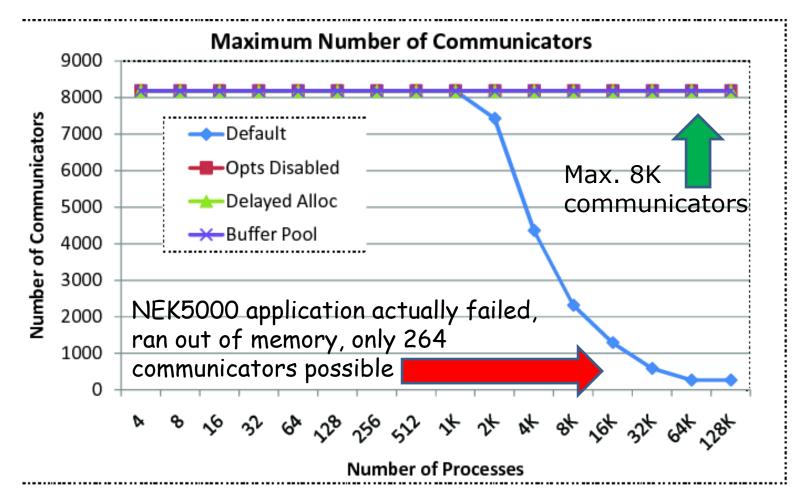
#### Note:

MPI specification puts no restrictions on the number and kind of communicators that can be created





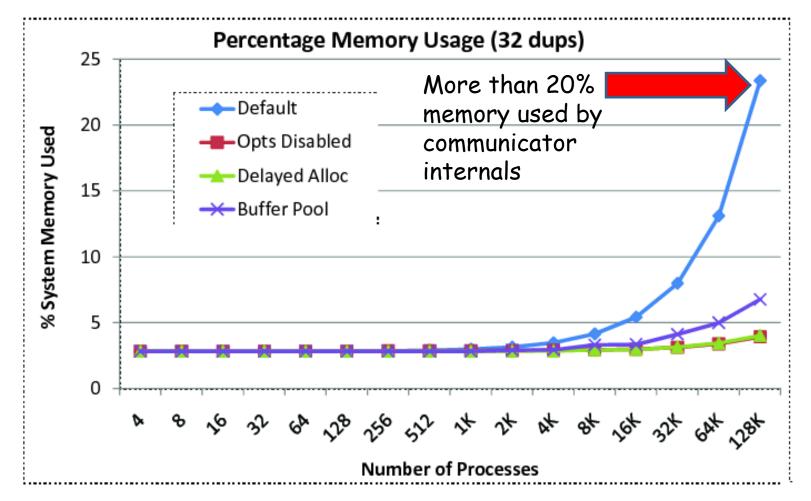
# Experiment (1) at Argonne National Lab BlueGene system, 2011







# Experiment (2) at Argonne National Lab BlueGene system, 2011







NEK5000: Computational fluid dynamics code, often used

(benchmark variant: NEKBONE), see

https://nek5000.mcs.anl.gov/





#### On scalability of MPI, especially communicators and interface

- P. Balaji, D. Buntinas, D. Goodell, W. Gropp, T. Hoefler, S. Kumar, E. L. Lusk, R. Thakur, Jesper Larsson Träff: MPI on millions of Cores. Parallel Processing Letters 21(1): 45-60 (2011)
- H. Kamal, S. M. Mirtaheri, A. Wagner: Scalability of communicators and groups in MPI. HPDC 2010: 264-275

Somewhat recent (now routine) result with many, many MPI processes (little detail): MPI has so far been able to scale

P. Barnes, C. Carothers, D. Jefferson, J. LaPre: Warp speed: executing Time Warp on 1,966,080 cores. ACM SIGSIM-PADS, 2013





#### Topological awareness

#### Orthogonal idea:

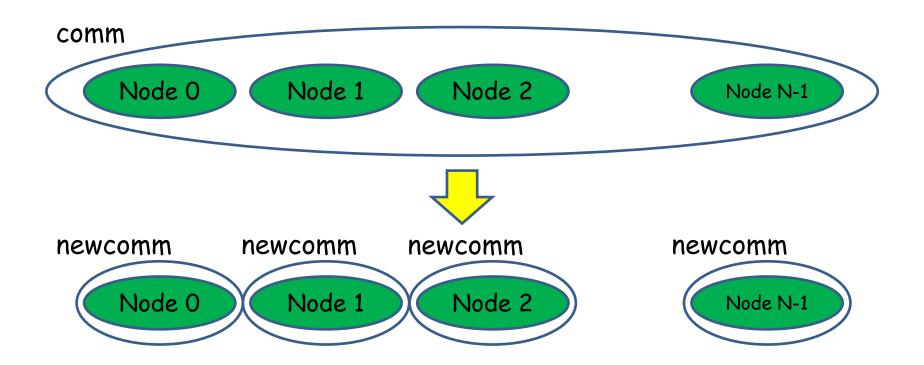
Instead of letting user specify communication requirements (communication topology), let system suggest good communicators subject to specified constraints

New in MPI 3.0, some support for better exploiting hierarchical systems/shared memory nodes

MPI\_COMM\_TYPE\_SHARED: Creates communicator for processes where a shared memory region can be created (processes on same shared-memory node)







More to come with MPI 4.0





#### New split types with MPI 4.0

MPI\_COMM\_TYPE\_HW\_UNGUIDED, MPI\_COMM\_TYPE\_HW\_GUIDED: Splitting according to other hardware characteristics (NUMA nodes), either per default, or as specified in MPI\_Info object

