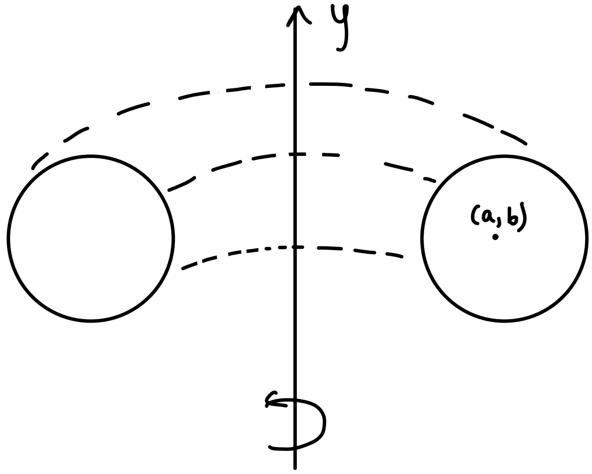
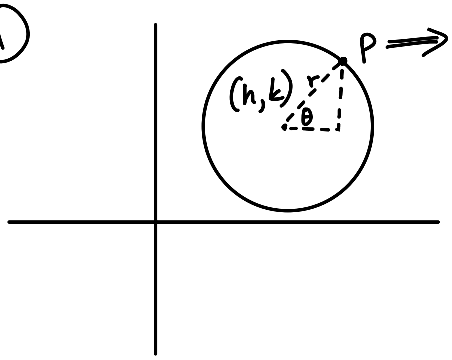
Formation of torus



$\Rightarrow (x-a)^2+(y-b)^2=r^2$

Getting equation of point on circle in parametric form:

(1)



$P \Rightarrow (h+r\cos\theta,\ k+r\sin\theta)$

$\hookrightarrow$ so $x = h + r\cos\theta$

$y = k + r\sin\theta$

(2) using trig identity and

$$(x-a)^2+(y-b)^2=r^2$$

$$\sin^2\theta+\cos^2\theta=1$$

$\times r^2 \qquad \times r^2$

$$r^2\sin^2\theta+r^2\cos^2\theta=r^2$$

$r^2\sin^2\theta=(y-b)^2$

$r\sin\theta=y-b$

$y=r\sin\theta+b$

$r^2\cos^2\theta=(x-a)^2$

$r\cos\theta=x-a$

$x=r\cos\theta+a$

Now the parametric equation for Circle 1 is:

$$x = r\cos\theta + a \qquad y = r\sin\theta$$

Rotation Matrices:

around x axis:
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

around y axis:
$$\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

around z axis:
$$\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so to rotate Circle 1 around y axis:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} r\cos\theta + a \\ r\sin\theta \\ 0 \end{bmatrix} = \begin{bmatrix} (r\cos\theta + a)\cos\phi \\ r\sin\theta \\ -(r\cos\theta + a)\sin\phi \end{bmatrix}$$

* where ϕ and θ are from 0 to 2π radians & plot point for every value of ϕ and θ.

Then to create rotating torus:

$$\underset{\text{(rotate } x)}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos x & -\sin x \\ 0 & \sin x & \cos x \end{bmatrix}} \times \underset{\text{(rotate } z)}{\begin{bmatrix} \cos z & -\sin z & 0 \\ \sin z & \cos z & 0 \\ 0 & 0 & 1 \end{bmatrix}} \times \underset{\text{(torus points)}}{\begin{bmatrix} (r\cos\theta + a)\cos\phi \\ r\sin\theta \\ -(r\cos\theta + a)\sin\phi \end{bmatrix}}$$

$$= \begin{bmatrix} (r\cos\theta + a)(\cos z \times \cos\phi + \sin x \times \sin z \times \sin\phi) - r\sin\theta \times \cos x \times \sin z \\ (r\cos\theta + a)(\cos z \times \cos\phi - \cos z \times \sin x \times \sin\phi) - r\sin\theta \times \cos x \times \cos z \\ (r\cos\theta + a)\cos x \times \sin\phi + r\sin\theta \times \sin x \end{bmatrix}$$